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CS-C2160 Theory of Computation (5 cr)
Course Exam 14 April 2021, 5–8:30 p.m.

Instructions

1. This online examination is carried out using the Quiz feature of MyCourses. The submission window opens at 5:00 p.m. and closes at 8:30 p.m. Late submissions are not accepted, but you may update an incomplete submission up to the closing time.

2. The examination has four problems. Write your solutions in clearly legible handwriting, using a pen with good contrast from the paper, and **start the answer to each problem on a new sheet of paper**. You can write down your answers in either Finnish, Swedish, or English.

3. **Number each answer sheet** at the top left corner of the sheet with the problem number as 1, 2, 3, 4. If you use several sheets per problem, then number those as 1.1, 1.2, 2.1, 2.2, 2.3 etc. (This is important to collate the sheets in correct order from the online submission server, if needed.) **Sign each answer sheet at the top.**

4. Scan/photograph the answer sheets for each problem, and upload them as an image file / files (jpg, png or pdf) to the MyCourses Quiz submissions server, as an answer to the corresponding Quiz question. To guarantee that you don't omit the uploads by mistake, you must upload at least one answer sheet per problem, even if empty.

5. Make sure that you have enough time at the end of the exam to complete the uploads, even if the server is busy or some other technical adversities arise. Remember that you can update your submissions until the server closes, so it is better to start uploading early rather than late. If you leave uploading to the end, reserve at least 30 minutes for this task.

6. To make sure that any possible issues arising from the arrangements can be resolved, please be present throughout the exam in the Zoom meeting `https://aalto.zoom.us/j/66313829636`. For any questions during the exam, first contact the meeting host privately via the Zoom chat.

7. Note that you must have completed your computerised "Astra" home assignments before taking the exam. Otherwise your exam will not be graded.

Materials, code of conduct

1. The exam is "open (course)book", i.e. all course material available via the course's MyCourses page can be used for reference, but **no other materials, notes or communications** are allowed. This includes searching for answers on the Internet or discussing the exam problems with other people. (Cf. the Aalto University Code of Academic Integrity at `https://into.aalto.fi/display/ensaannot/Aalto+University+Code+of+Academic+Integrity+and+Handling+Violations+Thereof`.) If you have any questions about the materials, please contact the invigilator via the Zoom chat.

Problems

1. Design **deterministic** finite automata for recognising the following languages:

   (a) $\{w \in \{0,1\}^* \mid w$ contains the substring 110$\}$,

   (b) $\{w \in \{0,1\}^* \mid w$ does **not** contain the substring 001$\}$,

   (c) $\{w \in \{0,1\}^* \mid w$ contains the substring 110 **or** contains the substring 001 (or both)$\}$,

   (d) $\{w \in \{0,1\}^* \mid w$ contains the substring 110 **but** does **not** contain the substring 001$\}$,

   (e) $\{w \in \{0,1\}^* \mid$ **if** w contains the substring 110 **then** it contains also the substring 001$\}$.

   15 points

2. For a string $w \in \{a,b\}^*$, let us denote by $n_a(w)$ the number of a's in w and by $n_b(w)$ the number of b's in w. Design context-free grammars for the following three languages:

   (a) $L_= = \{w \in \{a,b\}^* \mid n_a(w) = n_b(w)\}$,

   (b) $L_> = \{w \in \{a,b\}^* \mid n_a(w) > n_b(w)\}$,

   (c) $L_\neq = \{w \in \{a,b\}^* \mid n_a(w) \neq n_b(w)\}$.

   Give also parse trees in your grammars for the following strings: (a) $abba \in L_a$, (b) $baaba \in L_>$, (c) $bbaba \in L_\neq$. (*Hints:* In part (b), it might be helpful to first design a grammar for the language $L_\geq = \{w \in \{a,b\}^* \mid n_a(w) \geq n_b(w)\}$. In part (c), notice that for any two integers k and l, $k \neq l$ if and only if $k > l$ or $l > k$.) 15 points

3. For a string $w \in \{a,b,c\}^*$, let us denote by $n_a(w)$ the number of a's in w, by $n_b(w)$ the number of b's in w, and by $n_c(w)$ the number of c's in w.

   (a) Design a general (unrestricted) grammar for the language

   $$L_{abc} = \{w \in \{a,b\}^* \mid n_a(w) = n_b(w) = n_c(w)\}.$$

   Give a derivation in your grammar for the string $cabacb \in L_{abc}$.

   (b) Prove (precisely!) that the language L_{abc} is not context-free. 15 points

4. Which of the following claims are true and which are false? Provide a brief justification for each of your answers, based on results introduced at the course. (For example if the claim was: "The complement of any decidable language is semidecidable", your answer could be: "True. The complement of any decidable language is decidable (by switching the accepting and rejecting states in the recognising TM), and all decidable languages are by definition also semidecidable.")

   (a) The complement of any regular language is context-free.

   (b) The intersection of any two context-free languages is regular.

   (c) The union of any two context-free languages is context-free.

   (d) Nondeterministic Turing machines can recognise some undecidable languages.

   (e) The intersection of any two semidecidable languages is decidable. 15 points