Mathematics for Economists
ECON-C1000
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Problem Set 0: Getting Started

These exercises are designed to help you prepare for the course. They will be covered in the first review session, but you do not have to return them.

1. Vectors:

   (a) Familiarize yourself with the notion of a vector as an ordered list of real numbers so that a k -dimensional vector $\boldsymbol{x} = (x_1, x_2, ..., x_k)$, where each $x_i$for $i \in 1, ..., k$ is a real number.

      i. Draw vectors $\boldsymbol{x} = (3,5)$ and $\boldsymbol{y}(5,1)$ in the plane. Visualize vectors $t\boldsymbol{x} + (1-t)\boldsymbol{y}$ for $t \in [0,1]$. Visualize vectors $t\boldsymbol{x} + s\boldsymbol{y}$ for $t, s \geq 0$. Draw vector $\boldsymbol{z} = (2,6)$ and determine by visual inspection if there are $s, t \geq 0$ such that $\boldsymbol{z} = t\boldsymbol{x} + s\boldsymbol{y}$. Find $t, s \in \mathbb{R}$ such that $\boldsymbol{z} = t\boldsymbol{x} + s\boldsymbol{y}$.

      ii. Write your grades in your first year courses as a vector. Write the study credits for your first year courses as another vector. Write the total number of credits in your first year as an inner product of the latter vector and another vector. Write your weighted GPA as a ratio of two inner products.

      iii. In your later studies, you may use register data to discover empirical regularities. Each individual i is described by a vector of individual characteristics (age, years of schooling, profession, etc.). How would you express the idea that the income of i, denoted by y_i depends linearly on the vector of characteristics $\boldsymbol{x}^i$?

   (b) In high school, the length also called the norm of vector $\boldsymbol{x}$ denoted by $\|\boldsymbol{x}\|$ is calculated using Pythagorean theorem

   $$\|\boldsymbol{x}\| = \sqrt{x_1^2 + x_2^2} \text{ or } \|\boldsymbol{x}\| = \sqrt{x_1^2 + x_2^2 + x_3^2}.$$

   How would you define the length or norm of a k -dimensional vector $\boldsymbol{x} = (x_1, x_2, ..., x_k)$?

   (c) One way of defining a distance $d(\boldsymbol{x}, \boldsymbol{y})$ between two vectors $\boldsymbol{x}, \boldsymbol{y}$ is by setting $d(x, y) = \|\boldsymbol{x} - \boldsymbol{y}\|$. Another possibility is to set $d(\boldsymbol{x}, \boldsymbol{y}) = |x_1 - y_1| + |x_2 - y_2|$. Why is this called the Manhattan distance? Is this a sensible measure for distance? What do you require of a distance? What about $d(\boldsymbol{x}, \boldsymbol{y}) = (\sqrt{x_1 - y_1} + \sqrt{x_2 - y_2})^2$?

2. Matrices

   (a) Recall some basics about matrix algebra.

      i. Let $\boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$ ja $\boldsymbol{B} = \begin{bmatrix} 5 & 3 \\ 7 & 2 \end{bmatrix}$. Compute $\boldsymbol{C} = \boldsymbol{A} + \boldsymbol{B}$.

      ii. Compute $\boldsymbol{AB}$ and $\boldsymbol{BA}$. What do you observe about the order of multiplication?

      iii. Is it true that $(\boldsymbol{AB})\boldsymbol{C} = \boldsymbol{A}(\boldsymbol{BC})$?

   (b) A permutation matrix is a matrix of zeros and ones with a single nonzero element in each row and each column.

      i. How many 2×2 permutation matrices are there, how many 3×3, how many $n \times n$ permutation matrices?

      ii. If $\boldsymbol{A}$ is a permutation matrix and $\boldsymbol{B}$ an arbitrary matrix, what is $\boldsymbol{AB}$?

3. A system of linear equations can be written compactly using matrices and vectors: $\boldsymbol{Ax} = \boldsymbol{b}$, where $\boldsymbol{A}$ is a matrix, $\boldsymbol{x}$ is a vector of endogenous variables (i.e. the variables to be solved) and $\boldsymbol{b}$ is a vector of exogenous variables (i.e. variables that are not determined in the linear model but come from outside). Write the following in matrix notation:

$$\begin{aligned} 2x_1 - 3x_2 + 3x_3 &= 1 \\ 2x_1 - x_2 + 2x_3 &= 3 \\ 3x_1 - 2x_2 + x_3 &= 3 \end{aligned}$$

   Solve the system using elementary row operations.

4. Give an example of an economic model where prices are exogenous variables and of a model where prices are endogenous variables.

5. Find the derivative of the following functions:

   (a) $f(x) = x^4$

   (b) $f(x) = e^x$

   (c) $f(x) = (3x + 2)^3$

   (d) $f(x) = \frac{4}{x^2+1}$

   (e) $f(x) = x \ln x$

   (f) $f(x) = x^x$