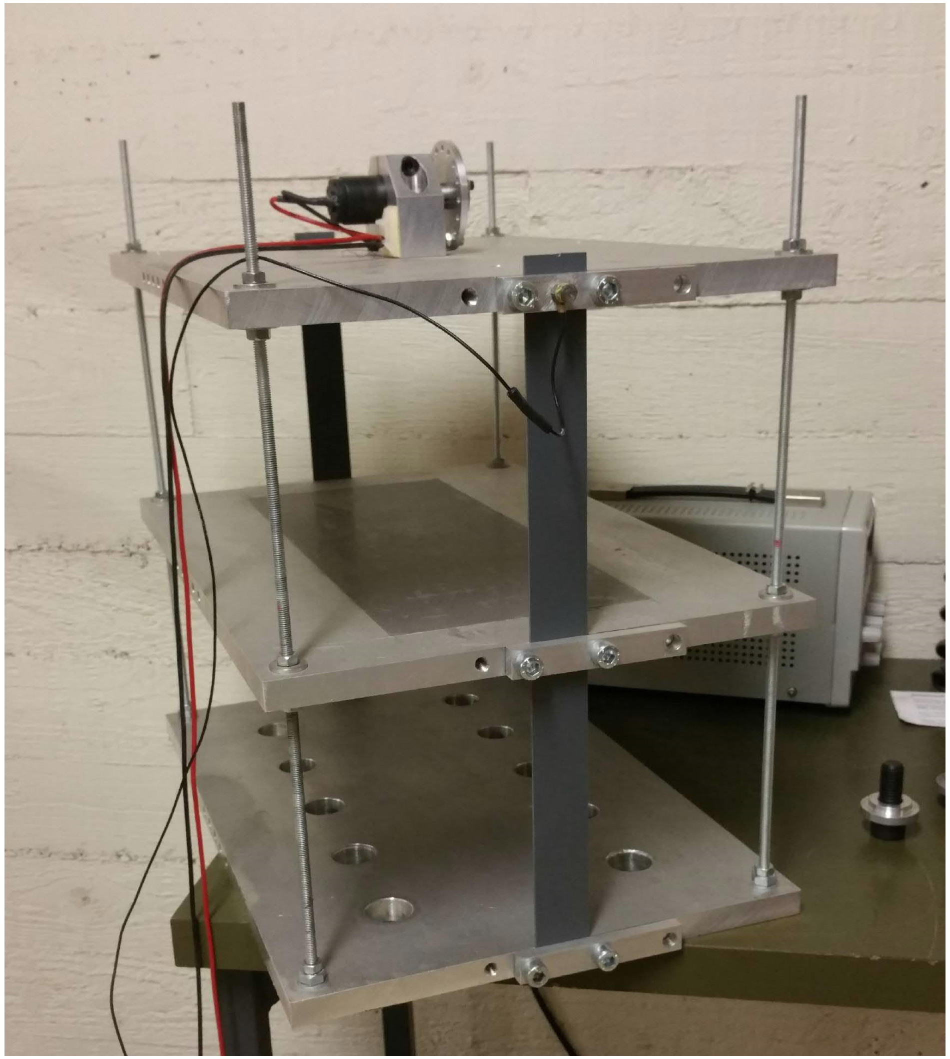
COE-C3005

Finite Element and Finite Difference Methods 2021

WEEK 16: INTRODUCTION

Fri 09:15-11:00 Calculation hours (JF & MÅ)

VIBRATION EXPERIMENT



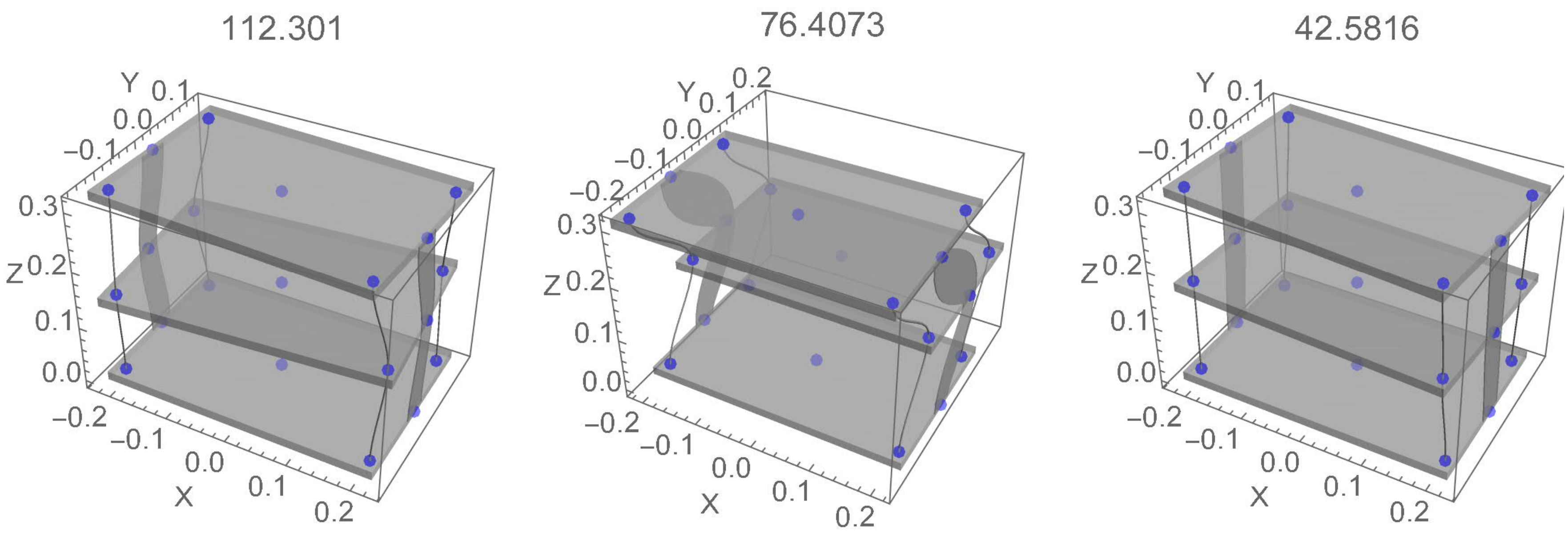
MODELLING ASSIGNMENT

In the modelling assignment, you will determine the two first frequencies of the free vibrations of the 3-story building using a model and

1. Particle Surrogate Method (PSM)
2. Finite Difference Method (FDM)
3. Finite Element Method (FDM)

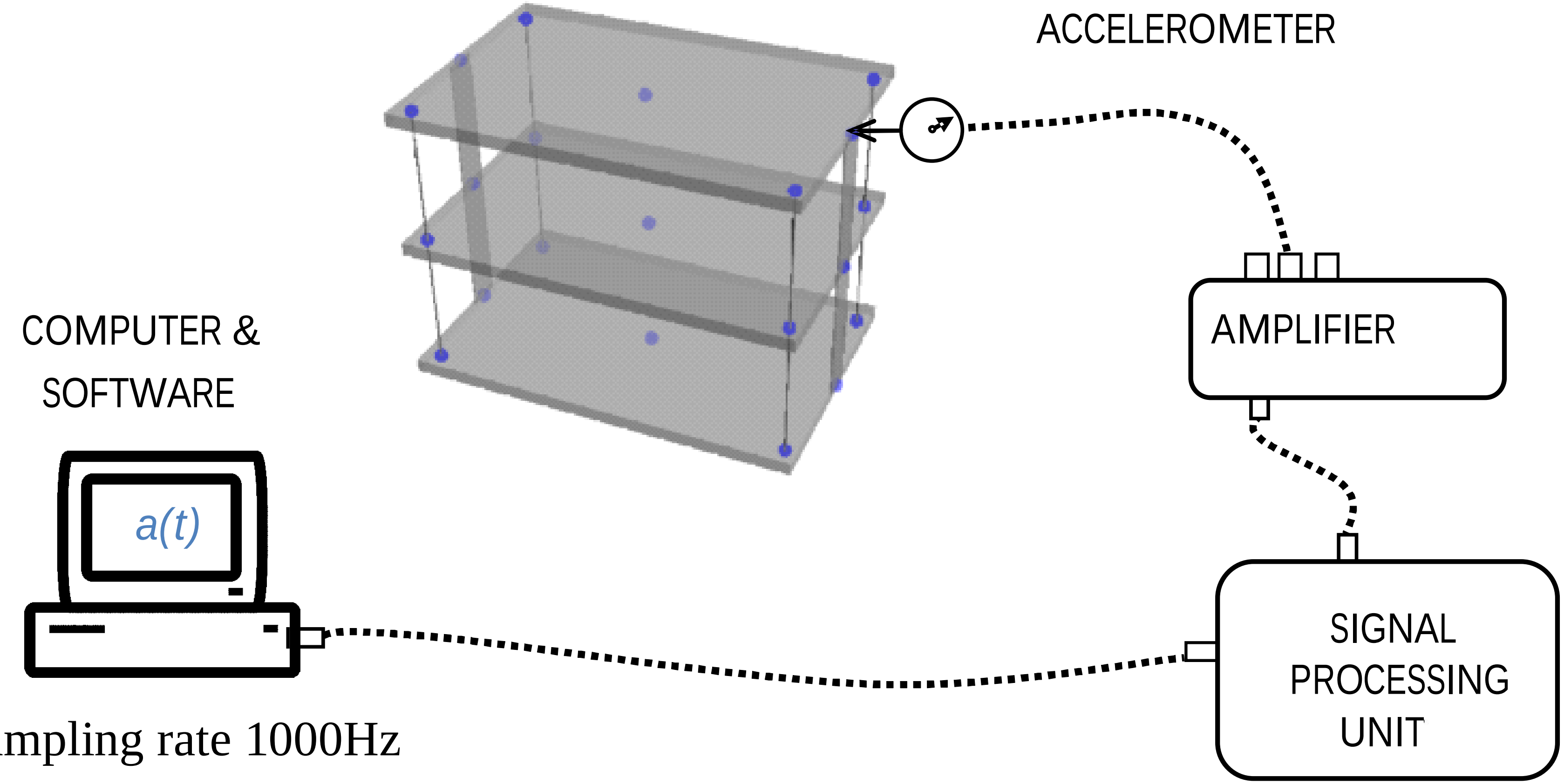
To report the outcome, supplement the assignment paper with experimental results and the outcome of calculations (table for results in light blue shading). Return your report (in PDF) on Sun 25.04.2021 23:55 at the latest (MyCourses).

FREE VIBRATIONS OF STRUCTURE



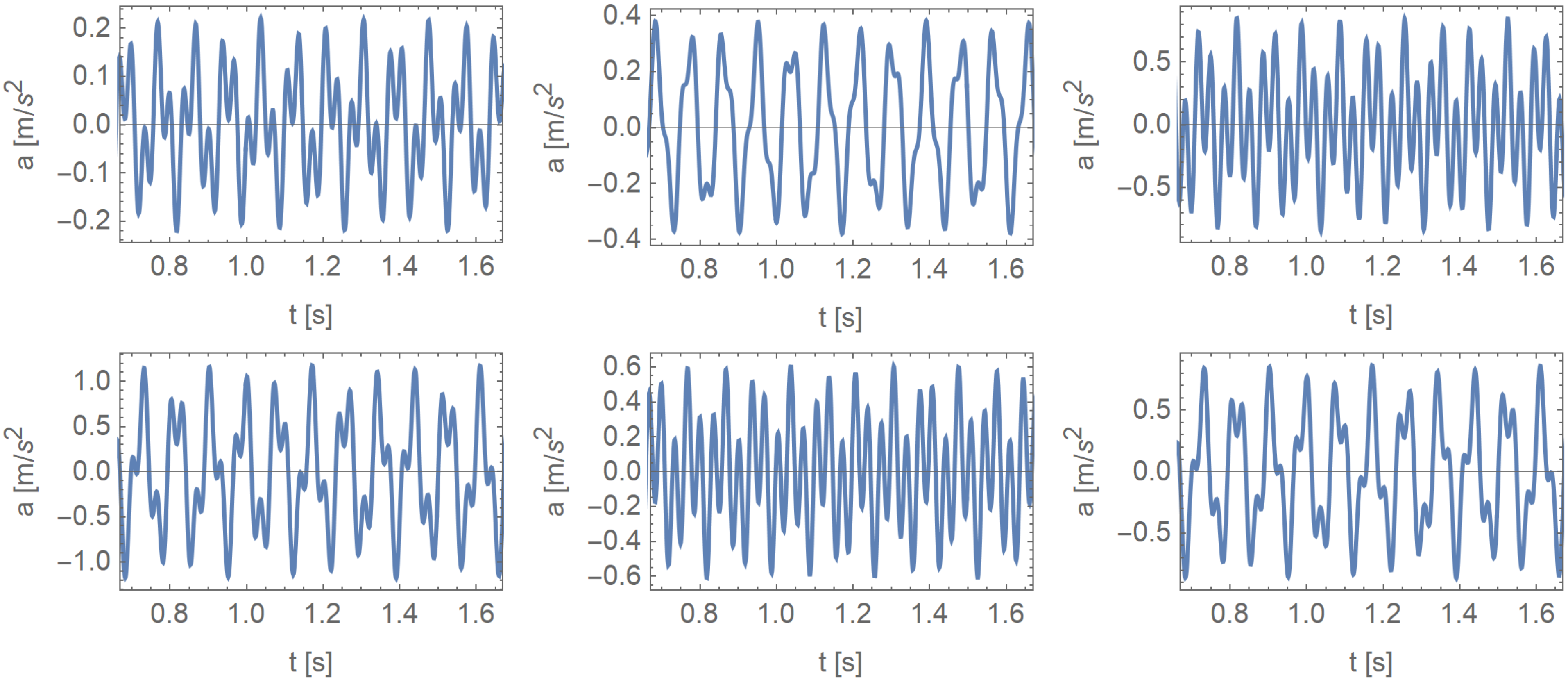
Assuming linearly elastic material, displacements $\mathbf{a}(t)$ at the grid points (in blue) can be represented as the sum $\mathbf{a}(t) = \sum \mathbf{A}_j[\alpha_j \sin(\omega_j t)/\omega_j + \beta_j \cos(\omega_j t)]$, where $\mathbf{A}_j$ are the modes (deformation patterns of the figure) and $\omega_j = 2\pi f_j$ the angular velocities associated with the modes.

VIBRATION EXPERIMENT



Sampling rate 1000Hz

ACCELERATION TIME-SERIES



Experimental data consists of the acceleration time-series measured by the accelerometer at one point. Experiment is repeated 6 times.

DISCRETE SINE SERIES

The discrete Fourier series (various forms exist) can be used to represent a list as the sum of lists of harmonic terms. For example, the sine-transformation pair for a list a_i $i \in \{1, 2, \ldots, n-1\}$ is given by

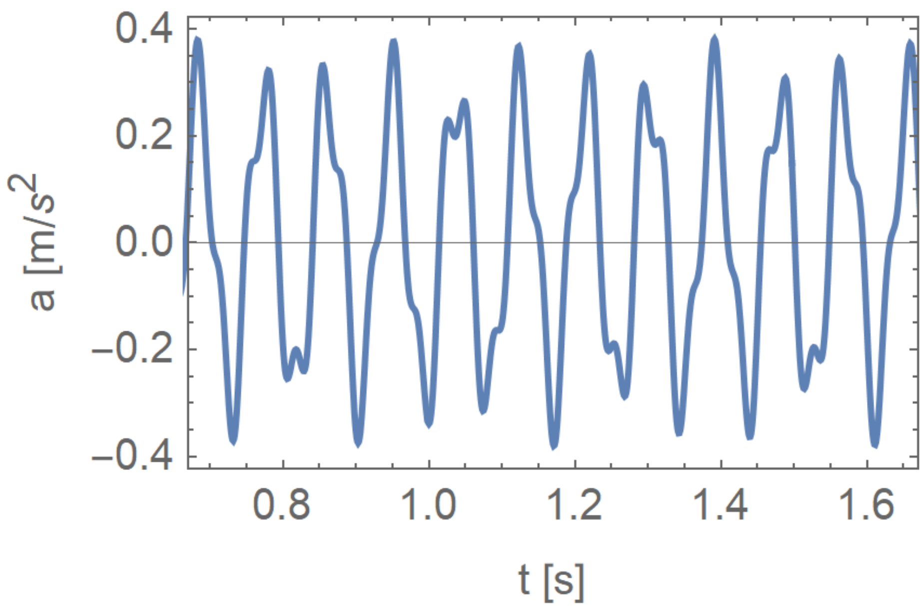
$$\alpha_j = \frac{2}{n} \sum_{i \in \{1,2,\ldots,n-1\}} \sin(j\pi \frac{i}{n}) a_i \quad j \in \{1, 2, \ldots, n-1\}$$

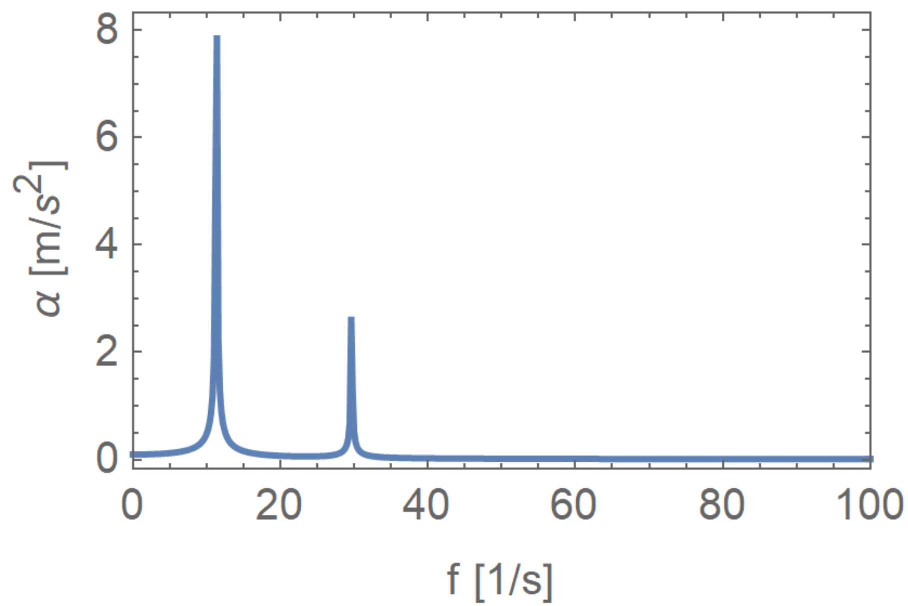
$$a_i = \sum_{j \in \{1,2,\ldots,n-1\}} \alpha_j \sin(j\pi \frac{i}{n}) \quad i \in \{1, 2, \ldots, n-1\}$$

The transformation pair is based on the orthogonality of the modes (Cronecker delta $\delta_{jl} = 1$ if $j = l$ and $\delta_{jl} = 0$ if $j \neq l$)

$$\sum_{j \in \{1,2,\ldots,n-1\}} \sin(j\pi \frac{i}{n}) \sin(l\pi \frac{i}{n}) = \delta_{jl} \frac{n}{2}.$$

PROCESSING OF DATA





Experimental data consists of the acceleration time-series measured by the accelerometer at one point. In processing of data, the time-acceleration representation is transformed to frequency-mode magnitude form by Discrete Fourier Transform (DFT).

MODELLING STEPS

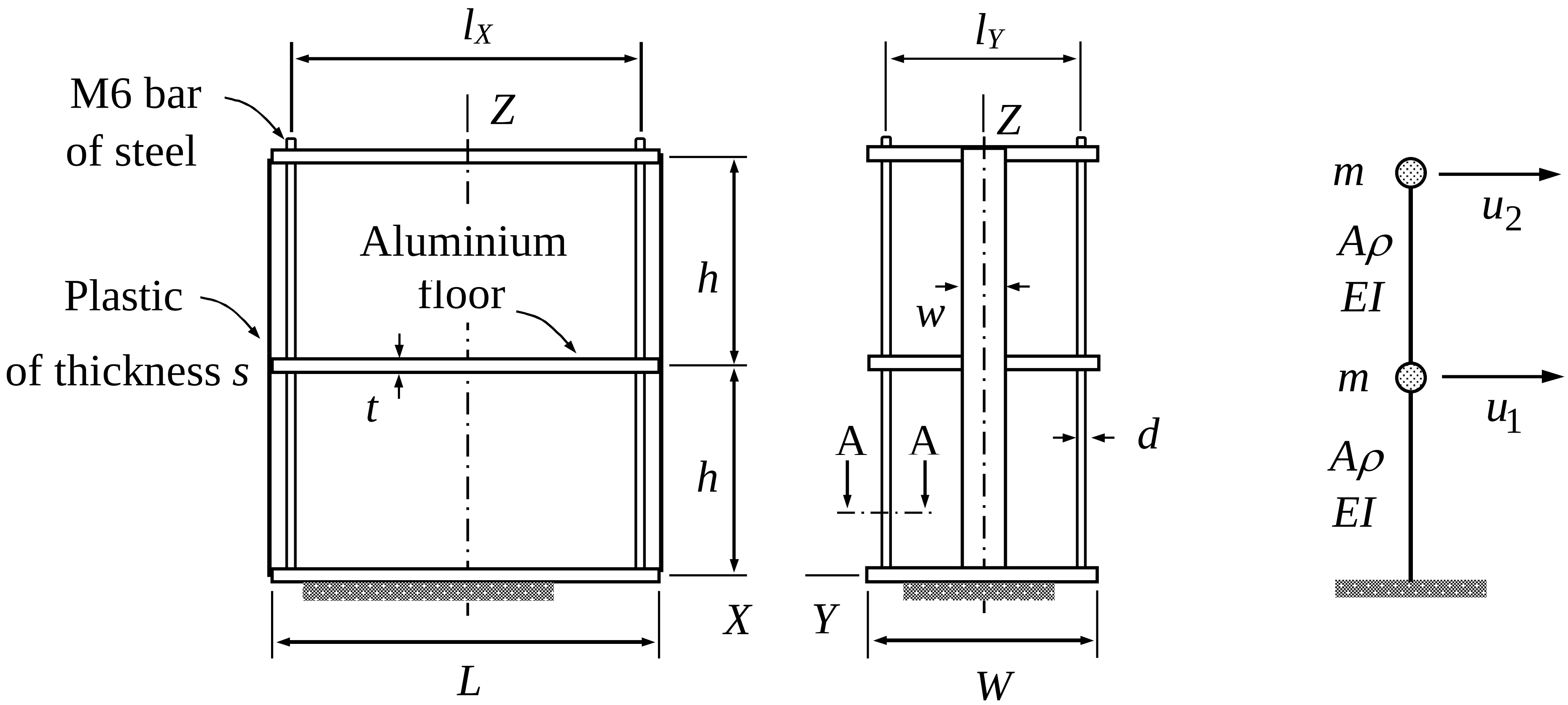
- **Crop:** Decide the boundary of a structure. Interaction with surroundings need to be described in terms of known forces, moments, displacements, and rotations. All uncertainties with this respect bring uncertainty to the model too.

- **Idealize and parameterize:** Simplify the geometry. Ignoring the details not likely to affect the outcome may simplify the analysis a lot. Assign symbols to geometric and material parameter of the idealized structure.

- **Model:** Write the equilibrium equations, constitutive equations, and boundary conditions of the structure.

- **Solve:** Use an analytical or approximate method and hand calculation or a code to find the solution.

STRUCTURE IDEALIZATION



The simplified model considers the columns as bending beams, floors as rigid bodies, omits the plastic strips, and assumes that the floors move horizontally in the $XZ-$plane. The horizontal displacements of the floors are denoted by $u_1(t)$ and $u_2(t)$.

PARAMETERIZATION

Parameter	**symbol**	**value**
Column thickness	d	0.0048 m
Room height	h	0.156 m
Column distance (x)	l_x	0.4 m
Column distance (y)	l_y	0.243 m
Floor length	L	0.44 m
Floor width	W	0.295 m
Floor thickness	t	0.015 m
Strip width	w	0.04 m
Strip thickness	s	0.002 m

APPROXIMATE METHODS

The simplest approximate equations of motion by Particle Surrogate Method, Finite Difference Method, and Finite Element Method, contain only the horizontal displacements of the first and second floors:

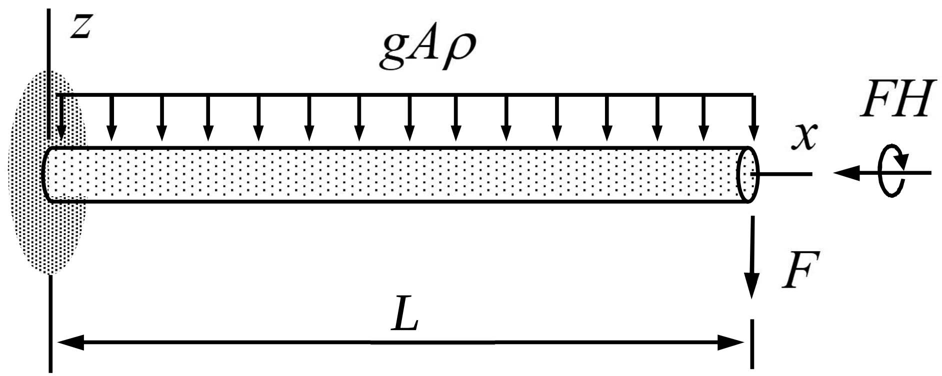
PSM: $$(m\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+4\times\frac{1}{2}\rho_s Ah\begin{bmatrix}2 & 0\\0 & 1\end{bmatrix})\frac{d^2}{dt^2}\begin{Bmatrix}u_1\\u_2\end{Bmatrix}+4\times12\frac{E_s I}{h^3}\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\u_2\end{Bmatrix}=0$$

FDM: $$(\frac{m}{h}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+4\times\rho_s A\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix})\frac{d^2}{dt^2}\begin{Bmatrix}u_1\\u_2\end{Bmatrix}+4\times12\frac{EI}{h^4}\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\u_2\end{Bmatrix}=0$$

FEM: $$(m\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+4\times\frac{\rho_s Ah}{6}\begin{bmatrix}4 & 1\\1 & 2\end{bmatrix})\frac{d^2}{dt^2}\begin{Bmatrix}u_1\\u_2\end{Bmatrix}+4\times12\frac{EI}{h^3}\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\u_2\end{Bmatrix}=0$$

Mode analysis for the frequencies assumes solution of the form $\mathbf{a}=\mathbf{A}\exp(\mathrm{i}\omega t)$ where $\omega=2\pi f$, $\mathrm{i}^2=-1$, $\mathbf{a}(t)=\{u_1 \;\; u_2\}^{\mathrm{T}}$, and $\mathbf{A}=\{A_1 \;\; A_2\}^{\mathrm{T}}$ (some constants).

BEAM THEORY



Domain: $-EI_{yy}\dfrac{d^4w}{dx^4}+b_z=0,\ \ GJ\dfrac{d^2\phi}{dx^2}=0\quad \text{in}\ \ (0,L)$

Free end: $-EI_{yy}\dfrac{d^2w}{dx^2}=\underline{M}_y,\ \ -EI_{yy}\dfrac{d^3w}{dx^3}=\underline{F}_z\ ,\ \ GJ\dfrac{d\phi}{dx}=\underline{M}_x\quad \text{at}\ \ x=L$

Clamped end: $w=0\ ,\ \ \theta=-\dfrac{dw}{dx}=0,\ \phi=0\quad \text{at}\ \ x=0$

MOMENTS OF AREA

Zero moment: $A = \int dA$

First moments: $S_z = \int ydA$ and $S_y = \int zdA$

Second moments: $I_{zz} = \int y^2 dA$, $I_{yy} = \int z^2 dA$, and $I_{zy} = I_{yz} = \int yzdA$

Polar moment: $J_B = I_{rr} = \int y^2 + z^2 dA = I_{zz} + I_{yy}$

The polar moment according to the standard model is usually (way) too large for profiles that do not actually remain planar in deformation.

FIRST TWO EIGENFREQUENCIES AND MODES

method	f_1 [Hz]	A_1 [–]	A_2 [–]	f_2 [Hz]	A_1 [–]	A_2 [–]
EXP		-	-		-	-
PSM						
FDM						
FEM						