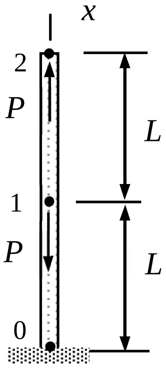
Name____________________________ Student number___________________

Home assignment 1



The bar shown is loaded by point forces of equal magnitudes P but opposite directions acting on points 1 and 2. Use the particle surrogate method (PSM) on the regular grid shown to write the equilibrium equations of points 1 and 2. After that, solve the equations for the axial displacements u_1 and u_2. Cross-sectional area A and Young's modulus E of the material are constants.

Solution

The equilibrium equations of the two free particles and one fixed for the bar model according to the particle surrogate method are given by (formulae collection)

$$u_0 = 0, \quad \frac{EA}{h}(u_0 - 2u_1 + u_2) - P = 0, \quad \frac{EA}{h}(u_1 - u_2) + P = 0$$

where $h = L$. Notice that the given point force needs to be taken into account in the sum of forces on the left hand side of equation of motion for particle 1 (in the formulae collection, only the effect of gravity is considered). The matrix notation uses only the equations of the free particles and the boundary condition given by particle 0 to eliminate u_0 from the equilibrium equations $-\mathbf{Ka} + \mathbf{F} = \mathbf{0}$

$$-\frac{EA}{L}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} + P\begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = 0 \quad \Leftrightarrow$$

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{PL}{EA}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}^{-1}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = \frac{PL}{EA}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = \frac{PL}{EA}\begin{Bmatrix} 0 \\ 1 \end{Bmatrix}. \quad \leftarrow$$