LECTURE ASSIGNMENT 1

Determine the eigenvalues $\lambda_1,\ \lambda_2$ and the corresponding eigenvectors $\mathbf{a}_1,\ \mathbf{a}_2$ of the 2×2 matrix $\mathbf{A}$. Consider the possible $(\lambda, \mathbf{a})$ pairs giving solutions to linear equation system $(\mathbf{A} - \lambda\mathbf{I})\mathbf{a} = \mathbf{0}$ with

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix},\ \ \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ and } \mathbf{a} = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}.$$

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As the matrix needs to be singular for a non-zero solution to $\mathbf{a}$, the possible values of λ follow from the characteristic equation $\det(\mathbf{A}-\lambda\mathbf{I})=0$

$$\det\begin{bmatrix}1-\lambda & 0 \\ -3 & 2-\lambda\end{bmatrix} = (1-\lambda)(2-\lambda)=0 \quad \Rightarrow \quad \lambda_1 = 1 \quad \text{or} \quad \lambda_2 = 2.$$

Eigenvector $\mathbf{a}$ (non-zero) corresponding to a possible value of λ follows from $(\mathbf{A}-\lambda\mathbf{I})\mathbf{a}=0$ when the value of λ is substituted there:

$$\lambda_1 = 1 : \quad \begin{bmatrix}1-1 & 0 \\ -3 & 2-1\end{bmatrix}\begin{Bmatrix}a_1 \\ a_2\end{Bmatrix} = 0 \quad \Rightarrow \quad \mathbf{a}_1 = \begin{Bmatrix}a_1 \\ a_2\end{Bmatrix} = \begin{Bmatrix}1 \\ 3\end{Bmatrix}$$

$$\lambda_2 = 2 : \quad \begin{bmatrix}1-2 & 0 \\ -3 & 2-2\end{bmatrix}\begin{Bmatrix}a_1 \\ a_2\end{Bmatrix} = 0 \quad \Rightarrow \quad \mathbf{a}_2 = \begin{Bmatrix}a_1 \\ a_2\end{Bmatrix} = \begin{Bmatrix}0 \\ 1\end{Bmatrix}$$

Hence, the eigenvalue-eigenvector pairs of $\mathbf{A}$ are given by

$$(\lambda,\mathbf{a})_1 = (1,\begin{Bmatrix}1 \\ 3\end{Bmatrix}) \quad \text{and} \quad (\lambda,\mathbf{a})_2 = (2,\begin{Bmatrix}0 \\ 1\end{Bmatrix}).$$ ←