LECTURE ASSIGNMENT 2

Find the displacement $u(x)$ of a bar of length L using the boundary value problem

$$EA\frac{d^2u}{dx^2} + \rho Ag = 0 \quad x \in]0, L[\,, \quad u(0) = u(L) = 0$$

given by the continuum model. Assume that the cross-sectional area A, Young's modulus E of the material, density ρ of the material, and acceleration by gravity g are constants.

Name_____ Student number_____

First, repeated integrations with the differential equation are used to find the generic solution. Let the integration constants be a and b:

$$\frac{d^2u}{dx^2} = -\frac{\rho Ag}{EA} \quad \Rightarrow \quad \frac{du}{dx} = -\frac{\rho Ag}{EA}x + a \quad \Rightarrow \quad u(x) = -\frac{\rho Ag}{EA}\frac{1}{2}x^2 + ax + b\,.$$

Second, boundary conditions are used to find the values of the integration constants a and b:

$$u(0) = b = 0 \quad \text{and} \quad u(L) = -\frac{\rho Ag}{EA}\frac{1}{2}L^2 + aL + b = 0 \quad \Rightarrow \quad b = 0 \quad \text{and} \quad a = \frac{\rho Ag}{EA}\frac{1}{2}L$$

Finally, the values of the integration constants are substituted into the generic solution to get the solution:

$$u(x) = \frac{\rho Ag}{EA}\frac{1}{2}x(L - x)\,.$$