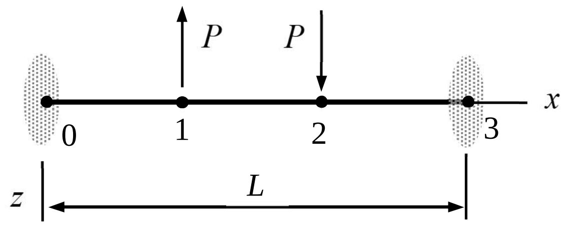
Name_________________________ Student number_________________

# Home assignment 1

A string of length $L$ and tightening $S$ is loaded by point forces of magnitudes $P$ as shown. If the ends are fixed and the initial geometry without loading is straight, find the transverse displacements at the grid points using the finite difference method (FDM) on a regular grid $i \in \{0,1,2,3\}$.




**Solution**

The boundary value problem is given by equilibrium equations for the regular interior points, jump conditions at the center point (non-regular point due to the point force), and boundary conditions for the end points

$$S\frac{d^2w}{dx^2} = 0 \quad x \in ]0,\frac{1}{3}L[ \quad \text{or} \quad x \in ]\frac{1}{3}L,\frac{2}{3}L[ \quad \text{or} \quad x \in ]\frac{2}{3}L,L[\,,$$

$$S\left[\!\left[\frac{dw}{dx}\right]\!\right] + P = 0\,,\ [\![w]\!] = 0 \quad x \in \{\frac{1}{3}L,\frac{2}{3}L\}\,,\ \text{and}\ w(x) = 0 \quad x \in \{0,L\}.$$

As the grid points inside the domain are at the locations of the point forces, the equations by the Finite Difference Method consist of displacement boundary conditions and jump conditions. Let us use the first order accurate backward and forward two-point difference approximations to the left and right derivatives, to get ($\Delta x = L/3$)

$$w_0 = 0\,,\ S(\frac{w_2 - w_1}{\Delta x} - \frac{w_1 - w_0}{\Delta x}) - P = 0\,,\ S(\frac{w_3 - w_2}{\Delta x} - \frac{w_2 - w_1}{\Delta x}) + P = 0\,,\ \text{and}\ w_3 = 0\,.$$

Using only the jump conditions with the known values of the boundary displacements

$$-\frac{S}{\Delta x}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} + P\begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = 0 \quad \Leftrightarrow \quad \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} = \frac{P\Delta x}{S}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = \frac{1}{9}\frac{PL}{S}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}.$$