LECTURE ASSIGNMENT 1

Polynomial interpolant $p(x)$ to dataset $\{\ldots,(x_{i-1}, f_{i-1}),(x_i, f_i),\ldots\}$ is the simplest possible continuous polynomial giving the prescribed values at the grid points, i.e., $p(x_i) = f_i$ for all indices. Use the Lagrange interpolation polynomial on a regular grid of spacing Δx to find the three point backward, central, and forward difference formulas for the first and second derivatives at point i.

Name______________________________ Student number______________

Backward difference approximations use dataset $\{(-2\Delta x, f_{i-2}),(-\Delta x, f_{i-1}),(0, f_i)\}$. Central difference approximations use dataset $\{(-\Delta x, f_{i-1}),(0, f_i),(\Delta x, f_{i+1})\}$. Forward difference approximations use dataset $\{(0, f_i),(\Delta x, f_{i+1}),(2\Delta x, f_{i+2})\}$. Interpolants to the datasets follow straightforwardly by using the Lagrange interpolation polynomial:

$$p_{\mathrm{b}}(x) = f_i + \frac{x}{2\Delta x}(f_{i-2} - 4f_{i-1} + 3f_i) + \frac{x^2}{2\Delta x^2}(f_{i-2} - 2f_{i-1} + f_i),$$

$$p_{\mathrm{c}}(x) = f_i + \frac{x}{2\Delta x}(-f_{i-1} + f_{i+1}) + \frac{x^2}{2\Delta x^2}(f_{i-1} - 2f_i + f_{i+1}),$$

$$p_{\mathrm{f}}(x) = f_i - \frac{x}{2\Delta x}(3f_i - 4f_{i+1} + f_{i+2}) + \frac{x^2}{2\Delta x^2}(f_i - 2f_{i+1} + f_{i+2}).$$

Thereafter, using the definitions $f_i' = p_{\mathrm{b}}'(0)$, $f_i'' = p_{\mathrm{b}}''(0)$, $f_i' = p_{\mathrm{c}}'(0)$, $f_i'' = p_{\mathrm{c}}''(0)$, $f_i'' = p_{\mathrm{f}}''(0)$, and $f_i'' = p_{\mathrm{f}}''(0)$ (origin is placed at point i):

	Backward	Central	Forward
f_i'	$\frac{f_{i-2} - 4f_{i-1} + 3f_i}{2\Delta x}$	$\frac{-f_{i-1} + f_{i+1}}{2\Delta x}$	$-\frac{3f_i - 4f_{i+1} + f_{i+2}}{2\Delta x}$
f_i''	$\frac{f_{i-2} - 2f_{i-1} + f_i}{\Delta x^2}$	$\frac{f_{i-1} - 2f_i + f_{i+1}}{\Delta x^2}$	$\frac{f_i - 2f_{i+1} + f_{i+2}}{\Delta x^2}$