LECTURE ASSIGNMENT 2

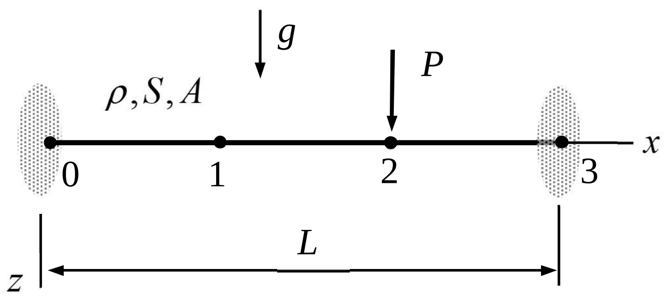
The continuum model for the string shown is given by equations

$$S\frac{d^2w}{dx^2}+\rho Ag=0 \quad x\in]0,\frac{2}{3}L] \text{ or } x\in]\frac{2}{3}L,L]$$

$$\left[\!\left[S\frac{dw}{dx} \right]\!\right]+P=0 \quad x=\frac{2}{3}L\,, \quad w=0 \quad x=0\,, \text{ and } w=0 \quad x=L\,.$$

Write the equations according to the Finite Difference Method using a regular grid $i\in\{0,1,2,3\}$, if the backward and forward difference approximations to the first derivative and the central difference approximation to the second derivative are given by

$$w_i'=\frac{1}{\Delta x}(w_i-w_{i-1})\,, \quad w_i'=\frac{1}{\Delta x}(w_{i+1}-w_i)\,, \text{ and } w_i''=\frac{1}{\Delta x^2}(w_{i-1}-2w_i+w_{i+1})\,.$$



Name______________________________ Student number______________

Finite difference Method uses the continuum model and difference approximations to derivatives at the grid points to discretize with respect to the spatial coordinate. Proper outcome requires a correct representation of the continuum model (obviously). Let us write the equations by considering the points one-by-one:

At point $i = 0$, one uses the boundary condition

$$w = 0 : \quad w_0 = 0 \quad \leftarrow$$

At the regular point $i = 1$, one uses the differential equation

$$S\frac{d^2w}{dx^2} + \rho Ag = 0 : \quad 9\frac{S}{L^2}(w_0 - 2w_1 + w_2) + \rho Ag = 0 \quad \leftarrow$$

At the non-regular point $i = 2$ of the point force, one uses the jump condition

$$\left[\!\left[S\frac{dw}{dx} \right]\!\right] + P = 0 : \quad 3\frac{S}{L}(w_3 - 2w_2 + w_1) + P = 0 \quad \leftarrow$$

At point $i = 3$, one uses the boundary condition

$$w = 0 : \quad w_3 = 0 \quad \leftarrow$$