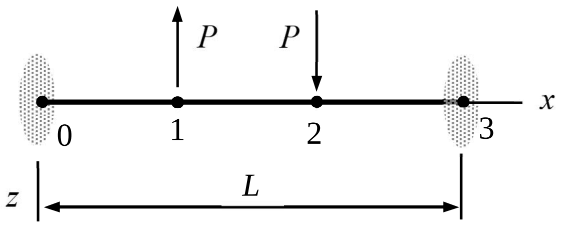
Name_________________________ Student number_________________

Home assignment 1

A string of length L and tightening S is loaded by point forces of magnitudes P as shown. If the ends are fixed and the initial geometry without loading is straight, find the transverse displacements at the grid points $i \in \{0,1,2,3\}$ using the Finite Element Method



Solution

The generic equation set for the string and bar models is given the Finite Element Method on a regular grid is

$$\frac{k}{\Delta x}(a_{i-1} - 2a_i + a_{i+1}) + F_i + f'\Delta x = m'\frac{\Delta x}{6}(\ddot{a}_{i-1} + 4\ddot{a}_i + \ddot{a}_{i+1}) \qquad i \in \{1,2,\ldots,n-1\}$$

$$\frac{k}{\Delta x}(w_1 - w_0) + F_0 + \frac{\Delta x}{2} f' - m'\frac{\Delta x}{6}(2\ddot{a}_0 + \ddot{a}_1) = 0 \quad \text{or} \quad a_0 = \underline{a}_0,$$

$$\frac{k}{\Delta x}(a_{n-1} - a_n) + F_n + \frac{\Delta x}{2} f' - m'\frac{\Delta x}{6}(2\ddot{a}_n + \ddot{a}_{n-1}) = 0 \quad \text{or} \quad a_n = \underline{a}_n,$$

$$a_i - g_i = 0 \quad \text{and} \quad \dot{a}_i - h_i = 0.$$

In the stationary case all time derivatives vanish and initial conditions are not used. With $a_i = w_i$, $k = S$, $m' = \rho A$, and $\Delta x = L/3$, the equations for the grid points $i \in \{0,1,2,3\}$ simplify to

$$w_0 = 0, \; 3\frac{S}{L}(w_0 - 2w_1 + w_1) - P = 0, \; 3\frac{S}{L}(w_1 - 2w_2 + w_3) + P = 0, \text{ and } w_3 = 0.$$

Using only the equations for points $i \in \{1,2\}$ simplified with the known values of the boundary displacements at points $i \in \{0,3\}$

$$-3\frac{S}{L}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} + P\begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = 0 \quad \Leftrightarrow \quad \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} = \frac{PL}{3S}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = \frac{1}{9}\frac{PL}{S}\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}. \quad \leftarrow$$