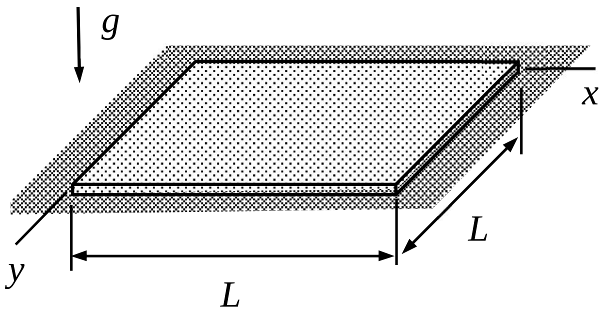
LECTURE ASSIGNMENT 1

A rectangular membrane of side length L, density ρ, thickness t, and tightening S' (force per unit length) is loaded by its own weight as shown. If the edges are fixed, find the transverse displacements at the grid points $(i, j) \in \{0,1,2,3\} \times \{0,1,2,3\}$ of a regular grid using the Finite Difference Method. Use symmetry to reduce the number of non-zero independent displacements to one.



Name______________________________ Student number______________

In a stationary problem, the discrete equations given by the Finite Difference Method on regular grid of spacing h are

$$\frac{S'}{h^2}[w_{(i-1,j)} + w_{(i,j-1)} - 4w_{(i,j)} + w_{(i+1,j)} + w_{(i,j+1)}] + f_i' = 0 \quad (i,j) \in I ,$$

$$w_{(i,j)} = 0 \quad (i,j) \in \partial I .$$

In the present problem, the set of interior points is given by

$$I = \{(1,1),(1,2),(2,1),(2,2)\}$$

the remaining of $(i,j) \in \{0,1,2,3\} \times \{0,1,2,3\}$ being boundary points ∂I of vanishing displacements. Due to symmetry, displacements at the interior points should be equal. Denoting the value by w_1, all equations for the interior point I boil down to

$$9\frac{S'}{L^2}(-4w_1 + w_1 + w_1) + \rho g t = 0$$

giving as the displacement at the interior points

$$w_1 = \frac{1}{18}\frac{\rho g t L^2}{S'} .$$