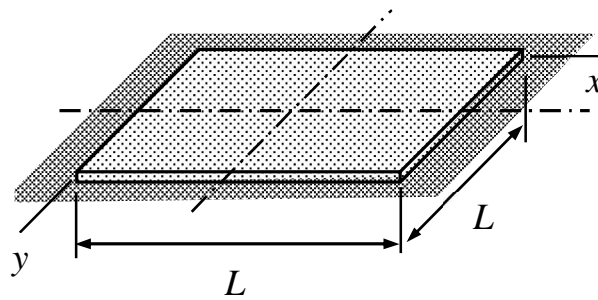


LECTURE ASSIGNMENT 2

Consider vibration of a rectangular membrane of side length L , density ρ , thickness t , and tightening S' (force per unit length). If the edges are fixed, find the angular velocity of the free vibrations using the Finite Difference Method on a regular grid of points $(i, j) \in \{0, 1, 2, 3\} \times \{0, 1, 2, 3\}$. Consider the mode, which is reflection symmetric with respect to the lines through the center point (figure).



Name _____ Student number _____

In a time-dependent membrane problem, the equations given by the Finite Difference Method on regular grid of spacing h are

$$\frac{S'}{h^2} [w_{(i-1,j)} + w_{(i,j-1)} - 4w_{(i,j)} + w_{(i+1,j)} + w_{(i,j+1)}] + f'_i = \rho t \ddot{w}_{(i,j)} \quad (i,j) \in I,$$

$$w_{(i,j)} = 0 \quad (i,j) \in \partial I.$$

Initial conditions are not needed in modal analysis of the problem. In the present problem, the set of interior points is given by

$$I = \{(1,1), (1,2), (2,1), (2,2)\}$$

the remaining of $(i,j) \in \{0,1,2,3\} \times \{0,1,2,3\}$ being boundary points ∂I of vanishing displacements. Due to the reflection symmetry, displacements at the interior points are equal, w_1 say. Consequently, all equations for the interior points $(i,j) \in I$ boil down to

$$\ddot{w}_1 + \omega^2 w_1 = 0 \quad \text{where} \quad \omega = \frac{3}{L} \sqrt{2 \frac{S'}{\rho t}}.$$

Therefore, the frequency of the assumed mode shape

$$f = \frac{3}{2\pi L} \sqrt{2 \frac{S'}{\rho t}}. \quad \leftarrow$$