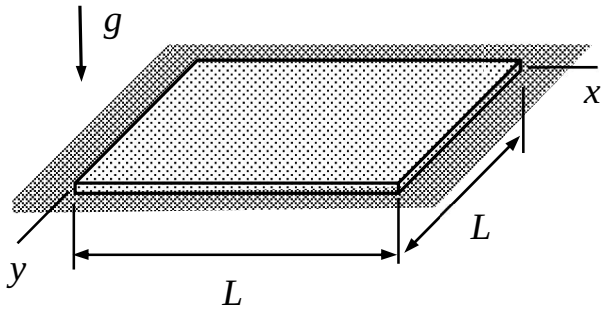
LECTURE ASSIGNMENT 1

A rectangular membrane of side length L, density ρ, thickness t, and tightening S' (force per unit length) is loaded by its own weight as shown. If the edges are fixed, find the transverse displacements at the grid points $(i, j) \in \{0,1,2,3\} \times \{0,1,2,3\}$ of a regular grid using the Finite Element Method. Use symmetry to reduce the number of non-zero independent displacements to one.



Name_____ Student number_____

In a stationary problem, the discrete equations given by the Finite Element Method on regular grid of spacing h and piecewise linear approximation on triangle elements

$$S'[w_{(i-1,j)} + w_{(i,j-1)} - 4w_{(i,j)} + w_{(i+1,j)} + w_{(i,j+1)}] + h^2 f_i' = 0 \quad (i,j) \in I ,$$

$$w_{(i,j)} = 0 \quad (i,j) \in \partial I .$$

Displacement vanishes at the boundary points and, due to the symmetry, displacements at the interior points should be equal. Denoting the common value by

$$w_{(1,1)} = w_{(1,2)} = w_{(2,1)} = w_{(2,2)} = w_1$$

all equations for the interior points boil down to

$$-S'2w_1 + \frac{L^2}{9}\rho g t = 0$$

giving as the displacement at the interior points

$$w_1 = \frac{1}{18}\frac{\rho g t L^2}{S'} .$$