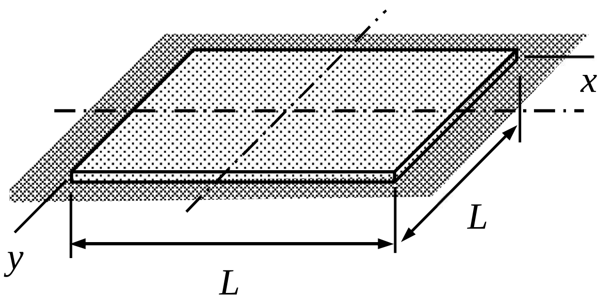
LECTURE ASSIGNMENT 2

Consider vibration of a rectangular membrane of side length L, density ρ, thickness t, and tightening S' (force per unit length). If the edges are fixed, find the angular velocity of the free vibrations using the Finite Element Method on a regular grid of points $(i,j) \in \{0,1,2,3\} \times \{0,1,2,3\}$. Consider the mode, which is reflection symmetric with respect to the lines through the center point (figure).



Notice: According to the principle of virtual work, the equation for a constrained displacement is the sum of equations for the constrained points.

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In a time-dependent membrane problem without external forces, the equations given by the Finite Element Method on regular grid of spacing h and piecewise linear approximation on a regular triangle division are

$$S'[w_{(i-1,j)} + w_{(i,j-1)} - 4w_{(i,j)} + w_{(i+1,j)} + w_{(i,j+1)}] =$$

$$m'h^2 \frac{1}{12}[\ddot{w}_{(i-1,j-1)} + \ddot{w}_{(i-1,j)} + \ddot{w}_{(i,j-1)} + 6\ddot{w}_{(i,j)} + \ddot{w}_{(i+1,j)} + \ddot{w}_{(i,j+1)} + \ddot{w}_{(i+1,j+1)}],$$

$$w_{(i,j)} = 0.$$

Initial conditions are not needed in modal analysis. In the present problem, displacement vanishes at the boundary points and, due to the symmetry, displacements at the interior points should be equal. Denoting the common value by

$$w_{(1,1)} = w_{(1,2)} = w_{(2,1)} = w_{(2,2)} = w_1.$$

equation for point $(1,1)$ simplifies to (equations for $(1,2)$, $(2,1)$, and $(2,2)$ may differ but that is omitted in the assignment)

$$\ddot{w}_1 + \omega^2 w_1 = 0 \quad \text{where} \quad \omega = \frac{2}{L}\sqrt{6\frac{S'}{\rho t}}.$$

Therefore, the frequency of the assumed mode shape

$$f = \frac{1}{\pi L}\sqrt{6\frac{S'}{\rho t}}.$$