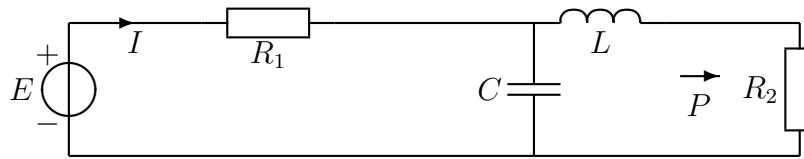
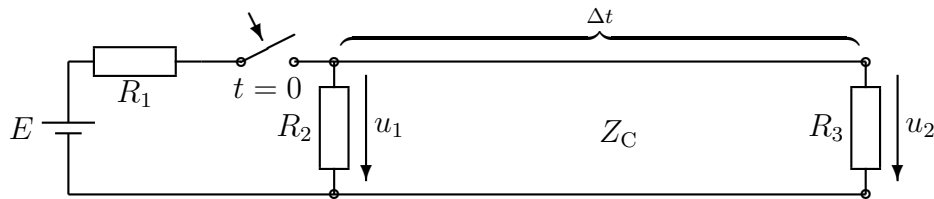


4. Jännitelähde $E = 3 + j20$ V syöttää piiriin virran $-1 + j6$ A. $R_1 = 3 \Omega$. Laske vastuksen R_2 kuluttama teho P .
4. Spänningen är $E = 3 + j20$ V och strömmen $-1 + j6$ A. $R_1 = 3 \Omega$. Hur stor är effekten P i resistansen R_2 ?

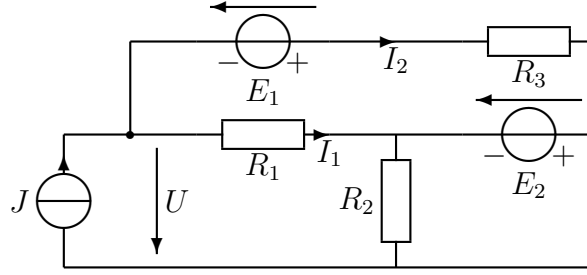


5. Laske jännite $u_2(t_2)$.
5. Beräkna spänningen $u_2(t_2)$.
 $R_1 = 25 \Omega$, $R_2 = 100 \Omega$, $R_3 = 60 \Omega$, $Z_C = 100 \Omega$, $E = 12$ V, $\Delta t = 2$ ms, $t_2 = 7$ ms.



Du får endast besvara fyra frågor! Resultat och svar kan hittas i MyCo.
Vastaa vain neljään tehtävään! Ratkaisut ja tulokset tulevat kurssin sivulle MyCoon.

1. Laske jännite U . $R_1 = 2 \Omega$, $R_2 = 4 \Omega$, $R_3 = 8 \Omega$, $E_1 = 14 \text{ V}$, $E_2 = 2 \text{ V}$, $J = 1,5 \text{ A}$.



$$-U + R_1 I_1 - E_2 = 0 \Rightarrow I_1 = \frac{U + E_2}{R_1} \quad (1)$$

$$-U - E_1 + R_3 \underbrace{I_2}_{J - I_1} = 0 \quad (2)$$

$$-U - E_1 + R_3 \left(J - \underbrace{I_1}_{\frac{U + E_2}{R_1}} \right) = 0 \quad (3)$$

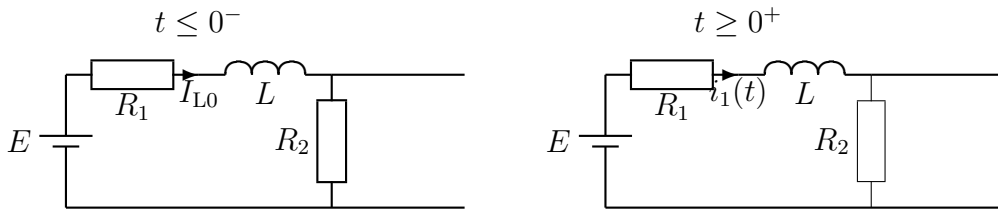
$$-U - E_1 + R_3 J - R_3 \frac{U + E_2}{R_1} = 0 \quad (4)$$

$$-U - \frac{R_3}{R_1} U = E_1 - R_3 J + \frac{R_3}{R_1} E_2 \quad (5)$$

$$U = -\frac{E_1 - R_3 J + \frac{R_3}{R_1} E_2}{1 + \frac{R_3}{R_1}} = -2 \text{ V} \quad (6)$$

Vastaus on siis -2 V , ei missään tapauksessa 2 V !

2. Kytkin suljetaan hetkellä $t = 0$. Laske virta $i_1(t)$. $E = 10 \text{ V}$, $L = 0,2 \text{ H}$, $R_1 = 2 \Omega$, $R_2 = 3 \Omega$.



$$t \leq 0^- : -E + R_1 I_{L0} + L \overbrace{\frac{dI_{L0}}{dt}}^0 + R_2 I_{L0} = 0 \Rightarrow I_{L0} = \frac{E}{R_1 + R_2} = 2 \text{ A} \quad (7)$$

$$t \geq 0^+ : -E + R_1 i_1(t) + L \frac{di_1(t)}{dt} = 0 \quad (8)$$

$$i_1(t) = B - Ae^{-t/\tau} \quad (9)$$

$$R_1(B - Ae^{-t/\tau}) + L \left(0 - A \cdot \frac{-1}{\tau} \cdot e^{-t/\tau} \right) = E \quad (10)$$

$$R_1 B - R_1 A e^{-t/\tau} = E - L \frac{A}{\tau} \cdot e^{-t/\tau} \quad (11)$$

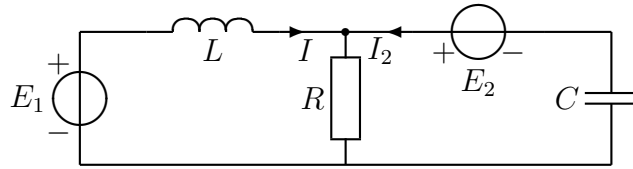
$$R_1 B = E \Rightarrow B = \frac{E}{R_1} = 5 \text{ A} \quad (= i_1(\infty)) \quad (12)$$

$$-R_1 A = -L \frac{A}{\tau} \Rightarrow \tau = \frac{L}{R_1} = 0,1 \text{ s} \quad (13)$$

$$I_{L0} = i_1(0) = B - Ae^{-0/\tau} = B - A \Rightarrow A = B - I_{L0} = 3 \text{ A} \quad (14)$$

$$i_1(t) = 5 - 3e^{-10t/\text{s}} \text{ A} \quad (15)$$

3. Laske virta I . $R = 2 \Omega$, $L = 0,4 \text{ H}$, $C = 0,2 \text{ F}$, $\omega = 2,5 \frac{\text{rad}}{\text{s}}$, $E_1 = 5 \angle 0^\circ \text{ V}$, $E_2 = 10 \angle -90^\circ \text{ V}$.



$$-E_1 + j\omega LI + R(I + I_2) = 0 \Rightarrow I_2 = \frac{E_1 - j\omega LI - RI}{R} \quad (16)$$

$$-R(I + I_2) + E_2 - \frac{1}{j\omega C} I_2 = 0 \quad (17)$$

$$-RI + E_2 - \left(R + \frac{1}{j\omega C}\right) \frac{E_1 - j\omega LI - RI}{R} = 0 \quad (18)$$

$$E_2 - \left(R + \frac{1}{j\omega C}\right) \frac{E_1}{R} = RI + \left(R + \frac{1}{j\omega C}\right) \frac{-j\omega LI - RI}{R} \quad (19)$$

$$I = \frac{E_2 - \left(R + \frac{1}{j\omega C}\right) \frac{E_1}{R}}{R - \left(R + \frac{1}{j\omega C}\right) \frac{j\omega L + R}{R}} \quad (20)$$

Lukuarvot voisi sijoittaa jo aikaisemmin, mutta lavennan nyt kuitenkin $j\omega CR$:illä:

$$I = \frac{j\omega CRE_2 - (j\omega CR + 1) E_1}{j\omega CR^2 - (j\omega CR + 1) (j\omega L + R)} \quad (21)$$

$$I = \frac{j\omega CRE_2 - (j\omega CR + 1) E_1}{j\omega CR^2 - (j\omega CR + 1) (j\omega L + R)} \quad (22)$$

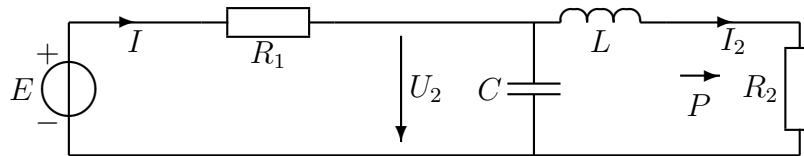
$$(23)$$

$\omega CR = 1$ ja $\omega L = 1$:

$$I = \frac{jE_2 - (j + 1) E_1}{j^2 - (j + 1) (j + 2)} = \frac{jE_2 - (j + 1) E_1}{j^2 + 1 - 3j - 2} = \frac{-j10j - (j + 1) 5}{j^2 + 1 - 3j - 2} \quad (24)$$

$$= \frac{5 - 5j}{-1 - j} = -5 \frac{1 - 1j}{1 + j} = -5 \frac{1 \angle -45^\circ}{1 \angle +45^\circ} = -5 \angle -90^\circ = 5 \angle 90^\circ \text{ A} \quad (25)$$

4. Jännitelähde $E = 3 + j20$ V syöttää piiriin virran $-1 + j6$ A. $R_1 = 3 \Omega$. Laske vastuksen R_2 kuluttama teho P .



$$S = EI^* = (3 + j20)(-1 - j6) = -3 + 120 - 20j - 18j = 117 - 38j \quad (26)$$

$$P_E = 117 = R_1|I|^2 + R_2|I_2|^2 = 3 * (1^2 + 6^2) + P_{R2} \quad (27)$$

$$P = P_{R2} = 117 - 111 = 6 \text{ W} \quad (28)$$

tai

$$U_2 = E - R_1I = 3 + j20 - 3(-1 + j6) = 6 + 2j \quad (29)$$

$$S_2 = U_2I^* = (6 + j2)(-1 - j6) = -6 + 12 - j38 = 6 - j38 \quad (30)$$

$$P = P_{R2} = \text{Re}[S_2] = 6 \text{ W} \quad (31)$$

Et tarvinnut muita lukuarvoja: $R_2 = 6 \Omega$, $\omega L = 2 \Omega$, $\omega C = 1 \text{ S}$, joita ei siis annettu. Tehtävää laatiessani valitsin lähtökohdaksi $I_2 = 1 \text{ A}$ ja mitoitin E :n ja I :n sen mukaan. Tarkistuksia:

$$E = R_1I + (j\omega L + R_2)I_2 \quad (32)$$

$$E = R_1I + \frac{1}{j\omega C}(I - I_2) \quad (33)$$

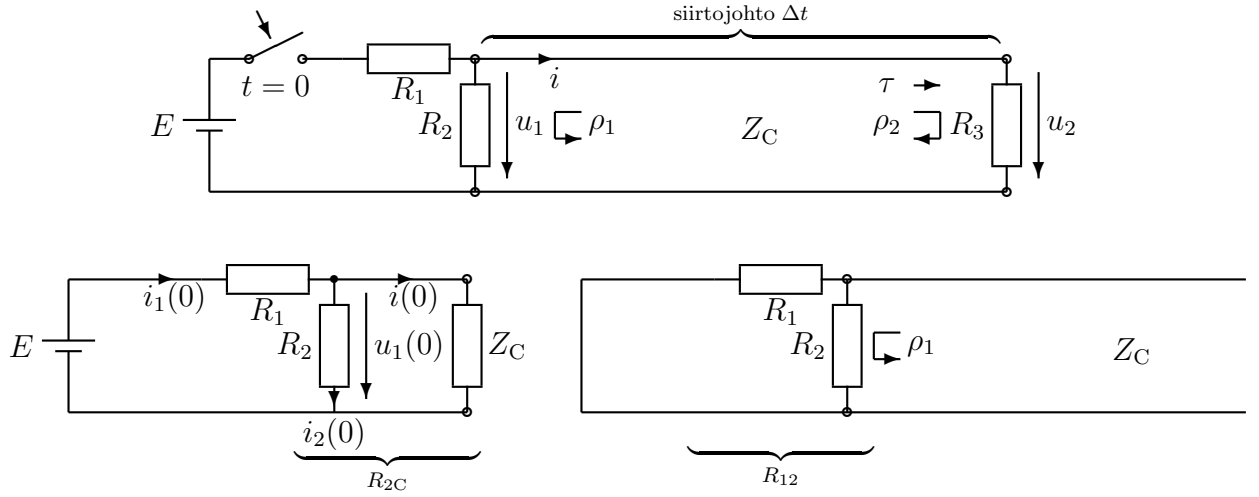
$$(j\omega L + R_2)I_2 = \frac{1}{j\omega C}(I - I_2) \quad (34)$$

$$\Rightarrow I = j\omega C(j\omega L + R_2)I_2 + I_2 = (-\omega C\omega L + j\omega C R_2) \cdot 1 + 1 = -1 + j6 \quad (35)$$

$$E = R_1I + (j\omega L + R_2)I_2 = 3(-1 + j6) + (j2 + 6)1 = 3 + j20 \quad (36)$$

5. Kytin suljetaan hetkellä $t = 0$. Laske jännite $u_2(t_2)$.

$R_1 = 25 \Omega$, $R_2 = 100 \Omega$, $R_3 = 60 \Omega$, $Z_C = 100 \Omega$, $E = 12 \text{ V}$, $\Delta t = 2 \text{ ms}$, $t_2 = 7 \text{ ms}$.



$$R_{2C} = \frac{R_2 Z_C}{R_2 + Z_C} = 50 \Omega \quad (37)$$

$$u_1(0) = \frac{R_{2C}}{R_1 + R_{2C}} \cdot E = \frac{2}{3} \cdot E \quad (38)$$

tai:

$$i_1(0) = i_2(0) + i(0) \quad (39)$$

$$\frac{E - u_1(0)}{R_1} = \frac{u_1(0)}{R_2} + \frac{u_1(0)}{Z_C} \quad (40)$$

$$\frac{E}{R_1} - \frac{u_1(0)}{R_1} = \frac{u_1(0)}{R_2} + \frac{u_1(0)}{Z_C} \quad (41)$$

$$\frac{E}{R_1} = \frac{u_1(0)}{R_1} + \frac{u_1(0)}{R_2} + \frac{u_1(0)}{Z_C} \quad (42)$$

$$u_1(0) = \frac{\frac{E}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{Z_C}} = \frac{E \frac{R_2}{R_1} Z_C}{\frac{R_2}{R_1} Z_C + Z_C + R_2} = \frac{400}{600} E = 8 \text{ V} \quad (43)$$

$$(44)$$

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = 20 \Omega \quad (45)$$

$$\rho_1 = \frac{R_{12} - Z_C}{R_{12} + Z_C} = -\frac{2}{3} \quad (46)$$

$$\rho_2 = \frac{R_3 - Z_C}{R_3 + Z_C} = -\frac{1}{4} \quad (47)$$

$$\tau_2 = 1 + \rho_2 = \frac{3}{4} \quad (48)$$

$$u_2(t_2) = u_2(3\Delta t) = u_1(0)\tau_2 + u_1(0)\rho_2\rho_1\tau_2 = 7 \text{ V} \quad (49)$$