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Benefit-to-cost analysis and optimization models in project portfolio selection

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1. Project portfolio selection
2. Benefit-to-cost analysis
3. Optimization models in project portfolio selection
4. Formulation of a knapsack model in military capital planning
5. Solving project allocation problems using spreadsheets

Feel free to ask questions etc. at any point

Project portfolio selection

- Common problem for many organizations regardless of their size, type or purpose is that they have multiple projects planned but limited resources
  - Sizes of the projects may vary e.g. from purchasing a new dishwasher to building a production department to increase capacity
  - Possible limitations include budget, time, expertise
- Choosing one project may require or prevent choosing some other project
- Finnish Transport Infrastructure Agency (Väylävirasto) decides on railway network maintenance projects but also on road and waterway maintenance etc.

Current and upcoming railway network maintenance projects

Source: https://vayla.fi/hankkeet-kartalla

Benefit-to-cost analysis

- ”First come first served” principle or intuitive decisions lead to poor choices with an outcome far from optimal
- Not even top-level decision makers can have complete understanding of every project
- Organizations with purely financial goals can choose projects with positive net revenue if there is no limitations but for example most public-sector organizations (military, hospitals etc.) have different goals
- Benefits could be defined over multiple objectives to reach organizations best interest

Benefit-to-cost analysis

$$b_i = \sum_{j=1}^{n} w_j v_j(y_{ij})$$

b_i = benefit measure for project i

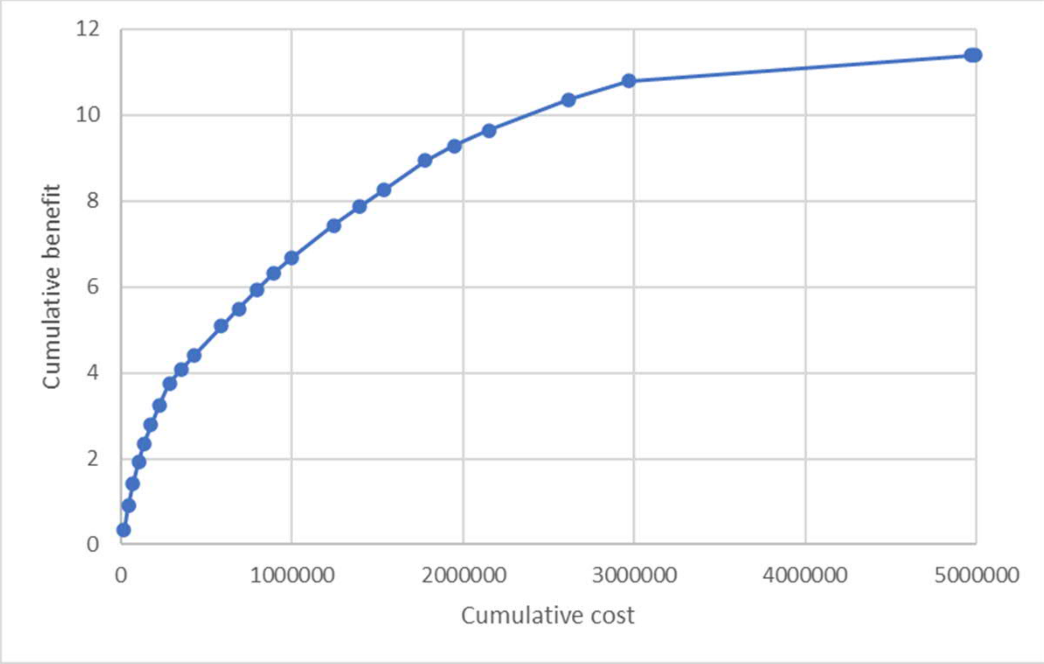
w_j = weight parameter to describe DM's assessment of relative importance of evaluation attribute j (usually $\sum_{j=1}^{n} w_j = 1$)

v_j = value function to describe performance of certain project on attribute j (usually scaled from 0 to 1)

Use of the formulation in benefit-to-cost analysis requires linear-additive form of the value function and assumption that project values are additive across the portfolio

Benefit-to-cost analysis

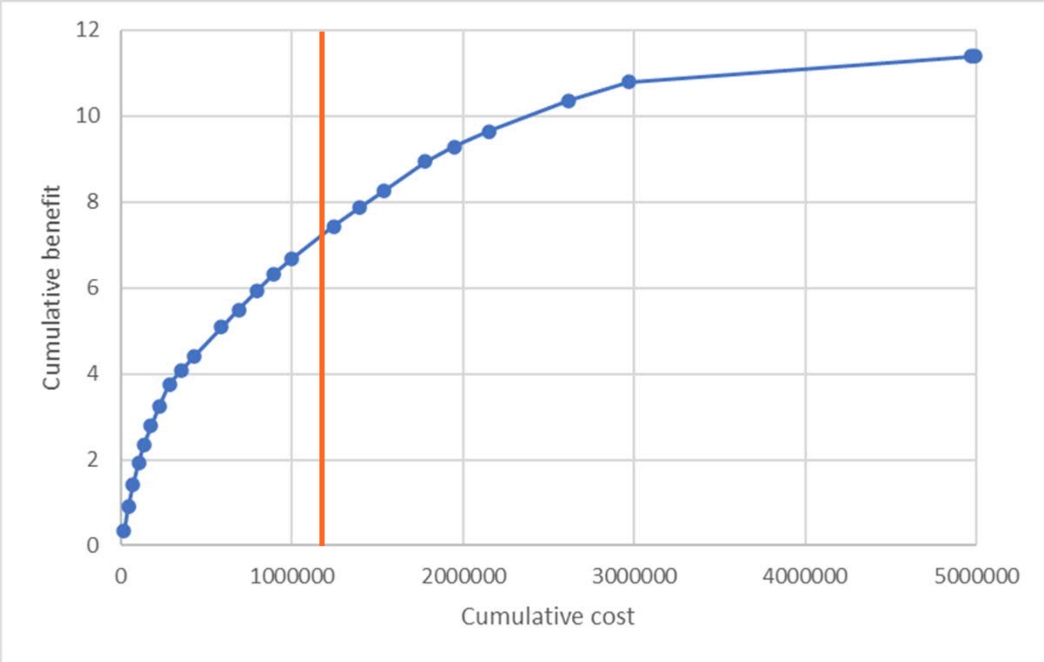
Project	Cost	Benefit	Benefit/Cost*100000
1	249000	0,75206	0,302032129
2	105790	0,43634	0,412458644
3	150000	0,4327	0,288466667
4	2000000	0,59733	0,0298665
5	200000	0,35885	0,179425
6	350000	0,42599	0,121711429
7	170000	0,33217	0,195394118
8	50000	0,44344	0,88688
9	28134	0,52413	1,862977181
10	23416	0	0
11	26000	0,566	2,176923077
12	15000	0,33637	2,242466667
13	75000	0,31954	0,426053333
14	39000	0,45272	1,160820513
15	35000	0,4908	1,402285714
16	140000	0,4	0,285714286
17	70000	0,33558	0,4794
18	100000	0,41267	0,41267
19	160000	0,6672	0,417
20	100000	0,346	0,346
21	100000	0,4008	0,4008
22	245000	0,6865	0,280204082
23	470000	0,72084	0,153370213
24	60000	0,52933	0,882216667
25	30000	0,41267	1,375566667



Benefit-to-cost analysis

- Benefit-to-cost analysis can be used to receive best possible benefit for the money used but usually there is a certain budget and this approach does not necessarily produce the best total benefit available within the constraints

- Structure of resource allocation problems is usually complex with e.g. yearly budgets and other constraints which makes benefit-to-cost analysis inconvenient



Optimization models in project portfolio selection

0-1 Linear Integer programming problem (also known as the knapsack problem)

$$\max \sum_{i=1}^{n} b_i x_i$$

$$\boxed{b_i = \sum_{j=1}^{n} w_j v_j\left(y_{ij}\right)}$$

$$\text{s.t.} \sum_{i=1}^{n} c_i x_i \leq C,$$

$$x_i \in \{0,1\}, i = 1, \dots, n$$

Possible constraints

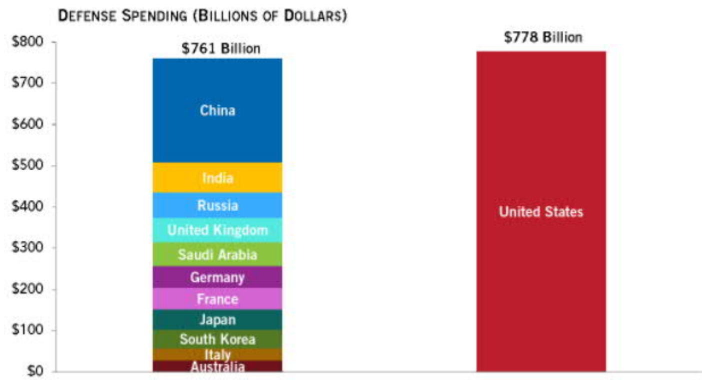
$$\sum\nolimits_{i \in S} x_i \leq 1, \qquad x_i - x_j \leq 0, \qquad x_i - x_j = 0, \dots$$

Assumption of additive costs and benefits

Formulation of a knapsack model in military capital planning

Motivation

- Mentality used to center on how to pay for what military needed, rather than whether the need was real
- Since mathematical optimization was introduced after WWII it has been utilized in US military capital planning
- Resulting decisions have committed trillions of dollars



Source: https://www.pgpf.org/chart-archive/0053_defense-comparison

MOE = measure of effectiveness

- Typically contributions to MOE are nonlinear -> can be approximated with piecewise linear functions

Basic idea of the problem

- Decide whether or not buy any of a weapon system and then how many to buy to maximize the contribution towards MOE within the limitations

Formulation of a knapsack model in military capital planning

Indices and index sets

a = acquisition option

w = weapon system

$w(a)$ = set of weapon system(s) procured under acquisition option a (continuous for simplicity)

Parameters

l_{aw} (u_{aw}) = lower (upper) limit on quantity $w \in w(a)$ under a

$fixedcontr_a$ = fixed contribution of a towards the MOE

$varcontr_{aw}$ = variable contribution per unit of $w \in w(a)$ under a towards the MOE

$fixedcost_a$ = fixed cost of selecting a

$varcost_{aw}$ = variable cost per unit of $w \in w(a)$ under a

$budget$ = available budget

Decision variables

$SELECT_a$ = 1 if any units are purchased under a, 0 otherwise

$QUANTITY_{aw}$ = number of units of $w \in w(a)$ purchased under a

Formulation of a knapsack model in military capital planning

Interactions among decisions and synergy:

As mentioned earlier, it is typical in real-life problems that there are restrictions such as ”select at most k of these options” etc. The interactions can also be synergetic. For example two different weapon systems produce marginal improvements to effectiveness on their own but together offer significant increase in effectiveness.

-> new variable $BOTH_{aa'}$ which has value of 1 when both a and a' are purchased.

Colours of money:

Usually the money spent on military assets is restricted to a spesific funding category -> new index c = funding category

Multiple-year planning horizon

Most capital planning extends over multiple-year horizon. The age of the weapon systems can also be tracked, especially if the planning horizon exceeds their service life. Converting costs to some present value is convenient (and important) to make options more comparable.

Formulation of a knapsack model in military capital planning

Indices and index sets

a = acquisition option

w = weapon system

$w(a)$ = set of weapon system(s) procured under acquisition option a

c = funding category, ”colour of money”

y = year (start year $\underline{y}$, stop year $\overline{y}$)

Parameters

l_{awy} (u_{awy}) = lower (upper) limit on quantity $w \in w(a)$ purchased in year y under option a

$fixedcontr_{awy}$ = fixed contribution of $w \in w(a)$ towards the MOE in year y under option a

$varcontr_{awy}$ = variable contribution per unit of $w \in w(a)$ towards the MOE in year y under option a

$fixedcost_{acy}$ = fixed cost of selecting a in category c in year y

$varcost_{acwy}$ = variable cost per unit of $w \in w(a)$ under a in category c in year y

$budget_{cy}$ = available budget in category c in year y

Formulation of a knapsack model in military capital planning

Decision variables

$SELECT_a$ = 1 if any units are purchased under a, 0 otherwise

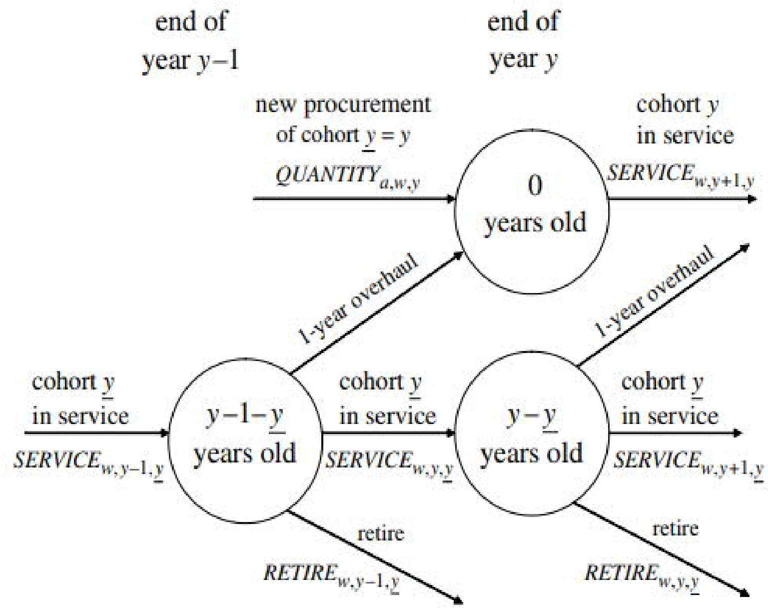
$QUANTITY_{aw\underline{y}}$ = number of units of $w \in w(a)$ purchased under a that begin operation at the end of year y

$SERVICE_{wy\underline{y}}$ = number of units of w in service during year y that first served at the end of year $\underline{y}$

$RETIRE_{wy\underline{y}}$ = number of units of w taken out of service at the end of year y that first served at the end of year $\underline{y}$

Cumulative budget

Excess budget from previous years could be allowed to be used during any current year



Brown et al. (2004)

Formulation of a knapsack model in military capital planning

Time Dependencies Among Decisions

Concurrent operation

Weapon set w procured under a is required to operate concurrently with w' under a'. Provided $\underline{y} \leq \underline{y}' \wedge \overline{y} \geq \overline{y}'$

$$SELECT_{a'} \leq SELECT_a$$

Prerequisite operation

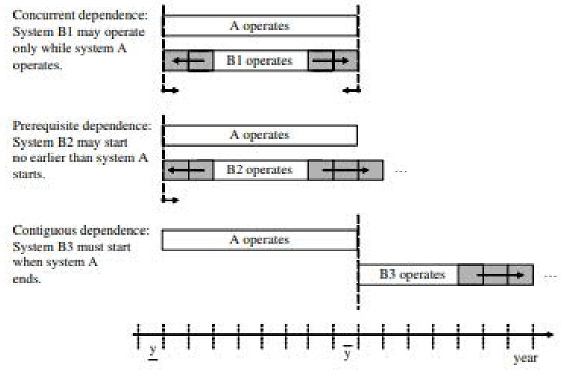
Weapon set w procured under one of options $a \in \Omega$ is required to operate prior to operating w' under a'. Provided $\underline{y} \leq \underline{y}'$

$$SELECT_{a'} \leq \sum_{a \in \Omega} SELECT_a$$

Contiguous operation

Exactly one weapon set w procured under one of options $a \in \Omega$ is required to operate immediately after operating w' under a'. Provided $\underline{y} = \overline{y}'$

$$SELECT_{a'} = \sum_{a \in \Omega} SELECT_a$$



Brown et al. (2004)

Formulation of a knapsack model in military capital planning

- Small changes in the inputs can lead to huge changes in the output
  - Hamming distance $\sum_{a|select_a^*=0} SELECT_a + \sum_{a|select_a^*=1}(1 - SELECT_a)$ sums the changes between legacy plan $select_a^*$ and revision and can be restricted to find alternative solutions with little higher costs etc.

- Uncertainty within war plan scenarios
  - Clients are not willing to commit to e.g. stochastic optimization. *"Military planners worry about what is possible, rather than what is likely."*

- Military capital planning problems can consist of dozens of different attributes to consider which complicates the decision-making process which highlights the importance of proper formulation of the problem and computational power

Solving project allocation problems using spreadsheets

Product development project

- 6 projects with different values (benefit)
- Projects 1 and 2 have both 3 alternatives and only one alternative can be chosen per project
- At least one of the projects 1, 4 and 5 must be chosen to develop product 1 and one of the projects 2 and 3 must be chosen to develop product 2
- Projects have varying costs over 5 years and the yearly budget is 10

		Required budget per year					Required work	
Project	Value	Year 1	Year 2	Year 3	Year 4	Year 5	P1	P2
P1.1	0,15	3	2	1	0	0	1	0
P1.2	0,12	2	1,5	1,5	1	0	1	0
P1.3	0,1	1,5	1,5	1,5	1	1	1	0
P2.1	0,3	4,5	5,5	6	6	5	0	1
P2.2	0,26	3	4	5	5	4	0	1
P2.3	0,18	1,5	2,5	2,5	2	0	0	1
P3	0,16	2	2	2	0	0	0	1
P4	0,19	0	1	1,5	1,5	1,5	1	0
P5	0,25	2	2,5	5	2,5	1	1	0
P6	0,21	0	1,5	3,5	1,5	0,5	0	0
Budget		10	10	10	10	10		

Recap

- It is typical that there is more projects on the planning table than there is resources to execute them
- Measuring the benefit of certain project can be difficult especially in the case of negative net revenue

➡ Need for analytical methods and optimization to solve project portfolio selection problems

- After the benefits are defined, we can choose the projects by using benefit-to-cost analysis or different optimization methods from which the 0-1 linear integer programming problem was the main focus of this presentation
- Sensitivity analysis and presence of uncertainty in real-life problems were only slightly discussed but crucial when solving these problems

References

Brown, G. G., Dell, R. F., Newman, A. M., 2004: Optimizing Military Capital Planning, Interfaces 34/6, p. 415-425.

Kirkwood, G. W., 1997: Strategic Decision Making: Multiobjective Decision Analysis with Spreadsheets, Duxbury Press, Wadsworth Publishing Company. p. 199-211, 216-222

Kleinmuntz, D. N., 2007: Resource Allocation Decisions, In book: Edwards ym. (eds): Advances in Decision Analysis - From Foundations to Applications. Cambridge University Press, p. 400-418.

Homework

Describe briefly a project portfolio selection problem that was not presented in this presentation. Choose at least **3** attributes that contribute to the total **benefit** of certain project and discuss how they could be measured and which stakeholders should be involved in the decision-making process.

Discuss also the **constraint(s)** and possible methods to assess the resulting optimal portfolio.

DL 24.9.2021 at 09:00

Send your answer to: miku.vilander@aalto.fi