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Preference Programming methods: Incomplete preference information for additive value functions; elicitation, modeling, decision recommendations

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Content

1. Additive value functions
2. Incomplete preference information
3. Preference Programming
4. Elicitation of scores and weights
5. Decision recommendations
6. Example

Recap: Additive value functions

- Additive value functions can be used for describing the preferences of the decision maker (DM)

$$\boxed{V(x) = V(x_1, \dots, x_n) = \sum_{i=1}^{n} w_i v_i(x_i)}$$

- Conditions required
  - Preferences are *transitive* and *complete*
  - All attributes are *mutually preferentially independent* and *difference independent*

Recap: Additive value functions

$$\boxed{V\left(x^j\right) = \sum_{i=1}^{n} w_i v_i\left(x_i^j\right)}$$

- Alternatives $X = \{x^1, \dots, x^m\}$
- Attributes $A = \{a_1, \dots, a_n\}$
- Attribute weights $w = [w_1, \dots, w_n] \in W = \{w \mid w_i \geq 0, \ \sum_{i=1}^{n} w_i = 1\}$
- Attribute-specific values (scores) $v_i\left(x_i^j\right) = v_i^j \in [0,1]$
- Overall values of the alternatives $V(x^j)$

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Job selection problem

- Similar to the house-buying problem from last week
  - Alternatives: different job opportunities
  - Attributes: salary, location, working hours

Alternative	a_1: Salary (per year)	a_2: Location	a_3: Working hours (per week)
Researcher	40 k€	Excellent	35 h
Data Scientist	60 k€	Good	45 h
Consultant	100 k€	Moderate	60 h
Medical Doctor	80 k€	Poor	45 h

Job selection problem

- We could try eliciting the attribute-specific value functions v_i with e.g. the bisection method
- Similarly, we could try eliciting the attribute weights w_i with e.g. the trade-off method

What if we don't have complete information?

- What if the DM cannot specify levels of attributes so that they are indifferent between the transitions, as required by the bisection method?
  - *How do we obtain the attribute-specific value functions v_i or the resulting scores v_i^j?*
- What if the DM cannot specify equally preferred alternatives, as required by the tradeoff method?
  - *How do we model the trade-offs between different attributes?*

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Incomplete preference information

- When the model parameters regarding the DM's preferences (that is, the scores and/or weights) are not exactly specified, the preference information is said to be incomplete
  - *"Increasing salary from x to x' is at least as desirable as decreasing working hours from y to y'."*
- These statements are transformed into linear constraints on the model parameters

Incomplete preference information

- The constraints define feasible sets of scores and weights
  - The set of feasible scores $S_j \subset [0,1]^n, j = 1, \ldots, m$
  - The set of feasible weights $S_w \subset W = \{w \mid w_i \geq 0, \sum_{i=1}^n w_i = 1\}$

- To simplify things a bit, for the rest of the presentation we assume that the exact scores v_i^j are known but information about the attribute weights w_i is incomplete. We denote $S_w = S$.

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Preference Programming

- In Preference Programming, incomplete preference information is used to derive decision recommendations
- Salo et al. (2010) use the term Preference Programming of methods that fulfil at least two of the following:
  - *i. Accommodate incomplete preference information through set inclusion*
  - *ii. Offer decision recommendations based on dominance concepts and decision rules*
  - *iii. Support the iterative exploration of the DM's preferences.*
- Preference Programming methods are often extensions of the more conventional decision analytic methods

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Elicitation of scores and weights under incomplete information

- There are several different approaches to the elicitation of scores and weights under incomplete information
- The DM is often allowed to provide interval-valued statements instead of exact numerical estimates
  - ***PAIRS*** *(Salo and Hämäläinen, 1992) – interval statements about scores and ratios of attribute weights*
  - ***PRIME*** *(Salo and Hämäläinen, 2001) – interval statements about scores and ratios of attribute weights*
  - ***Interval SMART/SWING*** *(Mustajoki et al., 2005) – interval-valued ratio statements in SMART/SWING*
- Methods for utilizing incomplete ordinal information also exist

Elicitation of scores and weights under incomplete information

- In our job selection problem, the DM may state that the combination of annual salary of 40 k€ and excellent location is at least as desirable as 70 k€ salary and poor location, but not more desirable than 100 k€ and poor location

$$(\mathbf{70\ k€}, \text{Poor}, *) \preccurlyeq (40\ \text{k€}, \text{Excellent}, *) \preccurlyeq (\mathbf{100\ k€}, \text{Poor}, *)$$

$$\boxed{\Longrightarrow 0.5 w_1 \leq w_2 \leq w_1}$$

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Decision recommendations

- Even when the preference information is incomplete, meaningful decision recommendations can often be provided
  - Dominance
  - Decision rules

Dominance

$$V(x^j) = \sum_{i=1}^{n} w_i v_i^j$$

- Alternative x^k dominates x^l in S, denoted by $x^k \succ_S x^l$ if

$$\begin{cases} V(x^k) \geq V(x^l) \text{ for all } w \in S \\ V(x^k) > V(x^l) \text{ for some } w \in S \end{cases}$$

- Dominance relations can thus be established by computing the minimum and maximum value difference

$$\begin{cases} \min_{w \in S}[V(x^k) - V(x^l)] \geq 0 \\ \max_{w \in S}[V(x^k) - V(x^l)] > 0 \end{cases}$$

Dominance

- Based on dominance, the alternatives are divided into *dominated* and *non-dominated* alternatives
  - For a dominated alternative, there always exists a better alternative
  - A rational DM is only interested in non-dominated alternatives
- The set of non-dominated alternatives might contain multiple alternatives. Then, there are a few approaches
  - Eliciting additional information
  - Decision rules

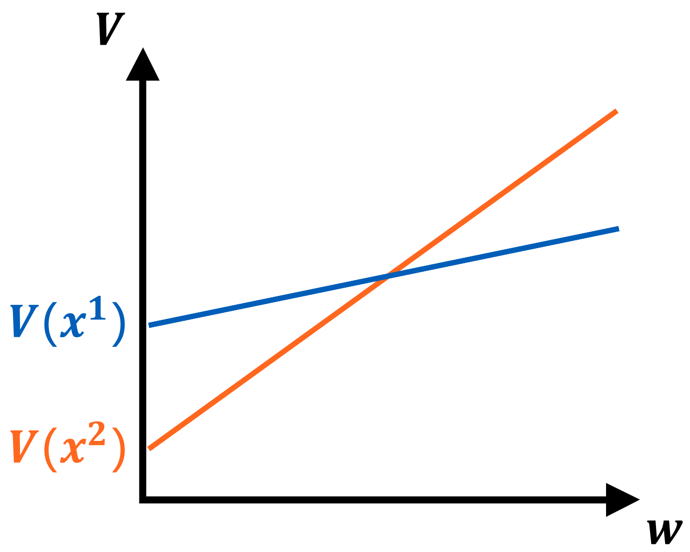
Decision rules

- Decision rules can be used to derive decision recommendations when there are multiple non-dominated alternatives
- Examples of decision rules

1) ***Maximax***: Select the alternative with the highest maximum value
2) ***Maximin***: Select the alternative with the highest minimum value
3) ***Central values***: Select the alternative with the highest average value
4) ***Minimax regret***: Select the alternative with the smallest maximum loss of value

Decision rules

- There are some issues with decision rules
  - *Scale-dependence*: changing the measurement scales $[x_i^0, x_i^*]$ can change the recommendations
  - Different rules may also give different recommendations
    - *Maximax:* x^2
    - *Maximin:* x^1



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Example

- Let's return to the job selection problem
- Assume that the exact scores $v_i(x_i^j)$ are known but information about attribute weights is incomplete

Alternative	a_1: Salary	a_2: Location	a_3: Working hours	$v_1(x_1)$	$v_2(x_2)$	$v_3(x_3)$
A) Researcher	40 k€	Good	35 h	0.00	0.67	1.00
B) Data Scientist	60 k€	Excellent	50 h	0.33	1.00	0.50
C) Consultant	100 k€	Moderate	60 h	1.00	0.33	0.00
D) Medical Doctor	80 k€	Poor	45 h	0.67	0.00	0.60

Example

- The DM states that
  - $(\mathbf{70\ k€}, \text{Poor}, *) \preccurlyeq (40\ \text{k€}, \text{Excellent}, *) \preccurlyeq (\mathbf{100\ k€}, \text{Poor}, *)$
  - $(*, \text{Poor}, \mathbf{45\ h}) \preccurlyeq (*, \text{Excellent}, 60\ \text{h}) \preccurlyeq (*, \text{Poor}, \mathbf{35\ h})$
  - $(60\ \text{k€}, *, \mathbf{50\ h}) \preccurlyeq (100\ \text{k€}, *, 60\ \text{h}) \preccurlyeq (60\ \text{k€}, *, \mathbf{40\ h})$

- These statements result in the following constraints
  - $0.5w_1 \leq w_2 \leq w_1$
  - $0.8w_3 \leq w_2 \leq w_3$
  - $0.6w_3 \leq w_1 \leq 0.9w_3$

Example

$0.5w_1 \leq w_2 \leq w_1$
$0.8w_3 \leq w_2 \leq w_3$
$0.6w_3 \leq w_1 \leq 0.9w_3$

- Computation of non-dominated alternatives

$$\boxed{x^k \succ_S x^l \Leftrightarrow \begin{cases} \min\limits_{w \in S}[V(x^k) - V(x^l)] \geq 0 \\ \max\limits_{w \in S}[V(x^k) - V(x^l)] > 0 \end{cases}}$$

- We find out that
  - **A** dominates **C** and **D**
  - **B** dominates **C** and **D**
- The non-dominated alternatives are thus **A** and **B**

Example

Alternative	$\max_{w \in S} V(x^j)$	$\min_{w \in S} V(x^j)$
A) Researcher	0.5897	0.5679
B) Data Scientist	0.5714	0.5556
C) Consultant	0.4321	0.4103
D) Medical Doctor	0.4444	0.4286

$\left.\begin{array}{l} A \\ B \end{array}\right\}$ Non-dominated

- Maximax: **A**
- Maximin: **A**

$\Longrightarrow$ We would thus select alternative **A**

Pros of Preference Programming

- Less effort required from the DM
- Can be used for *screening* purposes – especially if the number of alternatives and the costs of information elicitation are high
- *Group decision making* – different stakeholders do not need to agree on a single number but can provide interval statements etc.

Summary

- **Preference Programming** methods can be used for multi-attribute decision problems when only *incomplete information* is available. Several different methods exist.
- Information about **attribute scores and weights** can be represented e.g. as intervals which are modeled as *constraints* on the parameters of the additive value function.
- **Decision recommendations** are based on *dominance relations* and *decision rules*.

References

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Homework

Homework (1/3)

Let's return to out job selection example. Now we have eight alternatives of which to choose: **A, B, ..., H**. All the scores $v_i(x_i^j)$ are known exactly (see next slide).

The exact attribute weights are not known, but we know that

$$\begin{cases} w_1 \le w_2 \le 1.5 w_1 \\ w_3 \le w_2 \\ 0.5 w_1 \le w_3 \end{cases}$$

The weights are normalized such that $w_1 + w_2 + w_3 = 1$.

Homework (2/3)

Attribute scores $v_i(x_i^j)$ for alternatives **A-H**

Alternative	$v_1(x_1)$	$v_2(x_2)$	$v_3(x_3)$
A	0.00	0.50	0.30
B	0.40	0.70	0.10
C	1.00	0.10	0.30
D	0.80	0.00	0.50
E	1.00	0.40	0.00
F	0.20	0.60	0.60
G	0.00	0.20	1.00
H	0.20	1.00	0.10

Homework (3/3)

1. Which of the alternatives **A-H** are ***non-dominated***?
2. Based on the ***maximax*** decision rule, which of the non-dominated alternatives would you recommend?

Send your solution to *helmiina.kontio@aalto.fi* by next Friday 8.10.2021 (9:00).

Hints:
- MATLAB could be useful
- MS-E2134 slides (lecture 6) might also help