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Preference Programming in portfolio decision analysis:

The Robust Portfolio Modeling method

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Outline

1. **Motivate the development of the RPM methodology**
2. **RPM methodology**
   - *Incomplete information on weights and project-specific scores*
   - *Non-dominated portfolios*
   - *Robust project selection*
3. **Extended RPM**
   - *Project interdependencies*
   - *Incomplete cost information*
   - *Variable budget level*
   - *Benefit-cost analysis*
4. **The dynamic programming algorithm**

Motivation for the method

- Eliciting preference information is difficult
- We need ways to deal with incomplete preference information in a simple and transparent way
- *Preference Programming* accomodates incomplete information in multi-criteria weighting models but is not directly applicable to portfolio selection

Motivation for the method

- Preference Programming allows for incomplete information by *set inclusion* which can be used in portfolio selection as well
  - *We define a set of feasible weights and scores*
  - *No clear optimal choice in the portfolios P_F due to uncertainty*

- Non-dominated portfolios are the interesting ones

- Enumerating the non-dominated portfolios is intractable
  - The number of possible portfolios with m projects is 2^m
  - The RPM dynamic programming algorithm can find them!

Example problem

- 40 feature development projects (features)
- 3 customer sections with different value scores for each feature
- Incomplete information on values and weights
  - *Value score intervals*
  - *Customer 1 more important than customer 2*
- Costs c_j and budget R

Name	Customer 1 (v_1^j)	Customer 2 (v_2^j)	Customer 3 (v_3^j)	Cost (c_j)
Feature A7	[25, 28]	[22, 26]	[45, 52]	10
Feature A8	[10, 20]	[50, 60]	[20, 30]	30
Feature A9	[33, 40]	[23, 25]	0	60
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

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Robust Portfolio Modeling method

Outline of RPM methodology

1. Define feasible set of portfolios with incomplete information
2. Compute non-dominated portfolios (algorithm)
3. Identify robust projects and need for additional information
4. Narrow down your set of non-dominated portfolios

(Repeat 3 and 4 until satisfied)

5. Use decision rules and possible other intuitions to make final portfolio selection

Feasible portfolios

The goal to maximise overall value over *feasible portfolios*

$$\max_{p \in P_F} V(p,w,v) = \max_z \left\{ \sum_{i=1}^{m} z_j \sum_{i=1}^{n} w_i v_j^j \,\middle|\, \sum_{i=1}^{m} c_j z_j \leq R, z \in \{0,1\}^m \right\}$$

We value the portfolios with an additive value function

$$V(p,w,v) = \sum_{x^j \in p} V(x^j) = \sum_{x^j \in p} \sum_{i=1}^{n} w_i v_j^j$$

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$$P_F = \left\{ p \in P \,\middle|\, C(p) \leq R \right\}$$

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Incomplete preference information

Instead of having exact weights w_i and scores v_i^j, we have

- Set of feasible weights

$$S_w = \left\{ w \in \mathbf{R}^n \,\middle|\, \sum w_i = 1,\ A^w w \leq B^w \right\}$$

- Set of feasible scores

$$S_v = \left\{ v \in \mathbf{R}_+^{m \times n} \,\middle|\, v_i^j \in [\underline{v}_i^j, \bar{v}_i^j] \right\}$$

These form the *information set*.

$$S \equiv S_w \times S_v$$

Incomplete preference information

Instead of having exact weights w_i and scores v_i^j, we have

- Set of feasible weights

$$w_1 \geq w_2$$

$$S_w = \left\{ w \in \mathbf{R}^n \middle| \sum w_i = 1,\ A^w w \leq B^w \right\}$$

- Set of feasible scores

$$S_v = \left\{ v \in \mathbf{R}_+^{m \times n} \middle| v_i^j \in [\underline{v}_i^j, \bar{v}_i^j] \right\}$$

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- Set of feasible scores

$$S_v = \left\{ v \in \mathbf{R}_+^{m \times n} \middle| v_i^j \in [\underline{v}_i^j, \bar{v}_i^j] \right\}$$

Meaning we have two score matrices $\underline{v}$ and $\overline{v}$

These form the *information set*.

$$S \equiv S_w \times S_v$$

Dominance

A portfolio p dominates portfolio p' with regard to information set S if

$$V(p,w,v) \geq V(p',w,v) \quad \text{for all } (w,v) \in S \quad \text{and}$$
$$V(p,w,v) > V(p',w,v) \quad \text{for some } (w,v) \in S$$

Meaning:

$$p \succ_S p' \Leftrightarrow \left\{ \begin{array}{l} \min\limits_{w \in S_w} \sum\limits_{i=1}^{n} w_i \left(\sum\limits_{x^j \in p} \underline{v}_i^j - \sum\limits_{x^j \in p'} \bar{v}_i^j \right) \geq 0 \\ \max\limits_{w \in S_w} \sum\limits_{i=1} w_i \left(\sum\limits_{x^j \in p} \bar{v}_i^j - \sum\limits_{x^j \in p'} \underline{v}_i^j \right) > 0 \end{array} \right.$$

Non-dominated portfolios

Non-dominated portfolios are the ”best” portfolios. These are the interesting ones because a rational DM will choose a non-dominated portfolio.

$$P_N(S) = \{p \in P_F | p' \not\succ_S p \ \forall p' \in P_F\}$$

These are computed using the RPM dynamic programming algorithm.

Core index

Choosing between non-dominated projects is difficult hence, we may want to elicit additional preference information.

To identify robust projects and guide elicitation of additional information we can analyse the core index of projects.

$$CI(x^j, S) = \frac{\#\ \text{non-dominated portfolios including } x^j}{\#\ \text{non-dominated portfolios}}$$

Robust projects

Core (robust) projects: $\mathrm{CI}(x^j, S) = 1.$

- Select

Borderline projects: $0 < \mathrm{CI}(x^j, S) < 1$

- Focus on

Exterior projects: $\mathrm{CI}(x^j, S) = 0$

- Disregard

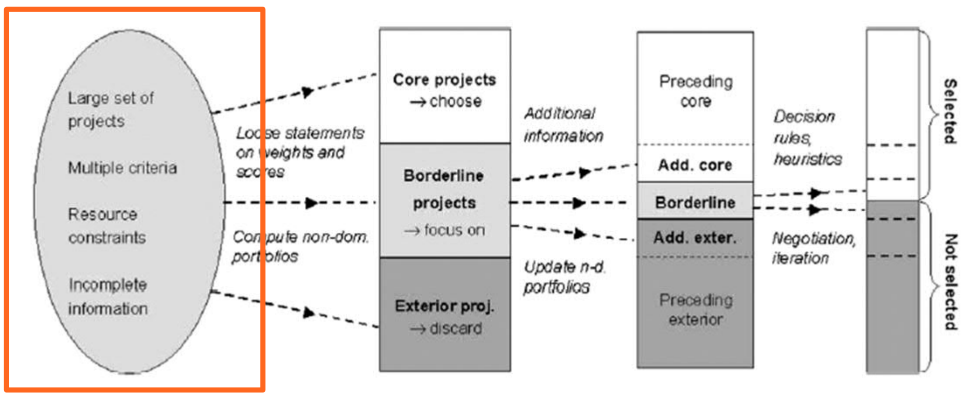
Example problem

Feature	Portfolio 1	Portfolio 2	Portfolio 3	$\mathrm{CI}(x^j)$
Feature A1	1	1	1	1
Feature A2	0	0	1	0.33
Feature A3	0	1	0	0.33
Feature A3	1	0	0	0.33
Feature A7	1	1	1	1
Feature A8	0	0	0	0
Feature A9	1	1	0	0.66
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Iterative decision support process

Eliciting more preference information reduces information set S.

This should be focused on borderline projects.



Source: Robust portfolio modeling with incomplete cost information and project interdependencies, Liesiö et al. (2008)

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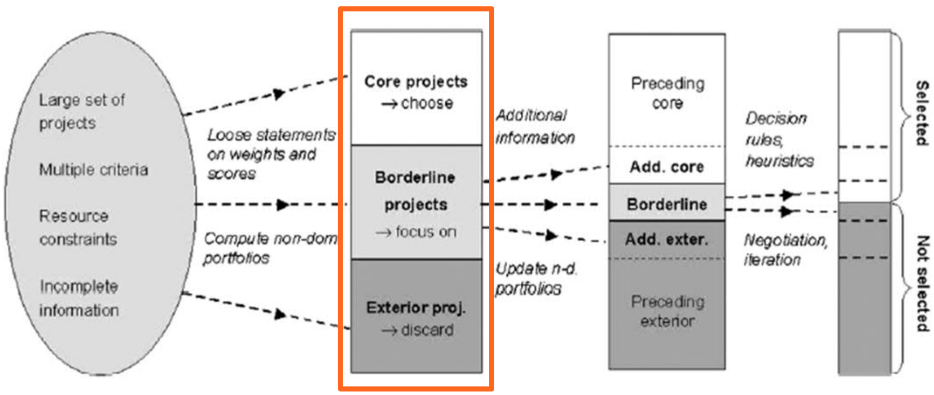

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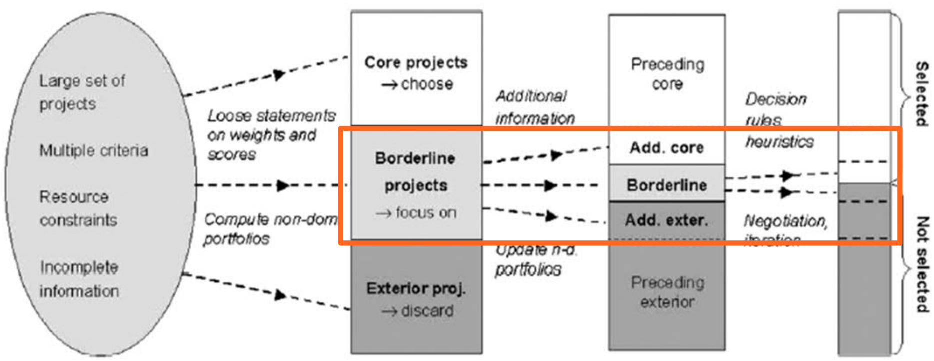

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But what about…

- Project interdependicies,
- Incomplete cost information,
- Variable budget levels,
- Benefit-cost analysis?

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RPM method - *Extended*

Project interdependecies with constraints

We can model interdependencies by manipulating the set of *feasible portfolios*

$$P_F = \left\{ p \in P \middle| Az \leq B \right\}$$

- *Budget constraint* $cz \leq R$
- *Logical constraints* – for example $z_1 + z_2 \leq 1$
- *Positioning constraints* – strategic requirements for the project composition
- *Threshold constraints* – minimum requirements to be fulfilled $(v_1)^T z > 40$

Project interdependencies with dummy projects

We can add dummy projects x^s into the model that capture the effects of interdependencies between other projects.

Then add *logical constraints* to ensure that the dummy project is included when the interacting projects are included

Name	Customer 1 (v_i^j)	Customer 2 (v_2^j)	Customer 3 (v_3^j)	Cost (c_j)
Feature A7	25	22	45	10
Feature A8	10	50	20	30
Feature A9	33	23	0	60
Synergy 1	0	0	0	-50
⋮	⋮	⋮	⋮	⋮

$$z_{A8} \geq z_{Synergy1}$$
$$z_{A9} \geq z_{Synergy1}$$
$$z_{A8} + z_{A9} - 1 \leq z_{Synergy1}$$

Cost uncertainties and soft budgets

Given the following set of costs

$$S_c = \left\{ c \in \mathbf{R}^m \middle| \underline{c} \leq c \leq \bar{c} \right\}$$

We can model cost uncertainties and ”soft” budgets by

(1) removing the budget constraint from feasibility constraints and

(2) adding costs as an attribute-specific score v_4^j for each portfolio and a weight w_4 which corresponds to it.

Name	Customer 1 (v_1^j)	Customer 2 (v_2^j)	Customer 3 (v_3^j)	Cost (v_4^j)
Feature A7	25	22	45	-10
Feature A8	10	50	20	-30
Feature A9	33	23	0	-60
⋮	⋮	⋮	⋮	⋮

Efficient portfolios

A portfolio is *efficient* if no other feasible portfolio gives a higher overall value at a lower cost.

Notice that efficient portfolios are analogous to non-dominated portfolios.

$$P_{\mathrm{E}}(S, S_c) = \{p \in P_{\mathrm{F}} \mid \nexists p' \in P_{\mathrm{F}} \text{ such that } \begin{cases} V(p', w, v) \geqslant V(p, w, v)\ \forall (w, v) \in S \\ C(p', c) \leqslant C(p, c)\ \forall c \in S_c \end{cases} \}$$

Since they are essentially non-dominated portfolios with a minimized cost attribute, an extended version of the RPM algorithm can be used to find them.

Finding non-dominated portfolios for all budget levels

Efficient portfolios help analyse how the portfolio overall value changes as a function of the budget level.

We can find the non-dominated portfolios $P_N(S) = P_N(S, c, R)$ for any costs c and budget level R from the set of efficient portfolios. We just

(1) Disregard efficient portfolios that do not meet budget constraint $\mathrm{C}(\mathrm{p}, \mathrm{c}) \leq R$

(2) Perform pairwise dominance checks

$$\begin{aligned} V(p, w, v) &\geq V(p', w, v) \quad \text{for all } (w, v) \in S \quad \text{and} \\ V(p, w, v) &> V(p', w, v) \quad \text{for some } (w, v) \in S \end{aligned}$$

Benefit-cost analysis GV(R)

We can for instance analyse how the *guaranteed overall value* $GV(R)$ develops when R increases, now that we have the sets of non-dominated portfolios $P_N(S, \bar{c}, R)$ for all budget levels R.

$$GV(R) = \max_{p \in P_N(S,\bar{c},R)} \min_{(w,v)\in S} V(p, w, v)$$

The guaranteed overall value for a budget level R is the highest of the minimum overall values of the non-dominated portfolios achieved with budget R.

Benefit-cost analysis MV(R)

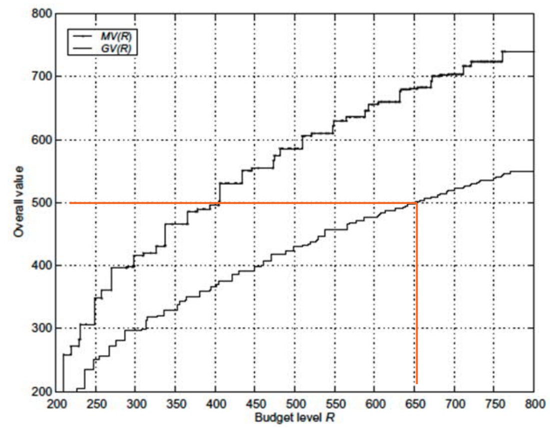
We could also inspect how the *maximal overall value* $MV(R)$ develops when R increases.

$$MV(R) = \max_{p \in P_N(S,\underline{c},R)} \max_{(w,v) \in S} V(p,w,v)$$

The *maximal overall value* shows the very maximum value that could be achieved with budget R, given information set S.

Benefit-cost analysis

- Budget may be set al level R where the guaranteed overall value $GV(R)$ first exceed a given threshold
- The budget can be set at a budget level when the benefit-cost ratio is high



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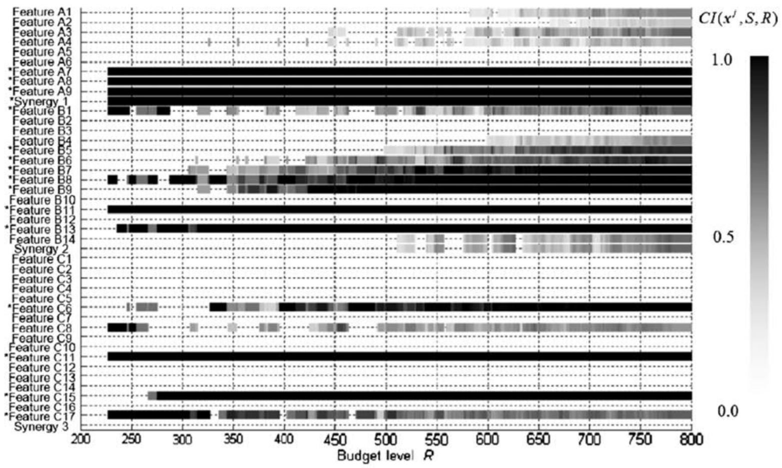
Budget dependent core index

We can analyse robustness of projects w.r.t. budget level R.

$$CI(x^j, S, R) = \frac{\#\text{ non-dominated portfolios including } x^j \text{ at budget level } R}{\#\text{ non-dominated portfolios at budget level } R}$$



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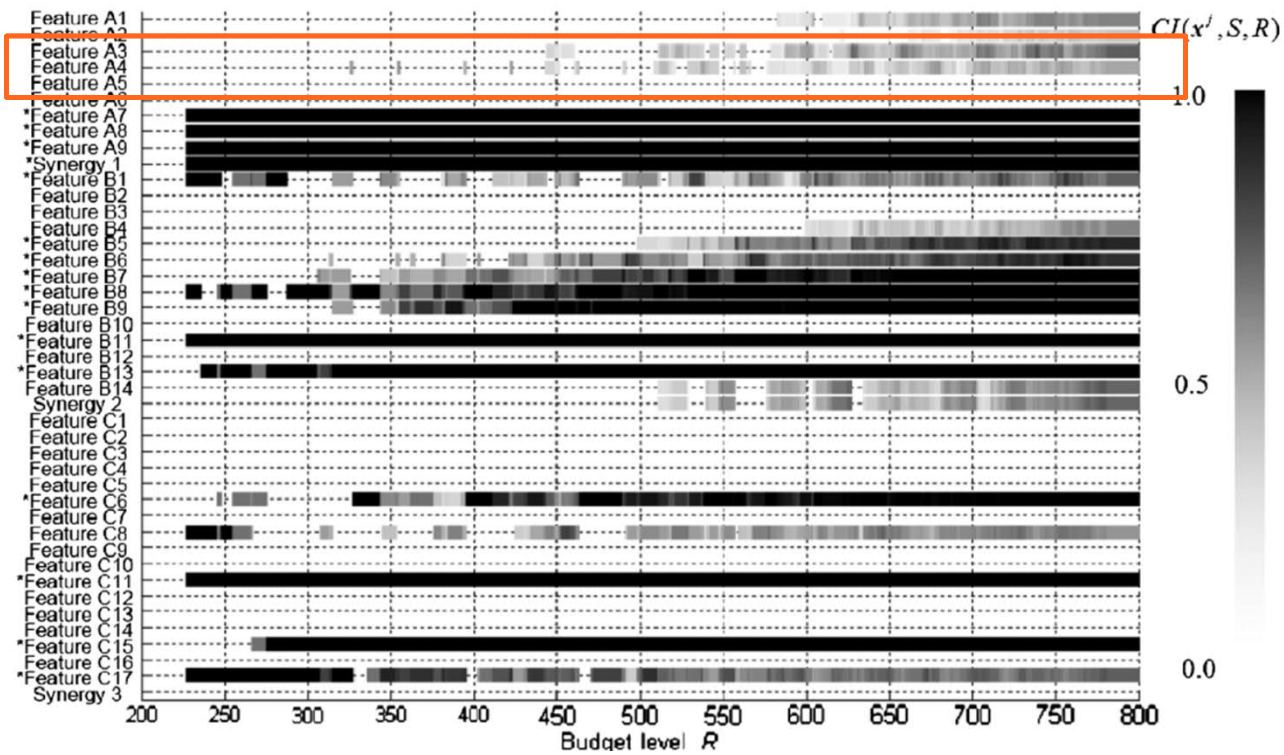

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The algorithm in short

Outline of dynamic programming algorithm

- Feasible portfolios using onlyx^1 ... x^k projects

$$P_F^k = \{p \in P_F | p \subseteq \{x^1, \ldots, x^k\}\}$$

- Non dominated portfolios when only using x^1 ... x^k projects

$$P_N^k = \{p \in P_F^k | \nexists p' \in P_F^k \text{ such that } p' \succ p, C(p') \leqslant C(p)\}$$

Outline of dynamic programming algorithm

(i) Portfolio p can only be non-dominated in set P^k if it is non-dominated in set P^{k-1}

$$p \in P_N^k \Rightarrow p \setminus \{x^k\} \in P_N^{k-1}$$

(ii) Portfolio p is feasible but dominated if it satisfies the budget constraint and there exist a portfolio p' that dominates it

$$p \in P_F^k \setminus P_N^k \Rightarrow \exists p' \in P_N^k \textit{ such that } p' \succ p, C(p') \leqslant C(p)$$

Outline of dynamic programming algorithm

1. $P_N^1 \leftarrow \{\{\emptyset\}, \{x^1\}\}$
2. **for** k = 1...m **do**
   a) $\tilde{P}_N^k \leftarrow \{p \in P_F \big| x^k \in p, p \setminus \{x^k\} \in P_N^{k-1}\}$
   b) $P_N^k \leftarrow \{p \in \tilde{P}_N^k \big| p' \in P_N^{k-1} \text{ such that } p' \succ p, C(p') \leq C(p)\ \}$
   c) $P_N^k \leftarrow \{p \in P_N^{k-1} \big| p' \in \tilde{P}_N^k \text{ such that } p' \succ p, C(p') \leq C(p)\ \}$

3 $P_N \leftarrow \{p \in P^m | p' \in P^m \text{ such that } p' \succ p\}$

Summary on the RPM method

- Incomplete weight, score, cost and budget information
- Project robustness
- Project interdependices
- Benefit-cost analysis
- Efficient algorithm for finding non-dominated portfolios

References

- Liesiö, J., Mild, P., Salo, A., 2007: Preference programming for robust portfolio modeling and project selection, European Journal of Operational Research 181/3, s. 1488-1505.

- Liesiö, J., Mild, P., Salo, A., 2008: Robust portfolio modeling with incomplete cost information and project interdependencies, European Journal of Operational Research 190/3, s. 679-695.

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Homework

Set inclusion – feasible set of weights and value matrices

1. Define feasible set of weights S_w for the example problem when Customer 1 is more important than Customer 2 and Customer 2 is more important than Customer 3. Assume that weights w_1, w_2 and w_3 correspond to the importance of the customers 1, 2 and 3 respectively.

2. Given the following score matrices, where the rows correspond to features F1, F2 and F3, and the columns to customers 1, 2 and 3. Modify the score matrices (and give a brief explanation of how you did it) so that they model the following preference information:

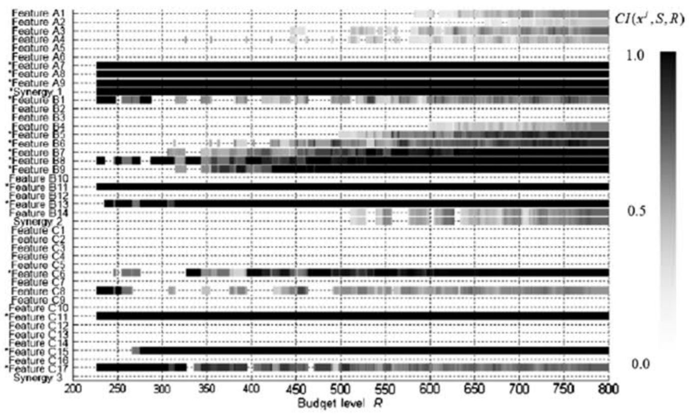
Customer 3 feels that there <u>might</u> be an increased benefit if both features F2 and F3 are implemented. They are unsure about this benefit but say that it would be an increase of at most 30 units.

3. Define the new feasible set of weights S_w corresponding to the new value matrices from part 2.

$$\underline{v} = \begin{bmatrix} 20 & 30 & 50 \\ 25 & 25 & 20 \\ 10 & 9 & 8 \end{bmatrix}, \quad \bar{v} = \begin{bmatrix} 25 & 38 & 56 \\ 30 & 29 & 24 \\ 12 & 12 & 12 \end{bmatrix}$$

Budget dependent core indices

4. Identify the core, borderline and exterior projects (features and synergies) at budget level R = 450 from this graph. Zooming in is easier on the original graph, which is found in Liesiö et al. (2008).



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Submit homework to helmi.hankimaa@aalto.fi by 9am on 15th of October.
Caption your email as ”Homework 6”.