Aalto University
School of Science

Preference information in DEA models: Value Efficiency Analysis (VEA) method + an application

Matti Staudinger
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Contents

Decision Making Unit, DMU

- DMU_i takes inputs, $\mathbf{x}_i$
- Produces outputs, $\mathbf{y}_i$
- Each DMU has *m* inputs and *p* outputs
- **X** input measures of all DMUs, $\mathbf{X} \in \mathbb{R}_+^{m \times n}$
- **Y** output measures of all DMUs, $\mathbf{Y} \in \mathbb{R}_+^{p \times n}$

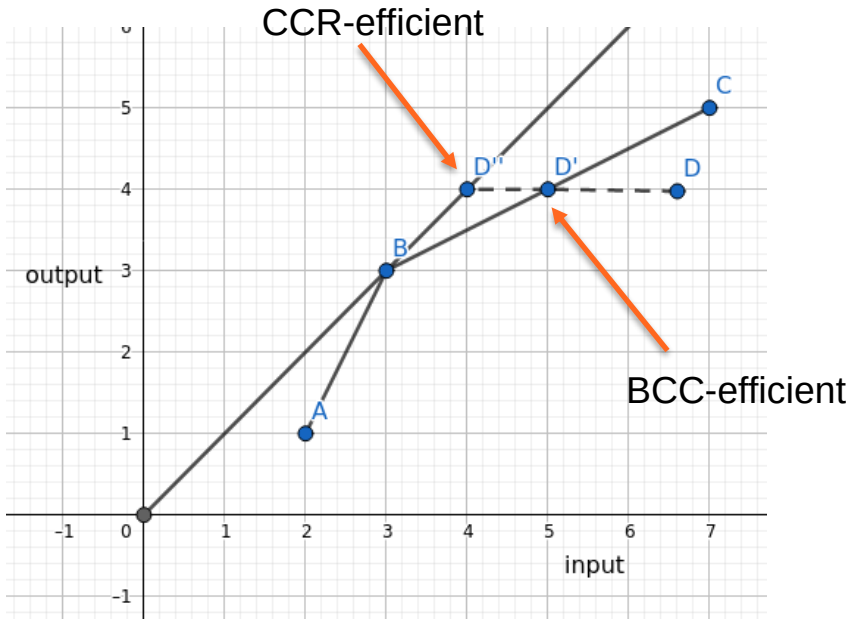
CCR-DEA and BCC-DEA

Output-Oriented CCR Primal (CCR_P-O)	Output-Oriented CCR Dual (CCR_D-O)
$\max Z_O = \theta + \epsilon(\mathbf{1}^T\mathbf{s}^+ + \mathbf{1}^T\mathbf{s}^-)$[1]	$\min W_O = \boldsymbol{\nu}^T\mathbf{x}_0$
s.t. (2.1a)	s.t. (2.1b)
$\mathbf{Y}\boldsymbol{\lambda} - \theta\mathbf{y}_0 - \mathbf{s}^+ = \mathbf{0}$	$\boldsymbol{\mu}^T\mathbf{y}_0 = 1$
$\mathbf{X}\boldsymbol{\lambda} + \mathbf{s}^- = \mathbf{x}_0$	$-\boldsymbol{\mu}^T\mathbf{Y} + \boldsymbol{\nu}^T\mathbf{X} \geq \mathbf{0}^T$
$\boldsymbol{\lambda}, \mathbf{s}^-, \mathbf{s}^+ \geq \mathbf{0}$	$\boldsymbol{\mu}, \boldsymbol{\nu} \geq \epsilon\mathbf{1}$
$\epsilon > 0$ ("Non-Archimedean")[2]	$\epsilon > 0$

Output-Oriented BCC Primal (BCC_P-O)	Output-Oriented BCC Dual (BCC_D-O)
$\max Z_O = \theta + \epsilon(\mathbf{1}^T\mathbf{s}^+ + \mathbf{1}^T\mathbf{s}^-)$	$\min W_O = \boldsymbol{\nu}^T\mathbf{x}_0 + u$
s.t. (2.2a)	s.t. (2.2b)
$\mathbf{Y}\boldsymbol{\lambda} - \theta\mathbf{y}_0 - \mathbf{s}^+ = \mathbf{0}$	$\boldsymbol{\mu}^T\mathbf{y}_0 = 1$
$\mathbf{X}\boldsymbol{\lambda} + \mathbf{s}^- = \mathbf{x}_0$	$-\boldsymbol{\mu}^T\mathbf{Y} + \boldsymbol{\nu}^T\mathbf{X} + u\mathbf{1}^T \geq \mathbf{0}^T$
$\mathbf{1}^T\boldsymbol{\lambda} = 1$	$\boldsymbol{\mu}, \boldsymbol{\nu} \geq \epsilon\mathbf{1}$
$\boldsymbol{\lambda}, \mathbf{s}^-, \mathbf{s}^+ \geq \mathbf{0}$	$\epsilon > 0$
$\epsilon > 0$	

Source: Halme et at. (1999)

input oriented DEA



DMU and Efficiency, General model

$$\mathbf{u} = \begin{bmatrix} y \\ -x \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} \mathbf{Y} \\ -\mathbf{X} \end{bmatrix}$$

A and **b** specify the set of feasible lambdas.

Output-Oriented GEN Primal (GEN_P-O)	Output-Oriented GEN Dual (GEN_D-O)
$\max Z_O = \theta + \epsilon(\mathbf{1}^T\mathbf{s}^+ + \mathbf{1}^T\mathbf{s}^-)$	$\min W_O = \nu^T\mathbf{x}_0 + \mathbf{u}^T\mathbf{b}$
s.t. (2.3a)	s.t. (2.3b)
$\mathbf{Y}\boldsymbol{\lambda} - \theta\mathbf{y}_0 - \mathbf{s}^+ = \mathbf{0}$	$\boldsymbol{\mu}^T\mathbf{y}_0 = 1$
$\mathbf{X}\boldsymbol{\lambda} + \mathbf{s}^- = \mathbf{x}_0$	$-\boldsymbol{\mu}^T\mathbf{Y} + \boldsymbol{\nu}^T\mathbf{X} + \mathbf{u}^T\mathbf{A} \geq \mathbf{0}^T$
$\mathbf{A}\boldsymbol{\lambda} \leq \mathbf{b}$	$\boldsymbol{\mu}, \boldsymbol{\nu} \geq \mathbf{u} \geq \mathbf{0}\ \epsilon\mathbf{1}$
$\boldsymbol{\lambda}, \mathbf{s}^-, \mathbf{s}^+ \geq \mathbf{0}$	$\epsilon > 0$
$\epsilon > 0$	

Source: Halme et at. (1999)

CCR-DEA

$$\lambda_i \geq 0$$

BCC-DEA

$$\sum \lambda_i = 1$$
$$\lambda_i \geq 0$$

$$\Lambda = \{\lambda | \lambda \in \mathbb{R}^n_+,\ \mathbf{A}\lambda \leq \mathbf{b}\}$$

$$T = \{\mathbf{u} | \mathbf{u} = \mathbf{U}\lambda,\ \lambda \in \Lambda\}$$

DMU and Efficiency, General model

A DMU is efficient if at optimal solution $\theta^* = 1$ and all slack variable equal zero. Otherwise it is inefficient.

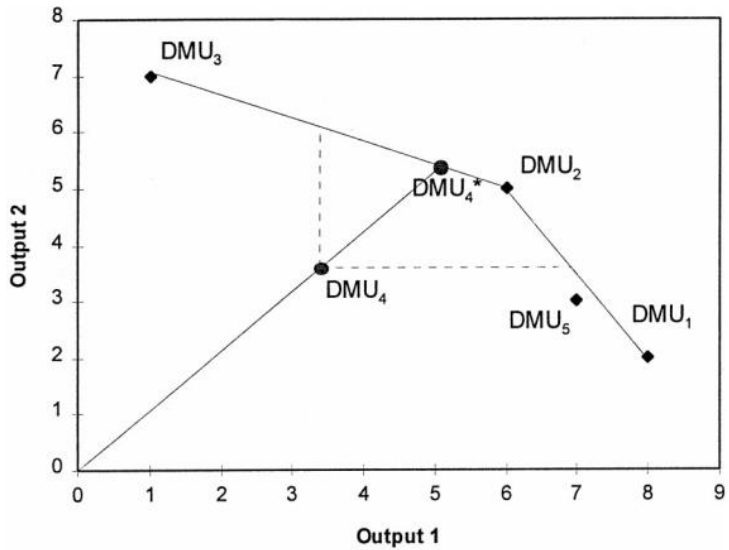
Output-Oriented GEN Primal (GEN_P-O)	Output-Oriented GEN Dual (GEN_D-O)
$\max Z_O = \theta + \epsilon(\mathbf{1}^T\mathbf{s}^+ + \mathbf{1}^T\mathbf{s}^-)$	$\min W_O = \nu^T\mathbf{x}_0 + \mathbf{u}^T\mathbf{b}$
s.t. (2.3a)	s.t. (2.3b)
$\mathbf{Y}\boldsymbol{\lambda} - \theta\mathbf{y}_0 - \mathbf{s}^+ = \mathbf{0}$	$\boldsymbol{\mu}^T\mathbf{y}_0 = 1$
$\mathbf{X}\boldsymbol{\lambda} + \mathbf{s}^- = \mathbf{x}_0$	$-\boldsymbol{\mu}^T\mathbf{Y} + \boldsymbol{\nu}^T\mathbf{X} + \mathbf{u}^T\mathbf{A} \geq \mathbf{0}^T$
$\mathbf{A}\boldsymbol{\lambda} \leq \mathbf{b}$	$\boldsymbol{\mu}, \boldsymbol{\nu} \geq \mathbf{u} \geq \mathbf{0}\ \epsilon\mathbf{1}$
$\boldsymbol{\lambda}, \mathbf{s}^-, \mathbf{s}^+ \geq \mathbf{0}$	$\epsilon > 0$
$\epsilon > 0$	

Source: Halme et at. (1999)

Point $\mathbf{U}\lambda^* = \mathbf{u}^*$ is efficient iff there does not exist another $\mathbf{u} \in T, \mathbf{u} \geq \mathbf{u}^*, \mathbf{u} \neq \mathbf{u}^*$

Data Envelopment Analysis, DEA

- Estimate the efficient frontier by measuring the relative efficiencies of the comparable DMUs
- Efficiency scores $\frac{|\mathrm{DMU}_i|}{|\mathrm{DMU}_i^*|}$ as performance indicators of DMUs
- DEA is value free
- How can we incorporate the preference of DM?



Source: Halme et at. (1999)

Most Preferred Solution, MPS

- MPS is the best input/output vector in DMs opinion. Such a vector is a convex combination of the input/output vectors of the DMUs under consideration.
- MPS, **u***, is a solution that is preferred to any other solution in T.
- MPS is assumed to be efficient.

Value Efficiency Analysis, VEA

- DM has an unknown value function $v(\mathbf{u})$
- Assist the DM to evaluate the value of each vector $\mathbf{u} = \begin{bmatrix} y \\ -x \end{bmatrix} \in T$

Assumptions:

- DM is able to come up with MPS, $\mathbf{u}^*$
- v is strictly increasing (increasing in y and decreasing in x)
- v obtains its maximum value $v(\mathbf{u}^*)$ at $\mathbf{u}^*$
- v is pseudoconcave

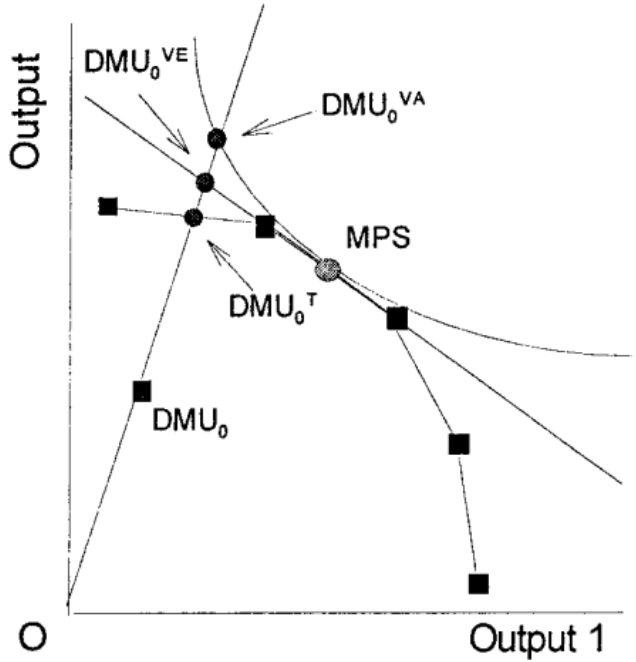
VEA visually

- DMU_0^{VA} not possible to find
- DMU_0^{VE} is the approximation
- DMU_0^{T} is what we got with DEA
- Value efficiency score:

$$\frac{|\mathrm{DMU}_0 - \mathrm{DMU}_0^{\mathrm{VE}}|}{|O - \mathrm{DMU}_0|}$$

- How can we find the approximation?

  -> Tangent hyperplane



Source: Halme et at. (1999)

Value Efficiency and weighted true Value Efficiency score

u is Value Efficient iff $v(\mathbf{u}) \geq v(\mathbf{u}^*)$, otherwise it is Value Inefficient.

Weighted true Value Efficiency score:

$$E_t^w(\mathbf{u}^0) = \gamma^t$$

If $\gamma > 0$, point $\mathbf{u}^0$ is truely Value Inefficient

If $\gamma = 0$, point $\mathbf{u}^0$ is Value Efficient

If $\gamma < 0$, point $\mathbf{u}^0$ is Value Superefficient

$$\sup \gamma$$

$$\text{s.t.}$$

$$\boldsymbol{u} - \gamma \boldsymbol{w} \geq \boldsymbol{u}^0,$$

$$\boldsymbol{u} \in V = \{\boldsymbol{u} | v(\boldsymbol{u}) \leq v(\boldsymbol{u}^*)\},$$

$$\boldsymbol{w} > \boldsymbol{0}.$$

Source: Halme et at. (1999)

Tangent Cones

X is a nonempty polytope, $x \in X$

D(x), Cone of feasible directions:

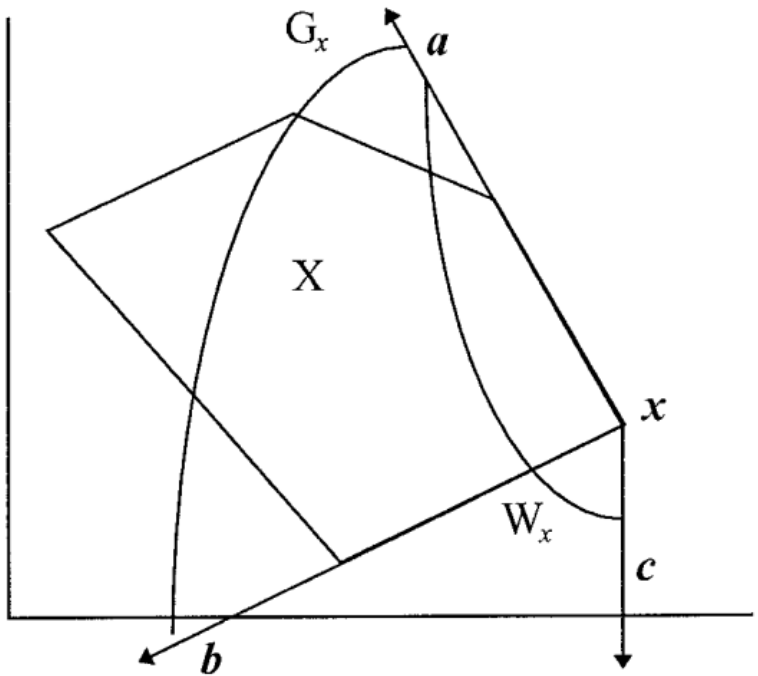
$$D(x) = \{d | x + \lambda d \in X \text{ for all } \lambda \in (0, \delta) \text{ for some } \delta > 0\}$$

G_x is tangent cone of X

$$G_x = \{y | y = x + d, d \in D(x)\}$$

W_x is augmented tangent cone of X

$$W_x = \{s | s = y + z, y \in G_x, z \in \mathbb{R}_-\}$$



Source: Halme et at. (1999)

Pseudoconcave function

f is pseudoconcave on a convex set *S* iff for all

$$x_1, x_2 \in S \text{ such that } \nabla f(x_1)^T (x_2 - x_1) \leq 0 \Rightarrow f(x_2) \leq f(x_1)$$

x* belongs to a non-empty polytope X. $\Xi(x^*)$ is the set of increasing pseudoconcave functions $\xi(x^*)$, which obtain their maximum in X at **x***.

Augmented tangent cone of X at **x*** is $W_{\mathbf{x}^*}$, Then

$$x \in W_{x^*}, \textit{iff } \xi(x) \leq \xi(x^*) \textit{ for all } \xi \in \Xi(x^*)$$

Value function approximation

When working with set T.
The set of all value functions $V = \{\mathbf{u} = \begin{bmatrix} y \\ -x \end{bmatrix} | v(\mathbf{u}) \leq v(\mathbf{u}^*)\}$

W_{u^*} is the largest cone with $W_{\mathbf{u}^*} \subset V = \{\mathbf{u} | v(\mathbf{u}) \leq v(\mathbf{u}^*)\}, \text{ for any } \Xi(\mathbf{u}^*)$

V is approximated by the cone W in such way that resulting scores are not greater than real ones.

$|\mathrm{DMU}_0 - \mathrm{DMU}_0^{\mathrm{VE}}| \leq |\mathrm{DMU}_0 - \mathrm{DMU}_0^{\mathrm{VA}}|$

Value Efficiency score

Value Efficiency score of point $\mathbf{u}^0$ is:

$$E^w(y^0, -x^0) = \sigma^s$$

If $\sigma^s = 0$ and s^-, $s^+ = 0$, $\mathbf{u}^0$ is Value Efficient, but not necessarily truly Value Efficient.

Increase output and decrease inputs by σ^s percentage to become Value Efficient.

$$\sigma^s = \frac{|\mathrm{DMU}_0 - \mathrm{DMU}_0^{\mathrm{VE}}|}{|O - \mathrm{DMU}_0|}$$

$$\max Z = \sigma + \epsilon(\mathbf{1}^T s^+ + \mathbf{1}^T s^-)$$

s.t.

$$\mathbf{Y}\boldsymbol{\lambda} - \sigma w^y - s^+ = y,$$

$$\mathbf{X}\boldsymbol{\lambda} + \sigma w^x + s^- = x,$$

$$\mathbf{A}\boldsymbol{\lambda} + \boldsymbol{\mu} = b,$$

$$s^-, s^+ \geq \mathbf{0},$$

$$\epsilon > 0 \text{ ("Non-Archimedean")},$$

Lift constraints corresponding to MPS

$$\lambda_j \geq 0, \quad \text{if } \lambda_j^* = 0, \; j = 1, 2, \ldots, n,$$

$$\mu_j \geq 0, \quad \text{if } \mu_j^* = 0, \; j = 1, 2, \ldots, k$$

where $\boldsymbol{\lambda}^* \in \Lambda$, $\boldsymbol{\mu}^*$ *correspond to the Most Preferred Solution:*

$$y^* = \mathbf{Y}\boldsymbol{\lambda}^*$$

$$x^* = \mathbf{X}\boldsymbol{\lambda}^*.$$

Source: Halme et at. (1999)

Example

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6
Output	1	4	7	9	12	8
Input	3	3	5	7	11	10

Problem, with BCC-model:

$$\max \lambda_1 + 4\lambda_2 + 7\lambda_3 + 9\lambda_4 + 12\lambda_5 + 8\lambda_6$$

$$\min 3\lambda_1 + 3\lambda_2 + 5\lambda_3 + 7\lambda_4 + 11\lambda_5 + 10\lambda_6$$

$$\text{s.t. } \mathbf{1}^T\boldsymbol{\lambda} = 1,$$

$$\boldsymbol{\lambda} \geq \mathbf{0}.$$

DM chooses MPS: (4, 5.5)

Value efficiency for DMU_5

$$\max \sigma + \epsilon(s^+ + s^-)$$

s.t.

$$\lambda_1 + 4\lambda_2 + 7\lambda_3 + 9\lambda_4 + 12\lambda_5 + 8\lambda_6 - 12\sigma - s^+ = 12$$

$$3\lambda_1 + 3\lambda_2 + 5\lambda_3 + 7\lambda_4 + 12\lambda_5 + 10\lambda_6 + 11\sigma - s^- = 11$$

$$\mathbf{1}^T\lambda = 1$$

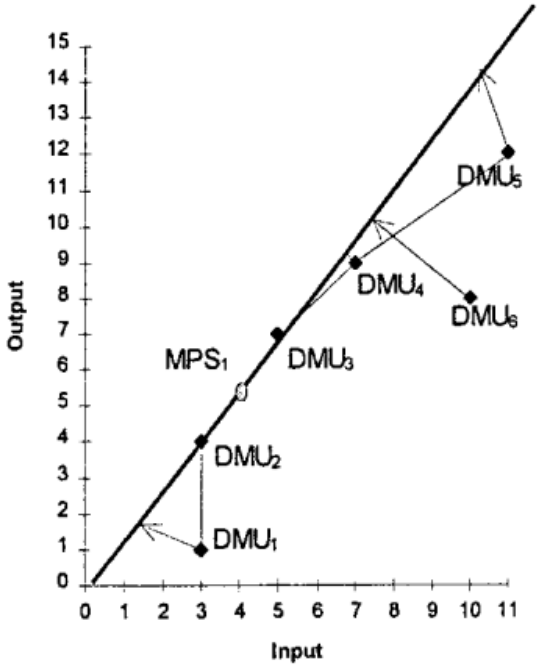
$$\lambda_1, \lambda_4, \lambda_5, \lambda_6 \geq 0, \ \lambda_2, \lambda_3 \text{ unrestricted}$$

General model

$$\boxed{\begin{aligned} \mathbf{Y}\boldsymbol{\lambda} - \sigma w^y - s^+ &= y, \\ \mathbf{X}\boldsymbol{\lambda} + \sigma w^x + s^- &= x, \\ \mathbf{A}\boldsymbol{\lambda} + \boldsymbol{\mu} &= b, \end{aligned}}$$

Source: Halme et at. (1999)

Example



Ineff. Units	Reference Units: DMU_1 λ_1	DMU_2 λ_2	DMU_3 λ_3	DMU_4 λ_4	DMU_5 λ_5	DMU_6 λ_6	Slacks: $s+$	$s-$	Efficiency Score σ
BCC-Model (Combined Score)									
DMU_1		1					3		0
DMU_6				0.77	0.23				0.21
Value Efficiency Model with MPS_1									
DMU_1		1.82	−0.82						0.55
DMU_4		−0.82	1.82						0.05
DMU_5		−2.23	3.23						0.14
DMU_6		−1.09	2.09						0.28

Source: Halme et at. (1999)

Application:
VEA of academic research

Application: VEA of academic research

- Helsinki School of Economics: 18 research units as DMUs
- Results-oriented approach for evaluating academic research

1. Definition of criteria and indicators
2. Collection of data
3. Calculate value efficiency scores for each unit

Research unit evaluation criteria and indicators

Quality of research

- Articles, books, citations

Research activity

- Conference papers and presentations

Impact of research

- Citations, invitations abroad, foreign co-authors

Activity in educating young scientists

- Doctoral students and degrees produced

~~**Activity in the scientific community**~~

- ~~Memberships in boards, expert servicies~~

Research unit indicators

- Values scaled to [0, 1], where 1 corresponds to best one.
- Relative weight of each indicator regard to each criterion
  - Average of 9 experts was used
- Weighted sum of the indicators was used as a scale for each criterion

Research units

outputs

input

Criterion values as the weighted sums of scaled indicators and resources

	Ref. Art.	Books	Citat	C_1	Art + books	Other	Conf Pres	C_2	Citat	Invit. Pres	For. Co-A	C_3	Dis.	Sup.	C_4	1000 Fmk/ month
Years:	92–96	92–96	90–95		92–96	94–96	95–96		90–95	95–96	92–96		92–96	92–96		
Weights:	0.55	0.27	0.18		0.60	0.23	0.17		0.48	0.27	0.25		0.65	0.35		
A	0.70	1.00	0.10	**67**	1.00	1.00	1.00	**100**	0.10	1.00	0.67	**48**	1.00	1.00	**100**	70
B	0.67	0.03	0.05	**38**	0.36	0.38	0.33	**36**	0.50	0.50	0.67	**32**	0.30	0.15	**25**	32
C	0.03	0.13	0.00	**5**	0.10	0.08	0.00	**8**	0.00	0.00	0.00	**0**	0.13	0.00	**9**	34
D	0.30	0.16	0.01	**21**	0.25	0.26	0.12	**23**	0.01	0.05	0.00	**2**	0.52	0.04	**35**	101
E	0.90	0.13	0.06	**54**	0.54	0.13	0.08	**37**	0.06	0.00	0.25	**9**	0.04	0.08	**6**	25
F	0.20	0.95	0.38	**43**	0.71	0.77	0.23	**64**	0.38	0.23	0.33	**33**	0.70	0.27	**55**	64
G	0.50	0.24	0.06	**35**	0.41	0.50	0.35	**42**	0.06	0.41	0.17	**18**	0.83	0.31	**65**	46
H	0.00	0.05	0.02	**2**	0.03	0.14	0.00	**5**	0.02	0.00	0.00	**1**	0.30	0.08	**23**	25
I	0.30	0.24	0.24	**27**	0.31	0.47	0.40	**36**	0.24	0.14	0.17	**19**	0.26	0.23	**25**	28
J	0.47	0.32	0.00	**34**	0.44	0.41	0.00	**36**	0.00	0.00	0.00	**0**	0.17	0.00	**11**	23
K	0.07	0.13	0.00	**7**	0.12	0.03	0.00	**8**	0.00	0.00	0.00	**0**	0.09	0.04	**7**	7
L	0.40	0.45	0.42	**42**	0.49	0.30	0.50	**45**	0.42	0.41	0.17	**35**	0.70	0.15	**51**	68
M	0.00	0.16	0.01	**4**	0.10	0.02	0.02	**7**	0.01	0.05	0.00	**2**	0.00	0.00	**0**	8
N	0.27	0.05	0.13	**18**	0.17	0.02	0.23	**15**	0.13	0.45	0.33	**27**	0.00	0.08	**3**	15
O	0.43	0.00	0.08	**25**	0.22	0.17	0.23	**21**	0.08	0.45	0.33	**25**	0.30	0.15	**25**	37
P	0.90	0.26	0.92	**74**	0.63	0.18	0.33	**47**	0.92	0.55	0.92	**82**	0.00	0.15	**5**	29
Q	0.13	0.74	0.01	**27**	0.54	0.37	0.62	**51**	0.01	0.73	0.00	**20**	0.00	0.00	**0**	12
R	1.00	0.11	1.00	**76**	0.58	0.58	0.44	**55**	1.00	0.05	1.00	**74**	0.00	0.08	**3**	119
$\sum$				**602**				**636**				**427**			**447**	

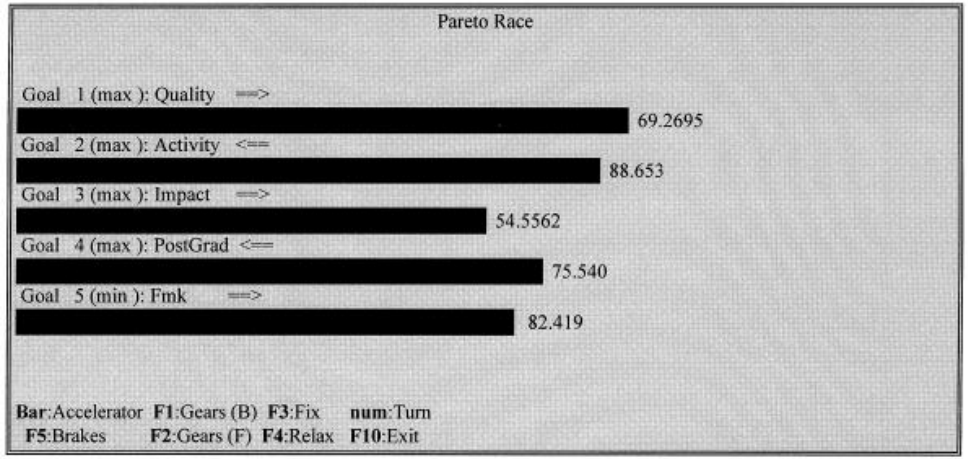
Source: Korhonen et al. (2001)

Most preferred solution

- Pareto Race was used to compute and search efficient frontier for MPS
- Convex combination of A (0,748) and R (0,252) was selected as MPS

A multiple objective linear programming model for finding the MPS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R		
Quality	67	38	5	21	54	43	35	2	27	34	7	42	4	18	25	74	27	76	→	max
Activity	100	36	8	23	37	64	42	5	36	36	8	45	7	15	21	47	51	55	→	max
Impact	48	32	0	2	9	33	18	1	19	0	0	35	2	27	25	82	20	74	→	max
Post-grad	100	25	9	35	6	55	65	23	25	11	7	51	0	3	25	5	0	3	→	max
1000 Fmk	70	32	34	101	25	64	46	25	28	23	7	68	8	15	37	29	12	119	→	min
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	≤	1



Source: Korhonen et al. (2001)

Value efficiency scores with output oriented BCC-model

- Efficiency of a DMU is determined by maximizing outputs subject to given input levels

$$\max \quad Z_0 = \theta + \varepsilon(\mathbf{1}^{\mathrm{T}}\mathbf{s}^{+} + \mathbf{1}^{\mathrm{T}}\mathbf{s}^{-})$$

s.t.

$$\boxed{\begin{aligned} &\mathbf{Y}\boldsymbol{\lambda} - \theta\mathbf{y}_0 - \mathbf{s}^{+} = \mathbf{0}, \\ &\mathbf{X}\boldsymbol{\lambda} + \mathbf{s}^{-} = \mathbf{x}_0, \\ &\mathbf{1}'\boldsymbol{\lambda} + z \leqslant 1, \end{aligned}}$$

$$\mathbf{s}^{-}, \mathbf{s}^{+}, \geqslant \mathbf{0},$$

$$\varepsilon > 0 \quad (\text{“Non-Archimedean”}),$$

$$\lambda_j \geqslant 0 \quad \text{if } \lambda_j^{*} = 0, \; j = 1, 2, \ldots, n,$$

$$z \geqslant 0 \quad \text{if } z^{*} = 0.$$

where $\boldsymbol{\lambda}^{*} \in \Lambda$, z^{*} correspond to the most preferred solution

$$\mathbf{y}^{*} = \mathbf{Y}\boldsymbol{\lambda}^{*},$$
$$\mathbf{x}^{*} = \mathbf{X}\boldsymbol{\lambda}^{*}.$$

General model

$$\boxed{\begin{aligned} \mathbf{Y}\boldsymbol{\lambda} - \sigma w^{y} - s^{+} &= y, \\ \mathbf{X}\boldsymbol{\lambda} + \sigma w^{x} + s^{-} &= x, \\ \mathbf{A}\boldsymbol{\lambda} + \boldsymbol{\mu} &= b, \end{aligned}}$$

Results and efficient units

Value efficiency analysis with an output-oriented model

Depts.	BCC-efficiency					BCC-value efficiency				
	Efficiency	A	P	Q	R	Efficiency	A	P	Q	R
A	1.00	1.00				1.00	1.00			
B	0.79	0.30	0.39			0.55	0.43	0.73		−0.16
C	0.17	0.49				0.08	1.04	0.38		−0.42
D	0.35	1.00				0.32	1.12			−0.12
E	0.88	0.02	0.81	0.03		0.75	0.05	1.01		−0.07
F	0.68	0.89	0.05	0.06		0.65	1.02	0.32		−0.34
G	0.98	0.66				0.54	1.19	0.16		−0.35
H	0.64	0.35				0.06	3.76	4.73		−7.49
I	0.77	0.32	0.13	0.17		0.41	0.85	0.56		−0.40
J	0.77	0.13	0.37	0.32		0.49	0.53	0.77		−0.30
K	0.86	0.08	0.04			0.11	0.63	0.91		−0.54
L	0.64	0.79	0.21			0.62	0.81	0.50		−0.31
M	0.23	0.00	0.07	0.52		0.07	1.12	0.63		−0.75
N	0.67	0.01	0.48			0.30	0.50	3.61		−3.11
O	0.58	0.42	0.27			0.38	0.64	0.95		−0.59
P	1.00		1.00			1.00		1.00		
Q	1.00			1.00		0.45	1.49	1.06		−1.55
R	1.00				1.00	1.00				1.00

Source: Korhonen et al. (2001)

Summary

- VEA extends DEA to incorporate DMs preferences
- These preferences are modelled via MPS
- Optimistic efficiency scores as relative difference between MPS and DMU
- Efficiency can be calculated with different models (CCR, BCC, input oriented, output oriented)
  - Results differ!
- Value efficincy frontier covers the efficiency frontier

References

1. Halme, M., Joro, T., Korhonen, P., Wallenius, J., 1999: A Value Efficiency Approach to Incorporating Preference Information in Data Envelopment Analysis, Management Science 45/1, s. 103-115.
2. Korhonen, P., Tainio, R., Wallenius, J., 2001: Value effciency analysis of academic research, European Journal of Operational Research 130/1, s. 121-132.

Homework

1. Explain why reseach unit H has such a drop in efficiency when moving from DEA to VEA (slide 25)?

2. Give examples of situations (one for a, one for b) where

a) It's better to use VEA than DEA

b) It's better to use DEA than VEA

Give also a brief explanation why.

Email answers to matti.s.staudinger@aalto.fi by 19.11. 9am.