MS-E2191 HW5 - Solution

Theory

As presented on the slides (slide 18), dominance is established by solving the following minimization and maximization problems pairwise for all pairs x^k and x^l:

$$\min_{w \in S} V(x^k) - V(x^l) \tag{1}$$

$$\max_{w \in S} V(x^k) - V(x^l) \tag{2}$$

$$\text{s.t. } Aw \leq 0, \tag{3}$$

$$\sum_{i=1}^{n} w_i = 1, \tag{4}$$

$$w_i \geq 0, \text{ for all } i = 1, \ldots, n. \tag{5}$$

The value difference can be written as

$$V(x^k) - V(x^l) = \sum_{i=1}^{n} w_i v_i(x_i^k) - \sum_{i=1}^{n} w_i v_i(x_i^l)) = \sum_{i=1}^{n} w_i (v_i(x_i^k) - v_i(x_i^l)), \tag{6}$$

and the matrix A contains the linear constraints on the attribute weights.
If $\min_{w \in S}[V(x^k) - V(x^l)] \geq 0$ and $\max_{w \in S}[V(x^k) - V(x^l)] > 0$, then x^k dominates x^l.

For the *maximax* decision rule, $\max_{w \in S} V(x^j)$ is calculated for each of the non-dominated alternatives, and the alternative with the highest maximum value is selected.

Answers

1. Based on dominance relations, we find out that alternatives C, E, F and H are non-dominated.
2. The maximum values for the non-dominated alternatives are presented in Table **??** table. Based on these values, our recommendation would be alternative H.

Alternative	C	E	F	H
Maximum value	0.50	0.56	0.50	0.58

Table 1: Maximum values of the non-dominated alternatives.

MATLAB code

```
labels = ["A", "B", "C", "D", "E", "F", "G", "H"];

% Scores
v = [0, 0.5, 0.3;
     0.4, 0.7, 0.1;
     1, 0.1, 0.3;
     0.8, 0, 0.5;
     1, 0.4, 0;
     0.2, 0.6, 0.6;
     0, 0.2, 1;
     0.2, 1, 0.1];

% Constraints
A = [1, -1, 0;
     -1.5, 1, 0;
     0, -1, 1;
     0.5, 0, -1];
b = zeros(4,1);
Aeq = [1, 1, 1];
beq = [1];
lb = [0; 0; 0];
ub = [1; 1; 1];

n = 3; % number of attributes
m = 8; % number of alternatives
minValDiff = zeros(m,m); % minimum value differences
maxValDiff = zeros(m,m); % maximum value differences
maxima = zeros(m,1);
dominance = zeros(m,m);

options = optimoptions('linprog', 'display', 'none');

for i=1:m
    [w, fval] = linprog(-v(i,:)', A, b, Aeq, beq,lb,ub,options);
    maxima(i) = -fval;

    for j=1+i:m
        [w, fval] = linprog((v(i,:)-v(j,:))', A, b, Aeq, beq,lb,ub,options);
        minValDiff(i,j) = fval;
        [w, fval] = linprog((v(j,:)-v(i,:))', A, b, Aeq, beq,lb,ub,options);
        maxValDiff(i,j) = -fval;
        minValDiff(j,i) = -maxValDiff(i,j);
        maxValDiff(j,i) = -minValDiff(i,j);

        % Print if xi dominates xj or vice versa
        if (minValDiff(i,j) >= 0 && maxValDiff(i,j) > 0)
            dominance(i,j) = 1;
            fprintf("%s dominates %s\n", labels(i), labels(j));
        elseif (minValDiff(j,i) >= 0 && maxValDiff(j,i) > 0)
            dominance(j,i) = 1;
            fprintf("%s dominates %s\n", labels(j), labels(i));
        end
    end
end
```