MS-E2191 Homework 9 solution

1. When should you use the additive-multilinear portfolio value function instead of the additive-linear one?

Additive-linear portfolio value function can't be used when adding a project into the portfolio doesn't result in the same value increase independent of the portfolio composition. So when assumption 4 doesn't hold.

Additive-multilinear portfolio value function can be used when changes in criterion-specific performance of a project remain equally preferred when other project's performances are fixed, so when assumption 5 holds but 4 doesn't.

2. Calculate criterion-specific V_i

a) Additive-linear

J={A}: $V_1(x_{J1}) = w_1 v_1(x_{A1}) = 0.1 \times 0.3 = 0.03$

J={B,C}: $V_1(x_{J1}) = w_1(v_1(x_{B1}) + v_1(x_{C1})) = 0.1 \times (0.8 + 0.5) = 0.13$

J={A,E}: $V_1(x_{J1}) = w_1(v_1(x_{A1}) + v_1(x_{E1})) = 0.1 \times (0.3 + 1) = 0.13$

J={A,B,C}: $V_1(x_{J1}) = w_1(v_1(x_{A1}) + v_1(x_{B1}) + v_1(x_{C1})) = 0.1 \times (0.3 + 0.8 + 0.5) = 0.16$

b) Additive-multilinear

J={A}: $V_1(x_{J1}) = w_1(1)v_1(x_{A1}) = 0.1 \times 0.3 = 0.03$

J={B,C}: $V_1(x_{J1}) = w_1(1)v_1(x_{B1})(1 - v_1(x_{C1})) + w_1(1)v_1(x_{C1})(1 - v_1(x_{B1})) + w_1(2)v_1(x_{B1}))v_1(x_{C1}) = 0.1{\times}0.8{\times}(1{-}0.5){+}0.1{\times}0.5{\times}(1{-}0.8){+}0.5{\times}0.8{\times}0.5 = 0.25$

J={A,E}: $V_1(x_{J1}) = w_1(1)v_1(x_{A1})(1 - v_1(x_{E1})) + w_1(1)v_1(x_{E1}))(1 - v_1(x_{A1}) + w_1(2)v_1(x_{A1}))v_1(x_{E1}) = 0.1{\times}0.3{\times}(1{-}1){+}0.1{\times}1{\times}(1{-}0.3){+}0.5{\times}1{\times}0.3 = 0.22$

$$
\begin{aligned}
&J = \{A, B, C\} : V_1(x_{J1}) = \\
&\quad w_1(1)v_1(x_{A1})(1 - v_1(x_{B1}))(1 - v_1(x_{C1})) \\
&\quad + w_1(1)v_1(x_{B1})(1 - v_1(x_{A1}))(1 - v_1(x_{C1})) \\
&\quad + w_1(1)v_1(x_{C1}))(1 - v_1(x_{A1}))(1 - v_1(x_{B1})) \\
&\quad + w_1(2)v_1(x_{A1}))v_1(x_{B1})(1 - v_1(x_{C1})) \\
&\quad + w_1(2)v_1(x_{A1}))v_1(x_{C1})(1 - v_1(x_{B1})) \\
&\quad + w_1(2)v_1(x_{B1}))v_1(x_{C1})(1 - v_1(x_{A1})) \\
&\quad + w_1(3)v_1(x_{A1}))v_1(x_{B1})v_1(x_{C1}) \\
&\quad = ... = 0.319
\end{aligned}
$$

c) What differences do you see?

With this choice of weighting function $w_1(1), w_1(2), w_1(3)$, both criterion-specific value functions give the highest value for portfolio $\{A, B, C\}$ and the lowest value for portfolio $\{A\}$. Additive-linear value function gives the same value for $\{B, C\}$ and $\{A, E\}$ although the additive-multilinear value function values $\{B, C\}$ higher. For a portfolio with only one project, the values are the same.