



Aalto University
School of Electrical
Engineering

CS-C3240 – Machine Learning D

Anomaly detection, Recommender Systems, Online learning

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Learning goals

- Collaborative Filtering
- Recommender Systems
- Batch gradient descent
- Online learning

Outline

Recommender systems

Stochastic Classification

Online learning

Elektronik



Alle Ergebnisse in Elektronik anzeigen

Computer & Zubehör



Alle Ergebnisse in Computer & Zubehör anzeigen

Musik

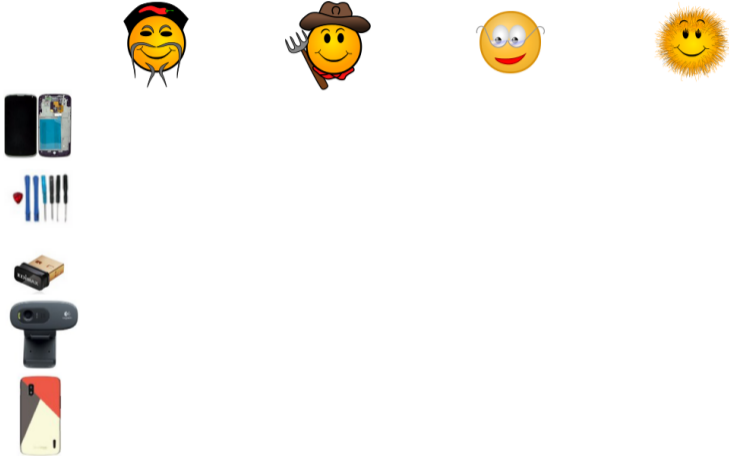


Alle Ergebnisse in Musik anzeigen

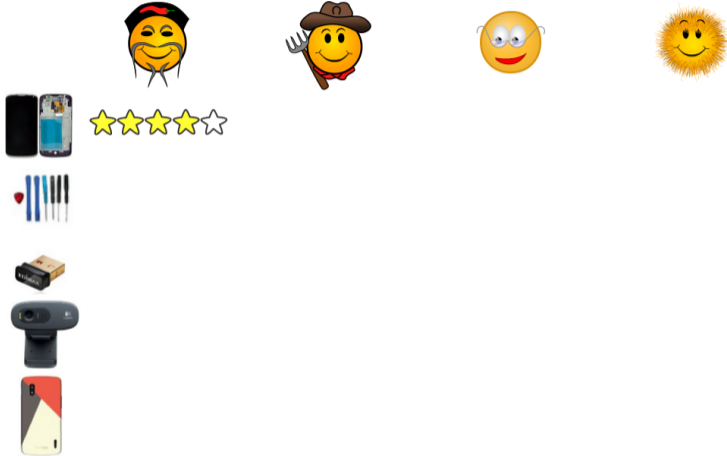
Bücher



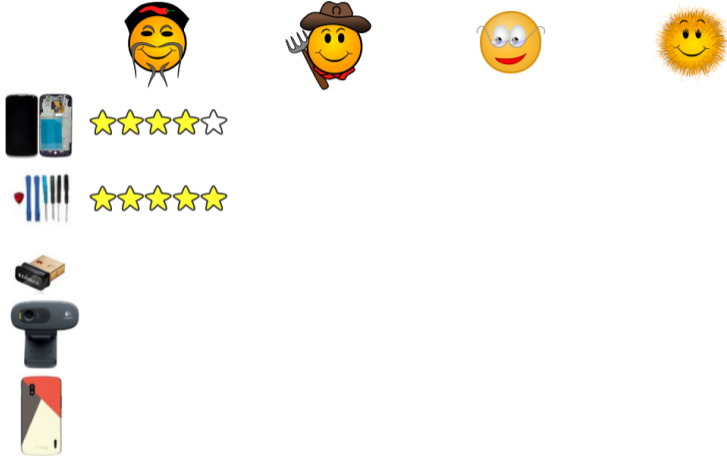
Recommender systems



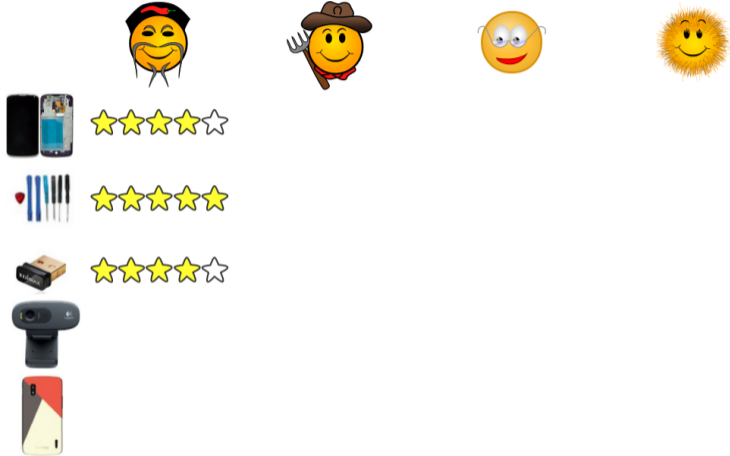
Recommender systems



Recommender systems



Recommender systems













Recommender systems



Recommender systems




Recommender systems










				
	☆☆☆☆☆	☆☆☆☆☆	?	☆☆☆☆☆
	☆☆☆☆☆	☆☆☆☆☆	?	☆☆☆☆☆
	☆☆☆☆☆	?	☆☆☆☆☆	?
	?	☆☆☆☆☆	☆☆☆☆☆	☆☆☆☆☆
	☆☆☆☆☆	☆☆☆☆☆	☆☆☆☆☆	?

Recommender systems

Task of Recommender systems

Given these ratings for a number of products, predict likely user-ratings for products that have not yet been rated

				
	★★★★☆	★★★★★	?	☆☆☆☆
	★★★★★	★★★★★	?	☆☆☆☆
	★★★★☆	?	☆☆☆☆	?
	?	☆☆☆☆	★★★★★	★★★★★
	☆☆☆☆	☆☆☆☆	★★★★★	?

					tools	accessories
	★★★★☆	★★★★★	?	☆☆☆☆☆	0.9	0.0
	★★★★★	★★★★★	?	★☆☆☆☆	1.0	0.3
	★★★★☆	?	★★☆☆☆	?	0.4	0.8
	?	☆☆☆☆☆	★★★★★	★★★★★	0.2	0.9
	★☆☆☆☆	★☆☆☆☆	★★★★★	?	0.0	1.0



tools

accessories

0.9

0.0

1.0

0.3

0.4

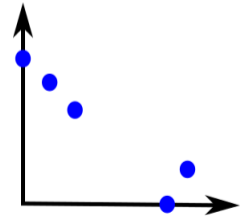
0.8

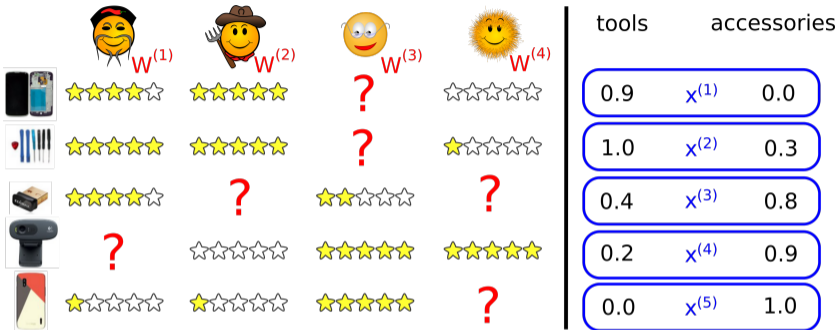
0.2










0.9

0.0

1.0



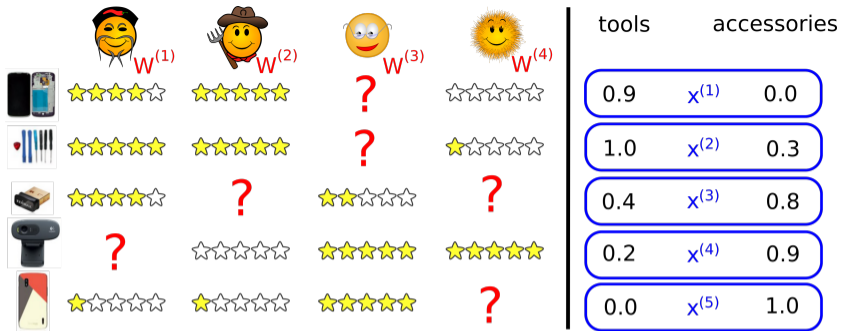


	 $w^{(1)}$	 $w^{(2)}$	 $w^{(3)}$	 $w^{(4)}$	tools	accessories
	★★★★☆	★★★★★	?	☆☆☆☆☆	0.9	$x^{(1)}$ 0.0
	★★★★★	★★★★★	?	★☆☆☆☆	1.0	$x^{(2)}$ 0.3
	★★★★☆	?	★★☆☆☆	?	0.4	$x^{(3)}$ 0.8
	?	☆☆☆☆☆	★★★★★	★★★★★	0.2	$x^{(4)}$ 0.9
	★☆☆☆☆	★☆☆☆☆	★★★★★	?	0.0	$x^{(5)}$ 1.0

Represent items in terms of weighted feature vectors

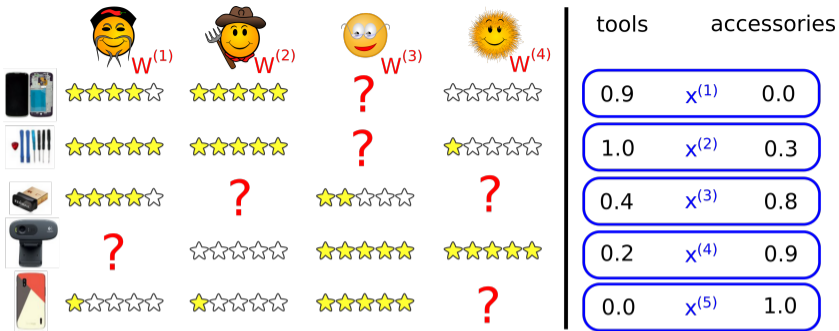
$$x^{(4)} = \begin{bmatrix} 0.2 \\ 0.9 \end{bmatrix} \quad w^{(1)} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$h(x) \Rightarrow (w^{(1)})^T x^{(4)} = 0.2 \cdot 5 + 0.9 \cdot 1 = 1,9$$



Learn weights from provided ratings for single user j (Logistic regression):

$$\min_{w^{(j)}} \frac{1}{2} \sum_{i: y^{(i,j)} \neq ?} \left(\left(w^{(j)} \right)^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{k=1}^m \left(w_k^{(j)} \right)^2$$



Learn weights from provided ratings for all users $1, \dots, N$:

$$\min_{w^{(1)}, \dots, w^{(N)}} \frac{1}{2} \sum_{j=1}^N \sum_{i: y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^N \sum_{k=1}^m (w_k^{(j)})^2$$

Recommender systems

Optimisation algorithm

$$\min_{w^{(1)}, \dots, w^{(N)}} \frac{1}{2} \sum_{j=1}^N \sum_{i: y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^N \sum_{k=1}^m (w_k^{(j)})^2$$

Recommender systems

Optimisation algorithm

$$\min_{w^{(1)}, \dots, w^{(N)}} \frac{1}{2} \sum_{j=1}^N \sum_{i: y^{(i,j)} \neq ?} \left((w^{(j)})^T \mathbf{x}^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^N \sum_{k=1}^m (w_k^{(j)})^2$$

Gradient descent update:

$$w_k^{(j)} = w_k^{(j)} - \delta \left(\sum_{i: y^{(i,j)} \neq ?} \left((w^{(j)})^T \mathbf{x}^{(i)} - y^{(i,j)} \right) \mathbf{x}_k^{(i)} + \lambda w_k^{(j)} \right)$$

Recommender systems

Optimisation algorithm

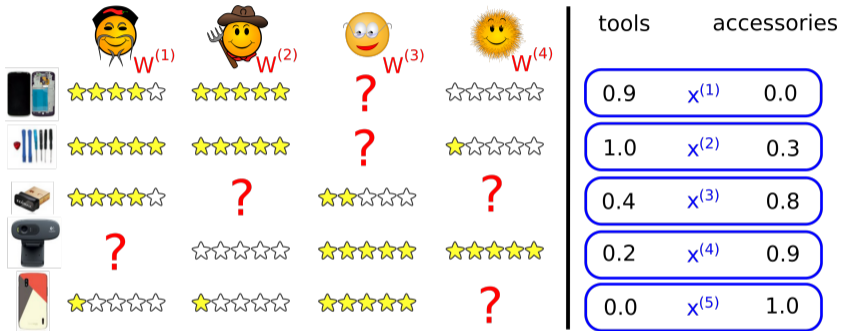
$$\min_{w^{(1)}, \dots, w^{(N)}} \frac{1}{2} \sum_{j=1}^N \sum_{i: y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^N \sum_{k=1}^m (w_k^{(j)})^2$$

Gradient descent update:

$$w_k^{(j)} = w_k^{(j)} - \delta \left(\underbrace{\sum_{i: y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right) x_k^{(i)}}_{\text{partial derivative}} + \lambda w_k^{(j)} \right)$$

Recommender systems

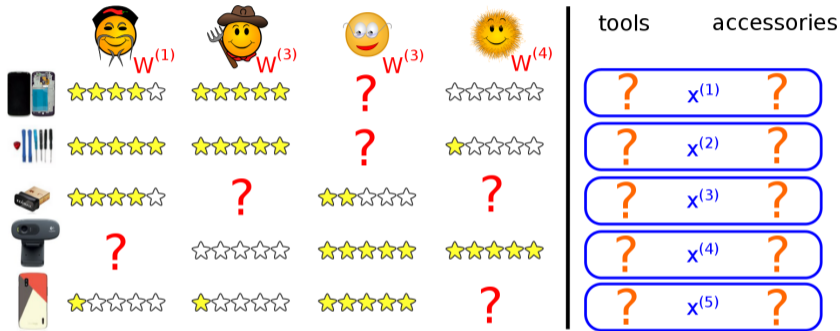
Collaborative filtering



We are able to calculate the weights given the feature vectors

Recommender systems

Collaborative filtering



We are able to calculate the weights given the feature vectors









→ But how do we obtain these feature vectors?

Recommender systems

Collaborative filtering

Users might tell their preferences

e.g. more interested in tools or accessories

	 $w^{(1)}$	 $w^{(2)}$	 $w^{(3)}$	 $w^{(4)}$	tools	accessories	
	☆☆☆☆☆	☆☆☆☆☆	?	☆☆☆☆☆	?	$x^{(1)}$?
	☆☆☆☆☆	☆☆☆☆☆	?	☆☆☆☆☆	?	$x^{(2)}$?
	☆☆☆☆☆	?	☆☆☆☆☆	?	?	$x^{(3)}$?
	?	☆☆☆☆☆	☆☆☆☆☆	☆☆☆☆☆	?	$x^{(4)}$?
	☆☆☆☆☆	☆☆☆☆☆	☆☆☆☆☆	?	?	$x^{(5)}$?







$$w^{(1)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad w^{(2)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad w^{(3)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \quad w^{(4)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

Recommender systems

Collaborative filtering

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	 $w^{(1)}$	 $w^{(2)}$	 $w^{(3)}$	 $w^{(4)}$	tools	accessories
	★★★★☆	★★★★★	?	☆☆☆☆☆	? $x^{(1)}$?	? $x^{(1)}$?
	★★★★★	★★★★★	?	☆☆☆☆☆	? $x^{(2)}$?	? $x^{(2)}$?
	★★★★★	?	☆☆☆☆☆	?	? $x^{(3)}$?	? $x^{(3)}$?
	?	☆☆☆☆☆	★★★★★	★★★★★	? $x^{(4)}$?	? $x^{(4)}$?
	☆☆☆☆☆	☆☆☆☆☆	★★★★★	?	? $x^{(5)}$?	? $x^{(5)}$?

$$w^{(1)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad w^{(2)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad w^{(3)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \quad w^{(4)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\left(w^{(1)}\right)^T x^{(1)} \approx 4; \quad \left(w^{(2)}\right)^T x^{(1)} \approx 5;$$

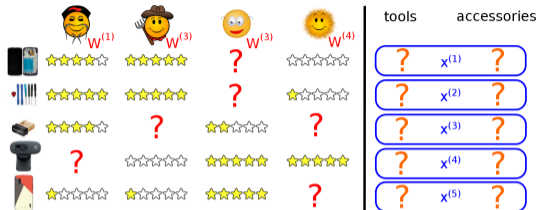
$$\left(w^{(3)}\right)^T x^{(1)} \approx ?; \quad \left(w^{(4)}\right)^T x^{(1)} \approx 0$$

Recommender systems

Collaborative filtering

Users might tell their preferences

e.g. more interested in tools or accessories



$$w^{(1)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad w^{(2)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad w^{(3)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \quad w^{(4)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\left(w^{(1)}\right)^T x^{(1)} \approx 4; \quad \left(w^{(2)}\right)^T x^{(1)} \approx 5;$$

$$\left(w^{(3)}\right)^T x^{(1)} \approx ?; \quad \left(w^{(4)}\right)^T x^{(1)} \approx 0$$

From these weights we can estimate the feature values

Recommender systems

Collaborative filtering

Optimisation algorithm

Given the weights/preferences $w^{(1)}, \dots, w^{(N)}$, we are able to infer a feature $x^{(i)}$

$$\min_{x^{(i)}} \frac{1}{2} \sum_{j: y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{k=1}^m \left(x_k^{(i)} \right)^2$$

Recommender systems

Collaborative filtering

Optimisation algorithm

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Given the weights/preferences $w^{(1)}, \dots, w^{(N)}$, we are able to infer $x^{(1)}, \dots, x^{(n)}$

$$\min_{x^{(1)}, \dots, x^{(n)}} \frac{1}{2} \sum_{i=1}^n \sum_{j: y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{i=1}^n \sum_{k=1}^m (x_k^{(i)})^2$$

Recommender systems

Collaborative filtering

Naive (iterative) Collaborative filtering algorithm

Given $x^{(1)}, \dots, x^{(n)}$, we are able to estimate $w^{(1)}, \dots, w^{(N)}$

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Recommender systems

Collaborative filtering

Naive (iterative) Collaborative filtering algorithm

Given $x^{(1)}, \dots, x^{(n)}$, we are able to estimate $w^{(1)}, \dots, w^{(N)}$

Given $w^{(1)}, \dots, w^{(N)}$, we are able to estimate $x^{(1)}, \dots, x^{(n)}$

Collaborative filtering (naive)

Init: Randomly initialise the $w^{(i)}$

- Repeat:**
- Estimate the $x^{(i)}$ from the $w^{(i)}$
 - Estimate the $w^{(i)}$ from the $x^{(i)}$

Recommender systems

Collaborative filtering

Naive (iterative) Collaborative filtering algorithm

Given $x^{(1)}, \dots, x^{(n)}$, we are able to estimate $w^{(1)}, \dots, w^{(N)}$

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Collaborative filtering (naive)

Init: Randomly initialise the $w^{(i)}$

Repeat:

- Estimate the $x^{(i)}$ from the $w^{(i)}$
- Estimate the $w^{(i)}$ from the $x^{(i)}$

- CF iteratively improves the estimates for $x^{(i)}$ and $w^{(i)}$
- Algorithm collaborates with users: by providing some information about their preferences, it computes and improves the features

Recommender systems

Collaborative filtering

$$\min_{w^{(1)}, \dots, w^{(N)}} \frac{1}{2} \sum_{j=1}^N \sum_{i: y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^N \sum_{k=1}^m \left(w_k^{(j)} \right)^2$$

$$\min_{x^{(1)}, \dots, x^{(n)}} \frac{1}{2} \sum_{i=1}^n \sum_{j: y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{i=1}^n \sum_{k=1}^m \left(x_k^{(i)} \right)^2$$

Recommender systems

Collaborative filtering

$$\min_{w^{(1)}, \dots, w^{(N)}} \frac{1}{2} \sum_{j=1}^N \sum_{i: y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^N \sum_{k=1}^m \left(w_k^{(j)} \right)^2$$

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Recommender systems

Collaborative filtering

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Minimize $w^{(i)}$ and $x^{(i)}$ simultaneously:

$$\min_{x^{(1)}, \dots, x^{(n)}, w^{(1)}, \dots, w^{(N)}} \frac{1}{2} \sum_{i,j: y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{i=1}^n \sum_{k=1}^m \left(x_k^{(i)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^N \sum_{k=1}^m \left(w_k^{(j)} \right)^2$$

Recommender systems

Init: Randomly initialise the $w^{(j)}$ and $x^{(i)}$

Optimisation: Simultaneously minimise the above function for $w^{(j)}$ and $x^{(i)}$

Gradient descent:

$$x_k^{(i)} = x_k^{(i)} - \delta \left(\sum_{j=y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right) w_k^{(j)} + \lambda x_k^{(i)} \right)$$
$$w_k^{(j)} = w_k^{(j)} - \delta \left(\sum_{i=y^{(i,j)} \neq ?} \left((w^{(j)})^T x^{(i)} - y^{(i,j)} \right) x_k^{(i)} + \lambda w_k^{(j)} \right)$$

Prediction: For a user i with parameters $w^{(j)}$ and an item with learned features x , estimate a rating of $(w^{(j)})^T x$

Outline

Recommender systems

Stochastic Classification

Online learning

Stochastic Classification

Model training becomes slow with increasing data size

Stochastic Classification

Model training becomes slow with increasing data size

- Because of repeated looping over the complete data set until convergence

Stochastic Classification

Model training becomes slow with increasing data size

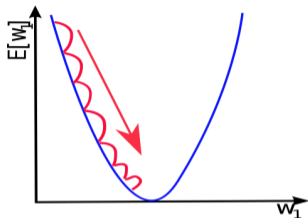
→ Because of repeated looping over the complete data set until convergence

Solution

- Randomly iterate the update only over a subset of items instead of repeatedly considering the whole data set.

Stochastic Classification

Example: Gradient descent



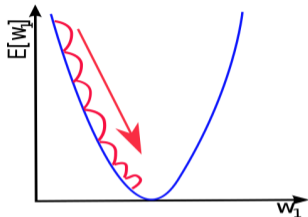
$$\text{minimize } L[(\mathcal{X}, \mathcal{Y}), h(\cdot)] = \frac{1}{2n} \sum_{i=1}^n \left(w^T x^{(i)} - y^{(i)} \right)^2$$

$$\text{Repeat } \forall j : w_j = w_j - \delta \cdot \frac{\partial L[(\mathcal{X}, \mathcal{Y}), h(\cdot)]}{\partial w_j}$$

$$\rightarrow \forall j : w_j = w_j - \delta \cdot \frac{1}{n} \sum_{i=1}^n \left(w_j x_j^{(i)} - y^{(i)} \right) \cdot x_j^{(i)}$$

Stochastic Classification

Example: Gradient descent



→ For single gradient descent-step, algorithms loops over all samples!

$$\text{minimize } L[(\mathcal{X}, \mathcal{Y}), h(\cdot)] = \frac{1}{2n} \sum_{i=1}^n \left(w^T x^{(i)} - y^{(i)} \right)^2$$

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Stochastic Classification

Example: Gradient descent

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$$\text{minimize } L[(\mathcal{X}, \mathcal{Y}), h(\cdot)] = \frac{1}{2n} \sum_{i=1}^n \left(w^T x^{(i)} - y^{(i)} \right)^2$$

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Stochastic Classification

Example: Gradient descent

→ For a single gradient descent-step, the algorithm loops over all samples!

To speed up the algorithm, compute gradient descent updates from individual training samples (randomly ordered)

$$\text{minimize } L[(\mathcal{X}, \mathcal{Y}), h(\cdot)] = \frac{1}{2n} \sum_{i=1}^n \left(w^T x^{(i)} - y^{(i)} \right)^2$$

$$\text{Repeat } \forall j : w_j = w_j - \delta \cdot \frac{\partial L[(\mathcal{X}, \mathcal{Y}), h(\cdot)]}{\partial w_j}$$

$$\rightarrow \forall j : w_j = w_j - \delta \cdot \frac{1}{n} \sum_{i=1}^n \left(w_j x_j^{(i)} - y^{(i)} \right) \cdot x_j^{(i)}$$

Stochastic Classification

Example: Gradient descent

Standard:

$$\text{minimize } L[(\mathcal{X}, \mathcal{Y}), h(\cdot)] = \frac{1}{2n} \sum_{i=1}^n \left(w^T x^{(i)} - y^{(i)} \right)^2$$

$$\text{Repeat } \forall j : w_j = w_j - \delta \cdot \frac{\partial}{\partial w_j} E[w_j]$$

$$\rightarrow \forall j : w_j = w_j - \delta \cdot \frac{1}{n} \sum_{i=1}^n \left(w_j x_j^{(i)} - y^{(i)} \right) \cdot x_j^{(i)}$$

Stochastic Classification

Example: Gradient descent

Standard:

$$\text{minimize } E[W] = \frac{1}{2n} \sum_{i=1}^n \left(w^T x^{(i)} - y^{(i)} \right)^2$$

$$\text{Repeat } \forall j : w_j = w_j - \delta \cdot \frac{\partial}{\partial w_j} E[w_j]$$

$$\rightarrow \forall j : w_j = w_j - \delta \cdot \frac{1}{n} \sum_{i=1}^n \left(w_j x_j^{(i)} - y^{(i)} \right) \cdot x_j^{(i)}$$

Stochastic:

Repeat over all training examples i (random order):

$$\Rightarrow \forall j : w_j = w_j - \delta \cdot \left(w_j x_j^{(i)} - y^{(i)} \right) \cdot x_j^{(i)}$$

Stochastic Classification

The stochastic implementation will generally move the parameters towards the global minimum (...but not always!)

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Greatly speeds up the gradient descent steps as it does not loop over all samples in each single iteration

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Tradeoff use $1 \leq k \leq n$ random examples for each gradient descent update

$l = 1, 1 + k, 1 + 2k, \dots$

$$\Rightarrow \forall j : \mathbf{w}_j = \mathbf{w}_j - \delta \cdot \frac{1}{l} \sum_{l=i}^{i+k} \left(\mathbf{w}_j x_j^{(i)} - y^{(i)} \right) \cdot x_j^{(i)}$$

Stochastic Classification

The stochastic implementation will generally move the parameters towards the global minimum (...but not always!)

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$l = 1, 1 + k, 1 + 2k, \dots$

$$\Rightarrow \forall j : \mathbf{w}_j = \mathbf{w}_j - \delta \cdot \frac{1}{l} \sum_{l=i}^{i+k} \left(\mathbf{w}_j x_j^{(i)} - y^{(i)} \right) \cdot x_j^{(i)}$$

$k > 1$ might be faster than $k = 1$ for parallelized code

Outline

Recommender systems

Stochastic Classification

Online learning

Online learning

Problem formulation

Online learning

Given a continuous stream of input data, update the parameters of your algorithm on-the-fly

Example: learn user behaviour from website users

Online learning

Problem formulation

Online learning

Given a continuous stream of input data, update the parameters of your algorithm on-the-fly

Example: learn user behaviour from website users

Similar to stochastic classification: Update the parameters based on individual training examples

$$\Rightarrow \forall j : w_j = w_j - \delta \cdot (h(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$

Online learning

Problem formulation

Online learning

Given a continuous stream of input data, update the parameters of your algorithm on-the-fly

Example: learn user behaviour from website users

Similar to stochastic classification: Update the parameters based on individual training examples

$$\Rightarrow \forall j : w_j = w_j - \delta \cdot (h(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)}$$

\Rightarrow Able to adapt to changing user behaviour over time

Questions?

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