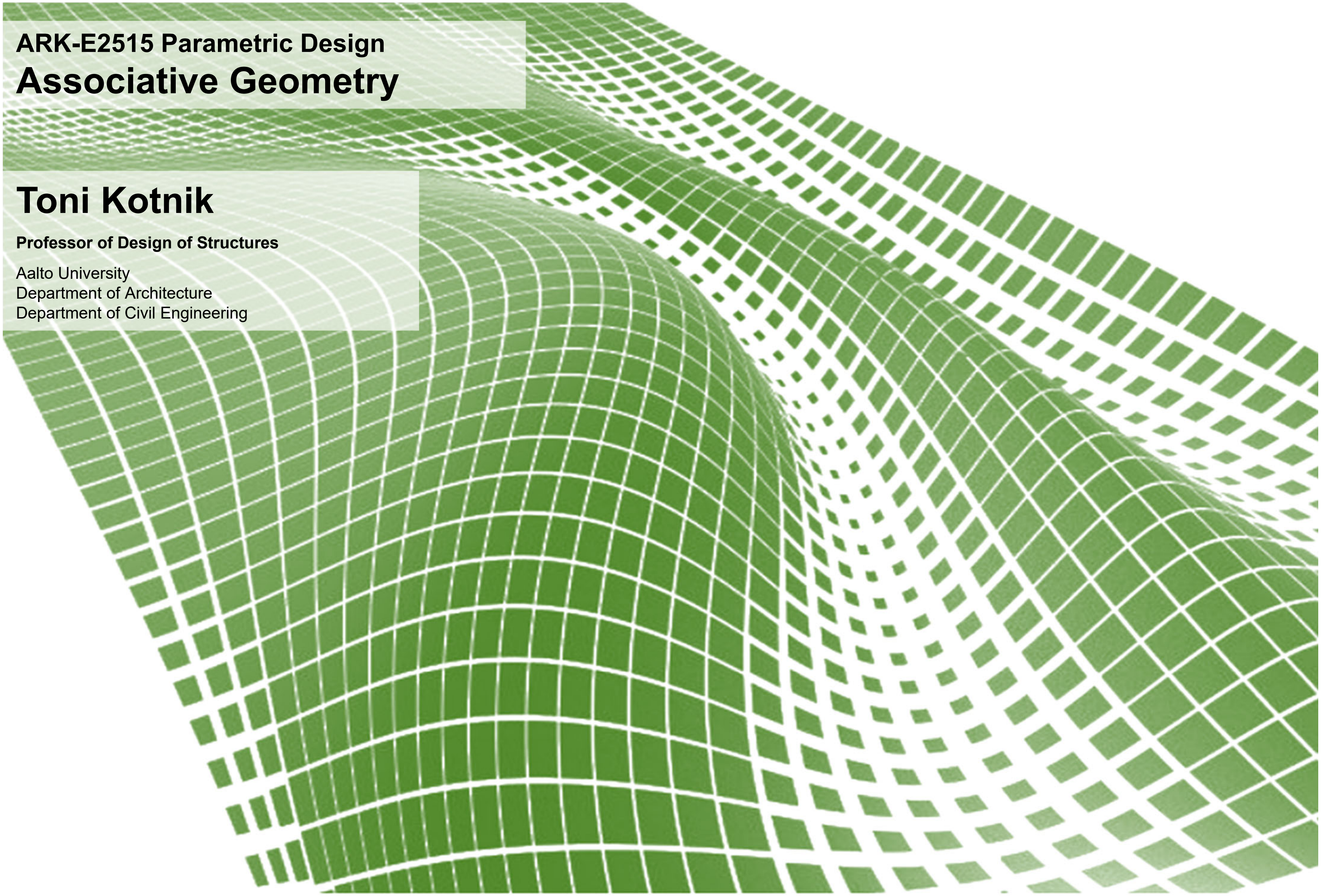


ARK-E2515 Parametric Design

Associative Geometry

Toni Kotnik

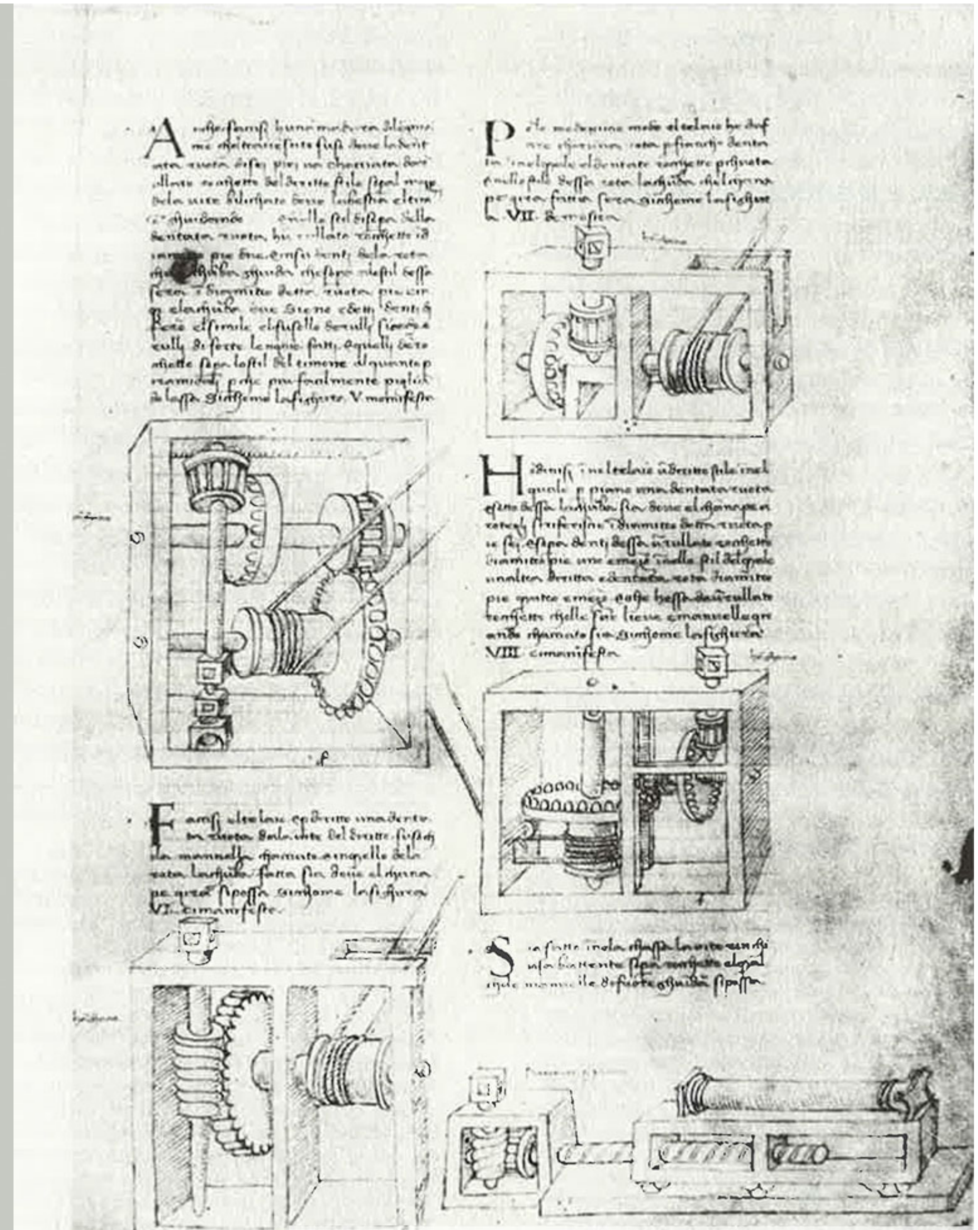
Professor of Design of Structures

Aalto University
Department of Architecture
Department of Civil Engineering

Filippo Brunelleschi: Cathedrale Santa Maria del Fiore
Florence; Italy, 1419-1436

Master-Builder

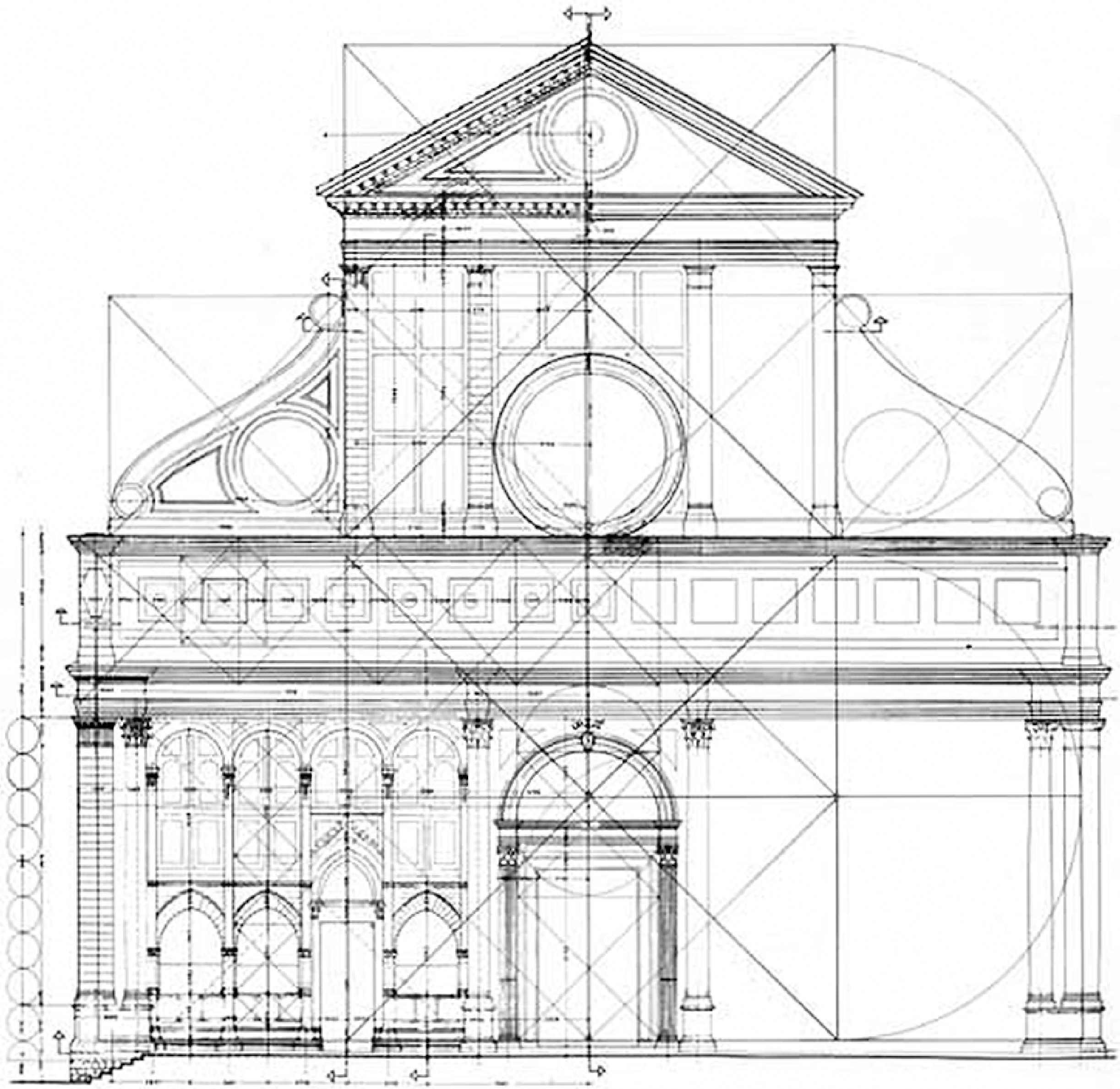
designing with models and machines on site



Filippo Brunelleschi: Cathedrale Santa Maria del Fiore
Florence; Italy, 1419-1436

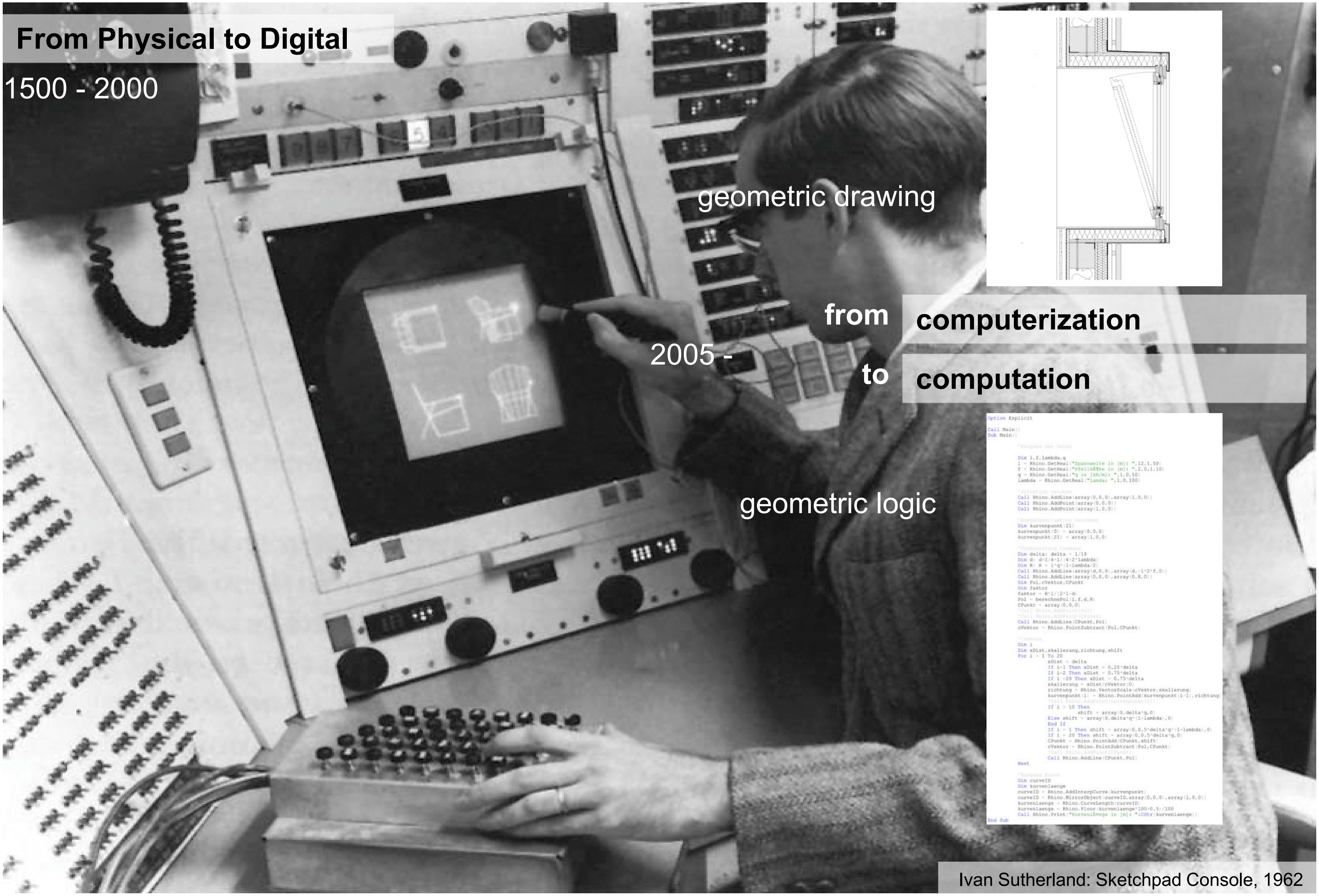
Leon Battista Alberti: Santa Maria Novella
Florence; Italy, 1456-1470

From Building to Drawing based on geometric relationships



Leon Battista Alberti: Santa Maria Novella
Florence; Italy, 1456-1470

From Physical to Digital



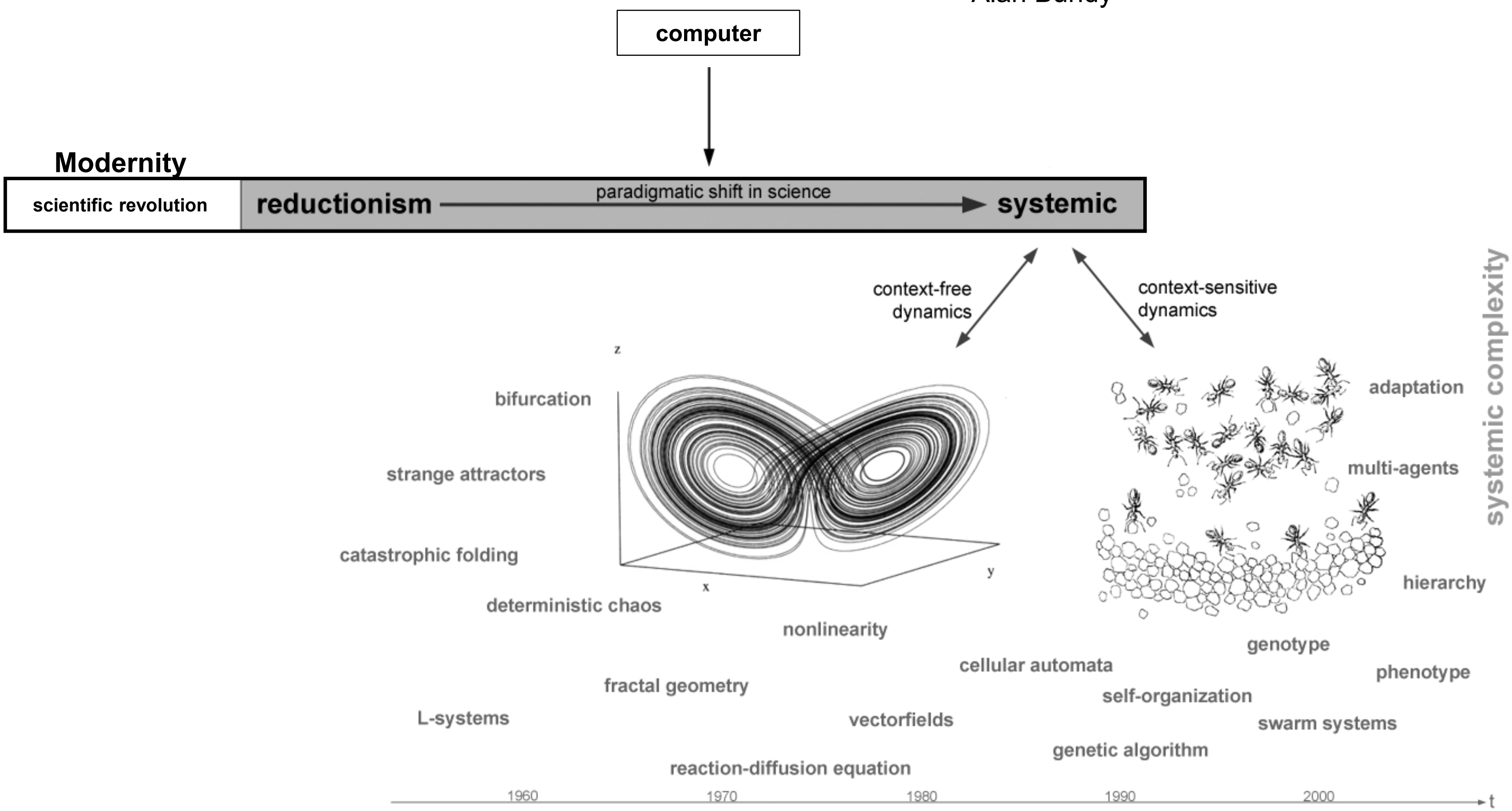

```
Option Explicit

Call Main()
Sub Main()

	'Eingabe der Daten

	Dim l,f,lambda,q
	l = Rhino.GetReal("Spannweite in [m]: ",12,1,50)
	f = Rhino.GetReal("PfeilhÃ¶he in [m]: ",2,0.1,10)
	q = Rhino.GetReal("q in [kN/m]: ",1,0,50)
	lambda = Rhino.GetReal("lamda: ",1,0,100)

	'Situation zeichen
	Call Rhino.AddLine(array(0,0,0),array(1,0,0))
	Call Rhino.AddPoint(array(0,0,0))
	Call Rhino.AddPoint(array(1,0,0))

	'Ausgangssituation zeichnen
	Dim kurvenpunkt(21)
	kurvenpunkt(0) = array(0,0,0)
	kurvenpunkt(21) = array(1,0,0)

	'Vorbereitung Cremona
	Dim delta: delta = 1/19
	Dim d: d=1/4+1/(4+2*lambda)
	Dim R: R = 1*q*(1+lambda/2)
	Call Rhino.AddLine(array(d,0,0),array(d,-1*2*f,0))
	Call Rhino.AddLine(array(0,0,0),array(0,R,0))
	Dim Pol,cVektor,CPunkt
	Dim faktor
	faktor = R*1/(2*1-d)
	Pol = berechnePol(1,f,d,R)
	CPunkt = array(0,0,0)
	'Call Rhino.AddPoint(Pol)
	'Call Rhino.AddPoint(CPunkt)
	Call Rhino.AddLine(CPunkt,Pol)
	cVektor = Rhino.PointSubtract(Pol,CPunkt)

	'Cremona
	Dim i
	Dim xDist,skalierung,richtung,shift
	For i = 1 To 20
		xDist = delta
		If i=1 Then xDist = 0.25*delta
		If i=2 Then xDist = 0.75*delta
		If i =20 Then xDist = 0.75*delta
		skalierung = xDist/cVektor(0)
		richtung = Rhino.VectorScale(cVektor,skalierung)
		kurvenpunkt(i) = Rhino.PointAdd(kurvenpunkt(i-1),richtung)
		'Call Rhino.AddPoint(kurvenpunkt(i))
		If i > 10 Then
			shift = array(0,delta*q,0)
		Else shift = array(0,delta*q*(1+lambda),0)
		End If
		If i = 1 Then shift = array(0,0.5*delta*q*(1+lambda),0)
		If i = 20 Then shift = array(0,0.5*delta*q,0)
		CPunkt = Rhino.PointAdd(CPunkt,shift)
		cVektor = Rhino.PointSubtract(Pol,CPunkt)
		'Call Rhino.AddPoint(CPunkt)
		Call Rhino.AddLine(CPunkt.Pol)
	Next

	'Ausgabe Kurve
	Dim curveID
	Dim kurvenlaenge
	curveID = Rhino.AddInterpCurve(kurvenpunkt)
	curveID = Rhino.MirrorObject(curveID,array(0,0,0),array(1,0,0))
	kurvenlaenge = Rhino.CurveLength(curveID)
	kurvenlaenge = Rhino.Floor(kurvenlaenge*100+0.5)/100
	Call Rhino.Print("KurvenlÃ¤nge in [m]: "&CStr(kurvenlaenge))
End Sub
```

Ivan Sutherland: Sketchpad Console, 1962

Digital Revolution

"An intellectual revolution is happening all around us, but few people are remarking on it. Computational thinking is influencing research in nearly all disciplines, both in the sciences and the humanities. … [The Computer] is changing the way we think. … If you want to understand the 21th century then you must first understand computation."

Alan Bundy

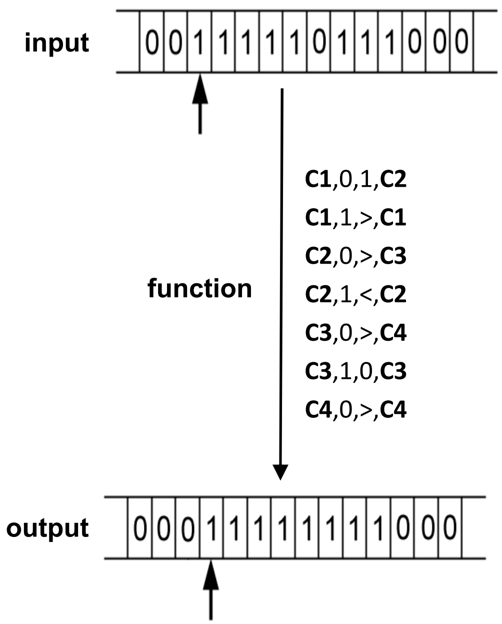


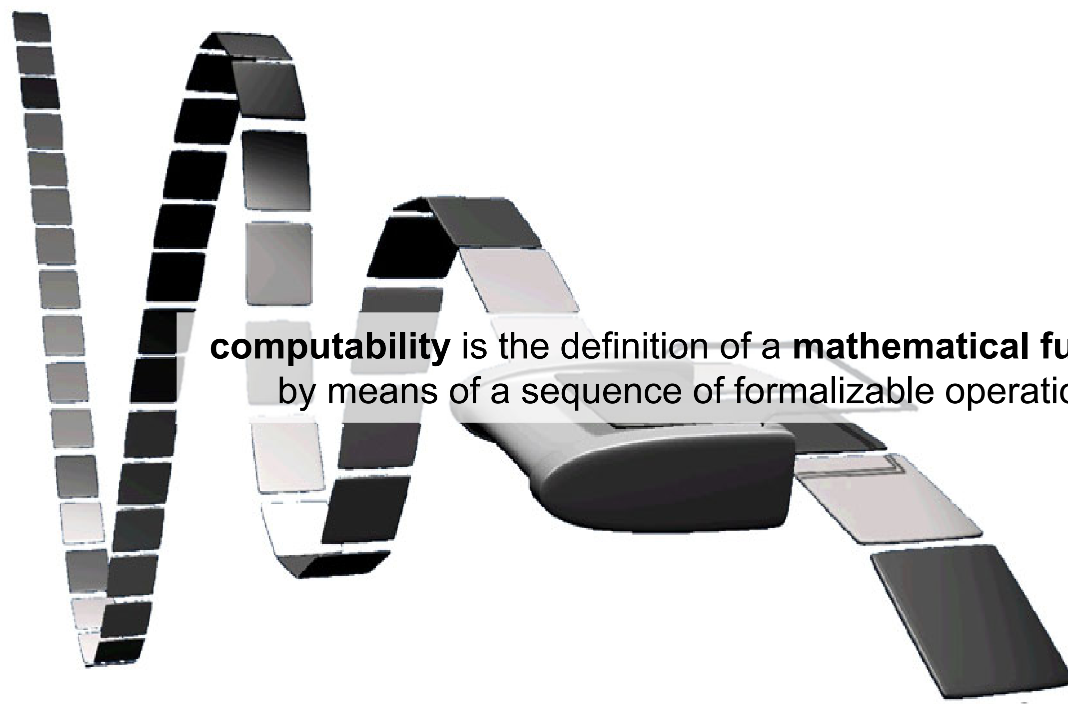
What is computation?

Computation

" Turing's 'Machines'. These machines are human *who calculate."*

Ludwig Wittgenstein

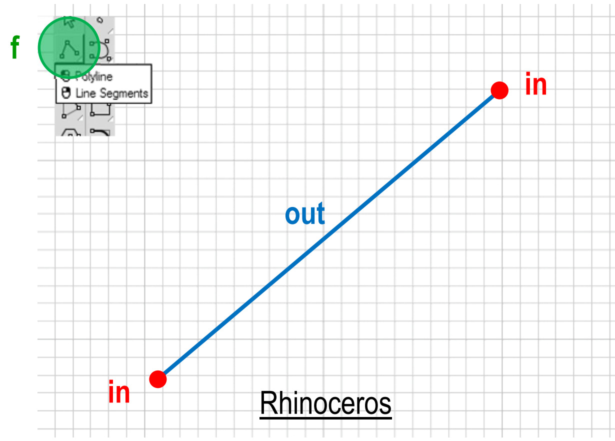




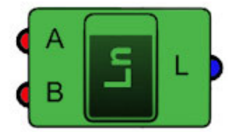
computability is the definition of a **mathematical function** by means of a sequence of formalizable operations

abstract machine model of computation based on Alan Turing

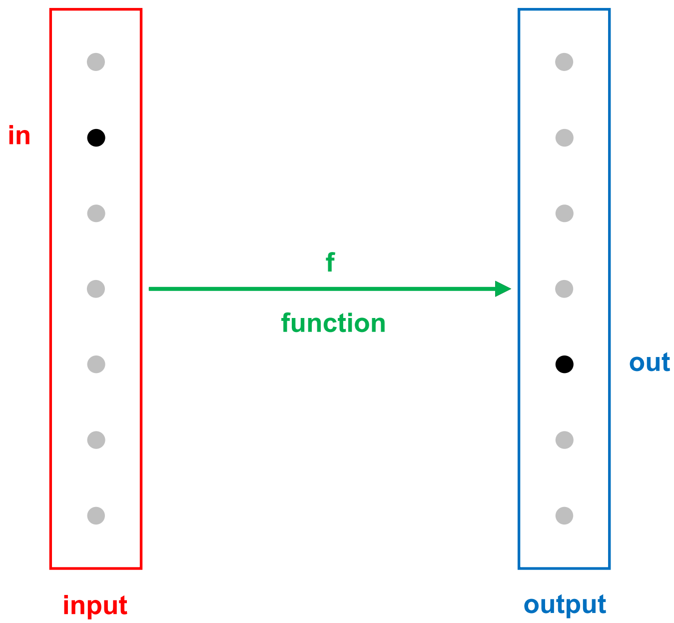
Computation



Rhinoceros



Grasshopper



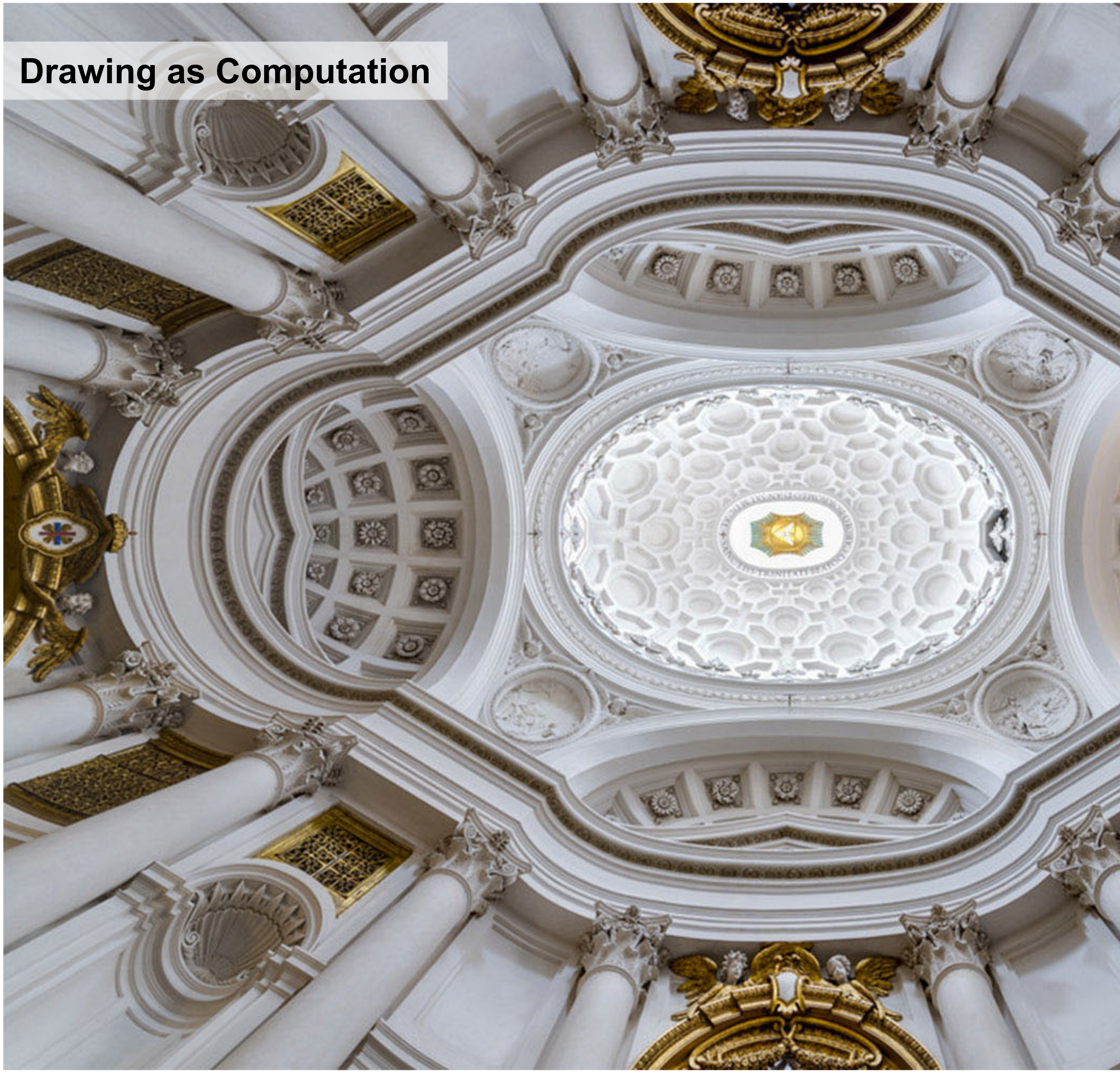
lineID = Rhino.AddLine (arrStart, arrEnd)

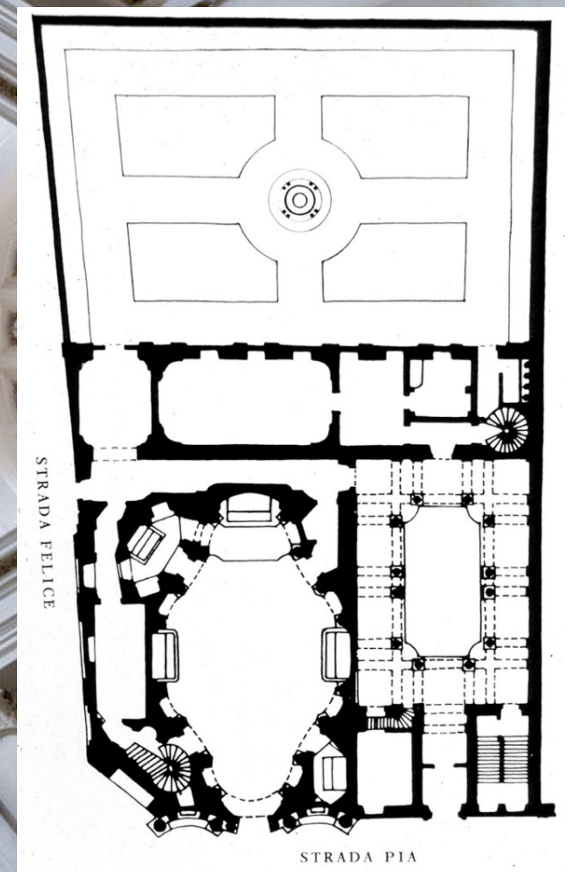
Rhinoscript

drawing a line as mathematical function

Drawing as Computation

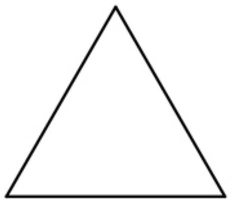
church at the monastery of the Trinitarian order

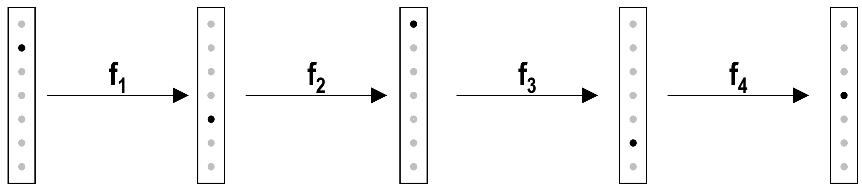




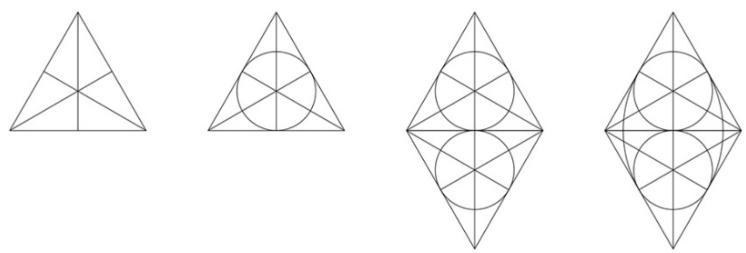
Borromini: San Carlo alle Quattro Fontane
Rome, Italy, 1638-1677

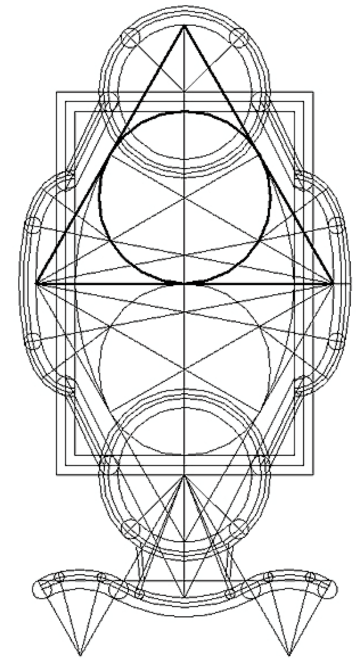
Drawing as Computation





Associative Geometry
sequence of geometric operations
that built upon each other

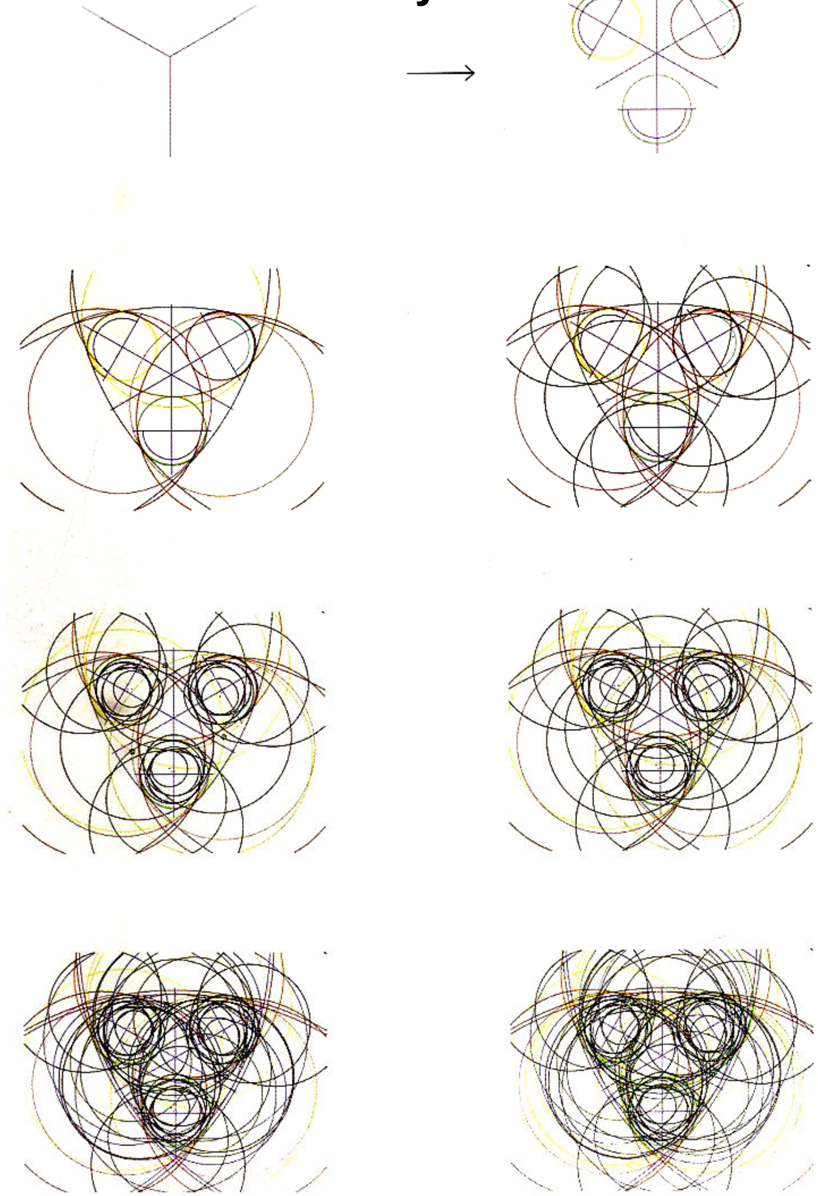


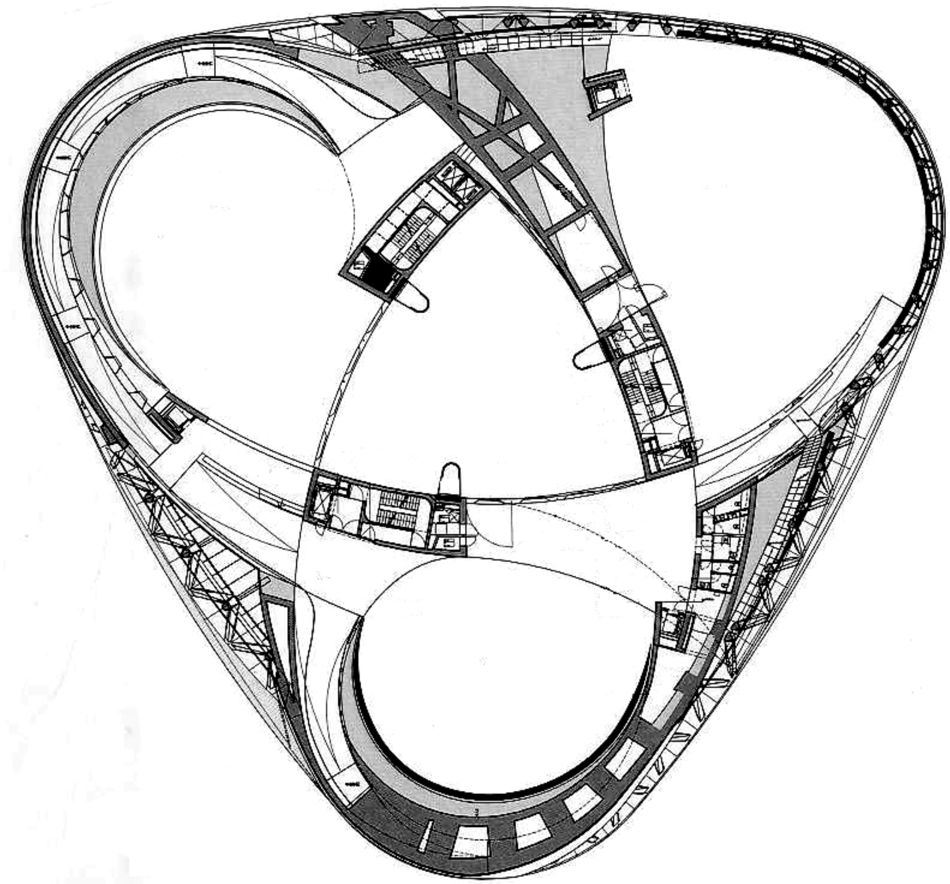


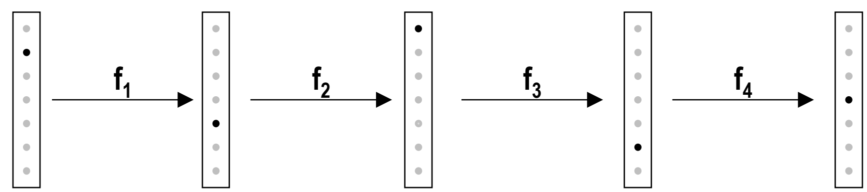
association (lat. *associare*: to unite, to ally)
uniting in a common purpose / work together for one goal

Borromini: San Carlo alle Quattro Fontane
Rome, Italy, 1638-1677

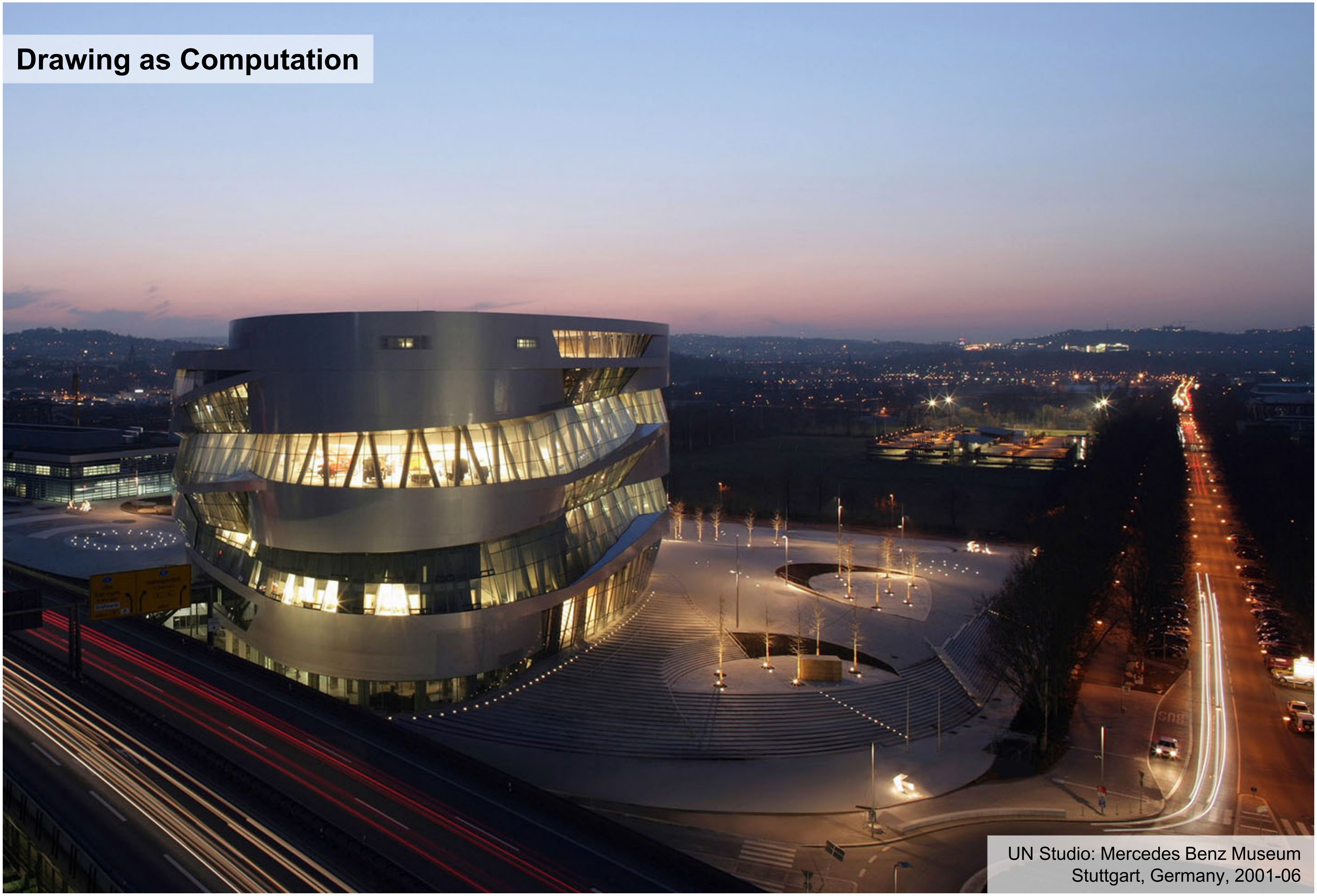
Associative Geometry





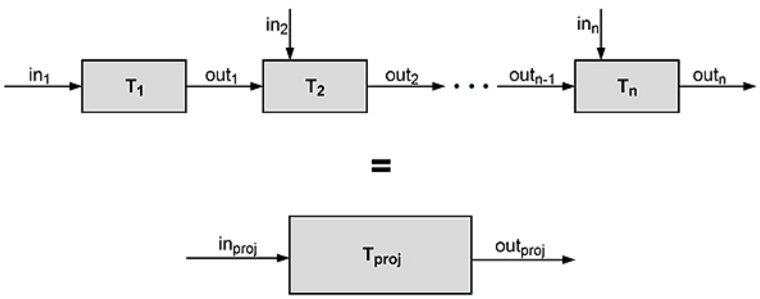


Drawing as Computation



UN Studio: Mercedes Benz Museum
Stuttgart, Germany, 2001-06

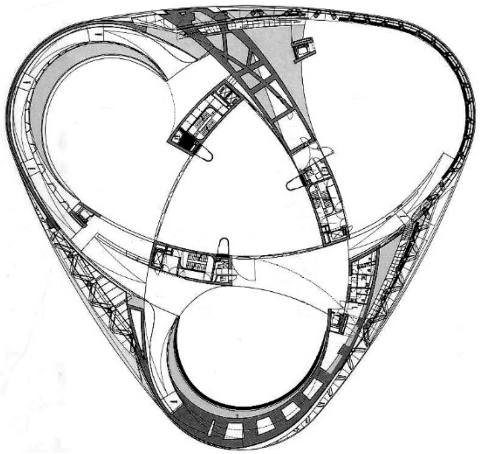
Drawing as Computation

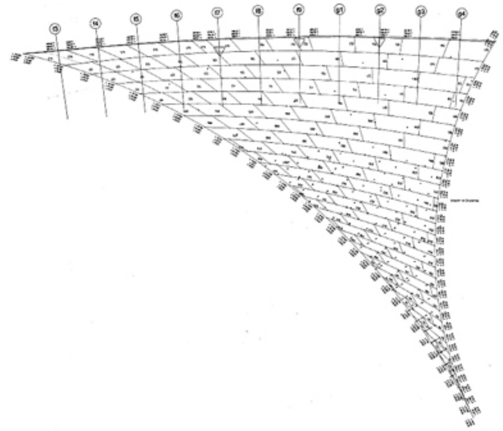


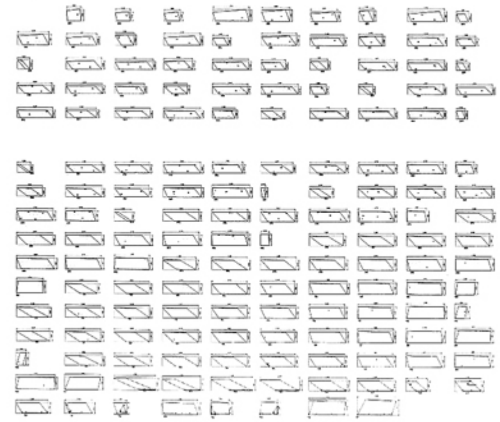
every architectural drawing carries an inherent logic defined by the sequence of geometric operations

architectural form
is a
mathematical function

From Drawing to Building







structure
materials

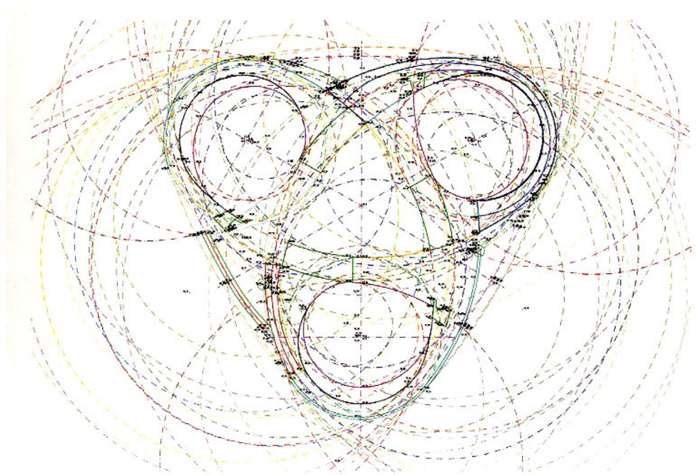
design
user data

fabrication
construction details

assembly
...

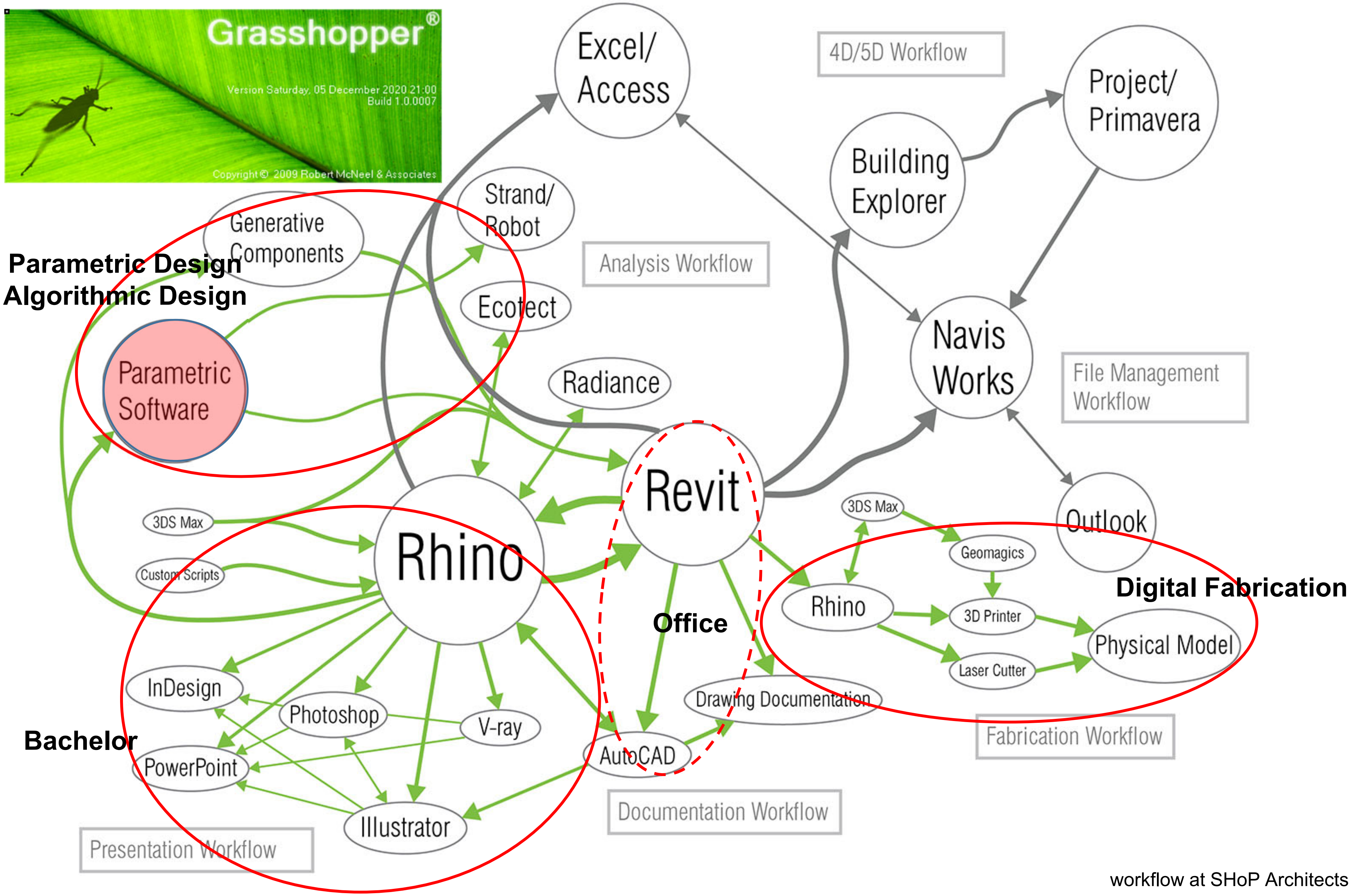
BIM

geometry



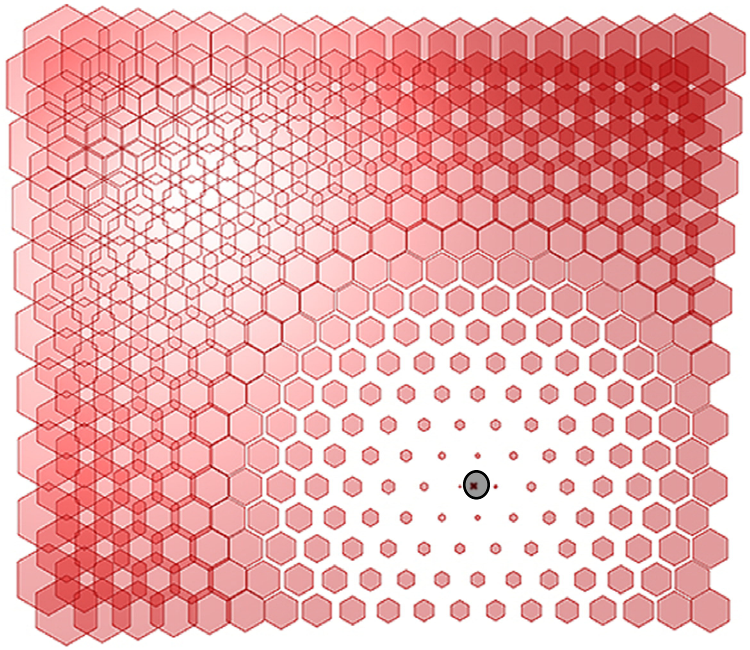
parametric design

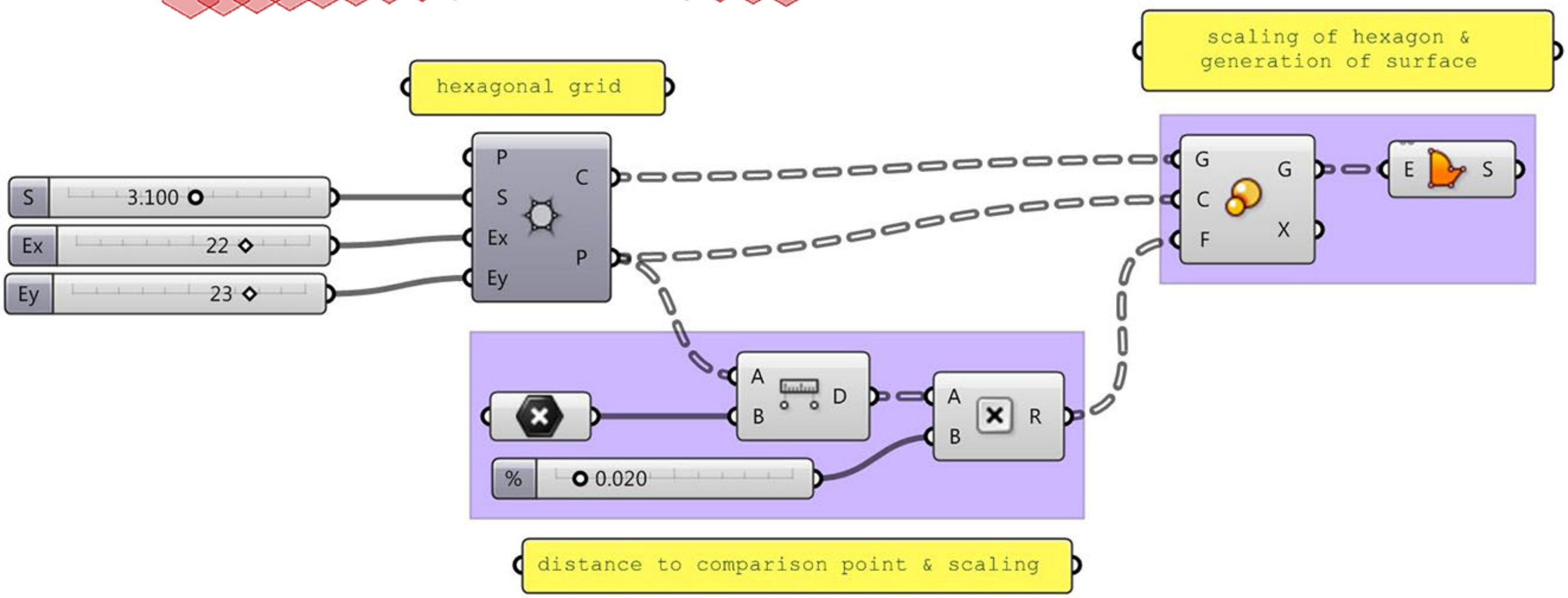
UN Studio: Mercedes Benz Museum
Stuttgart, Germany, 2001-06



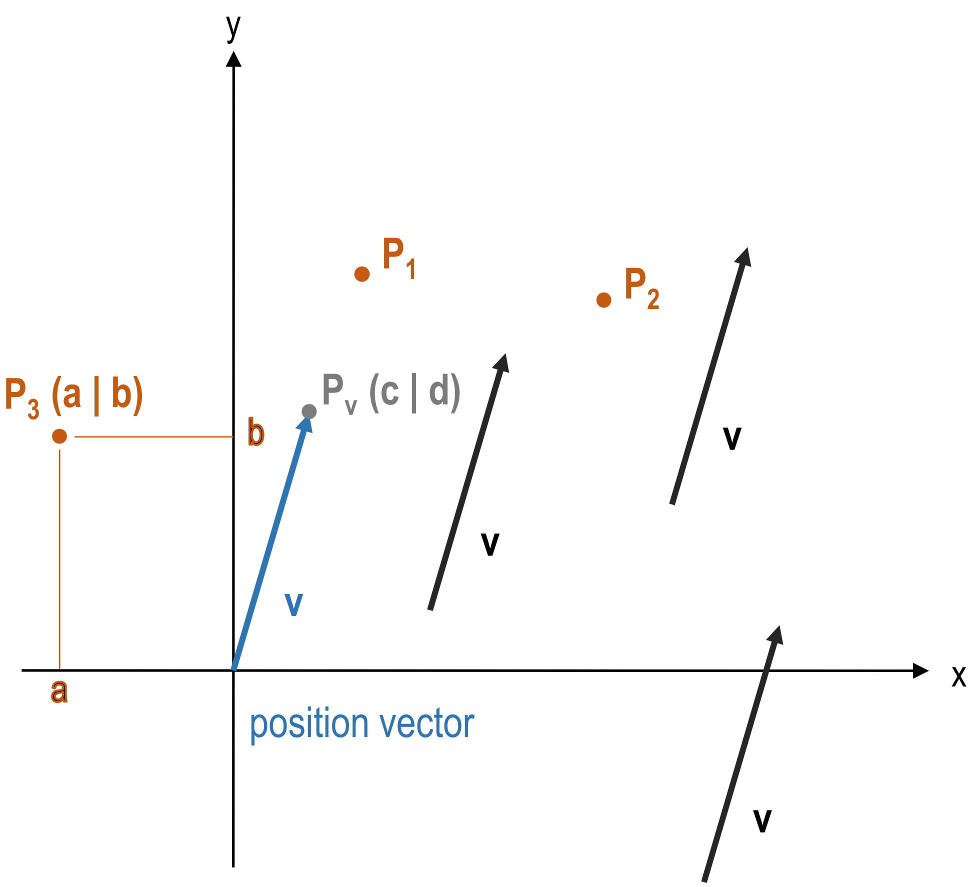
workflow at SHoP Architects

Check 1: construct a field of discs with radius increasing with the distance from a control point





Point & Vector



a vector is a mathematical object that has a direction and an intensity

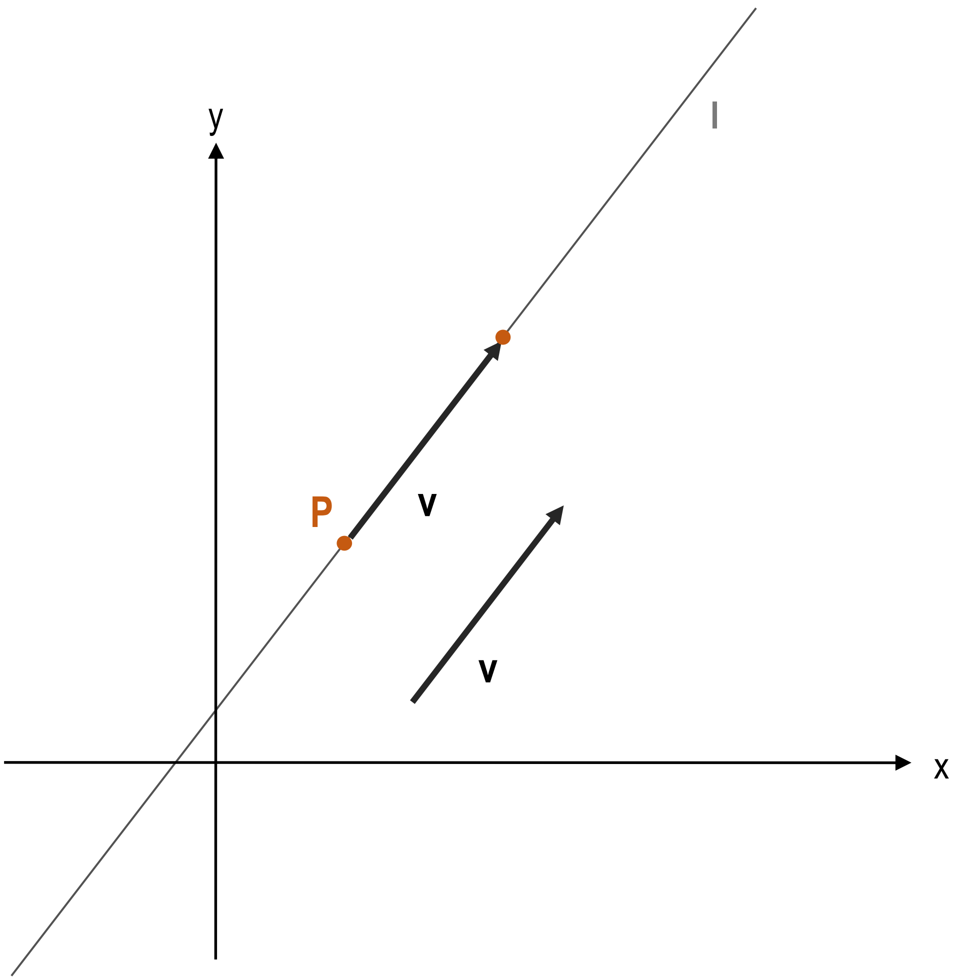
the position vector **v** can be identified with its endpoint P_v

v ≈ (c | d)

but the position vector **v** is not equal to its endpoint P_v

v ≠ (c | d)

Line



a line l is defined by a point **P** and a direction vector **v**

this results in a natural parametrization of the line with respect to the scaling factor t of **v**

$$l(t) = P + t \cdot \mathbf{v}$$

for the distance d between two points on the line this implies

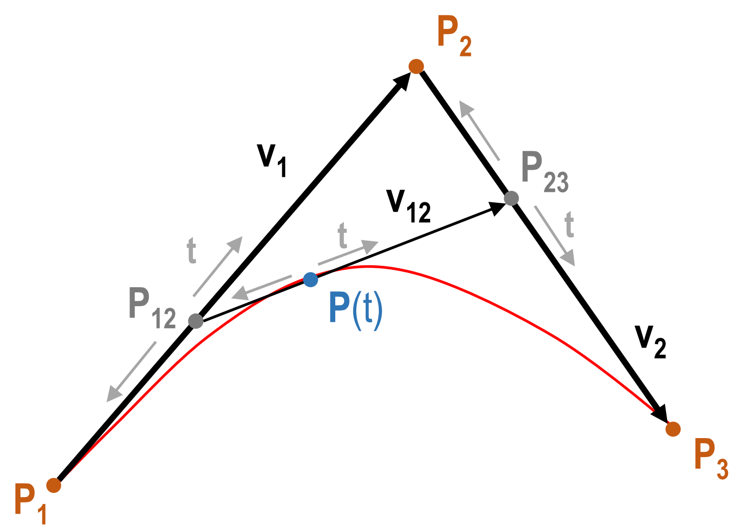
$$d(l(t_1), l(t_2)) = |t_1 - t_2| \cdot |\mathbf{v}|$$

attention: in Rhino the direction vector is always a unit vector, i.e.

$$d(l(t_1), l(t_2)) = |t_1 - t_2|$$

Bézier-Curve

$t \in [0,1]$



using the parametrization of two intersecting lines simultaneously

$$P_{12} = P_1 + t \cdot \mathbf{v_1}$$
$$P_{23} = P_2 + t \cdot \mathbf{v_2}$$

a unique point along the connecting line can be defined

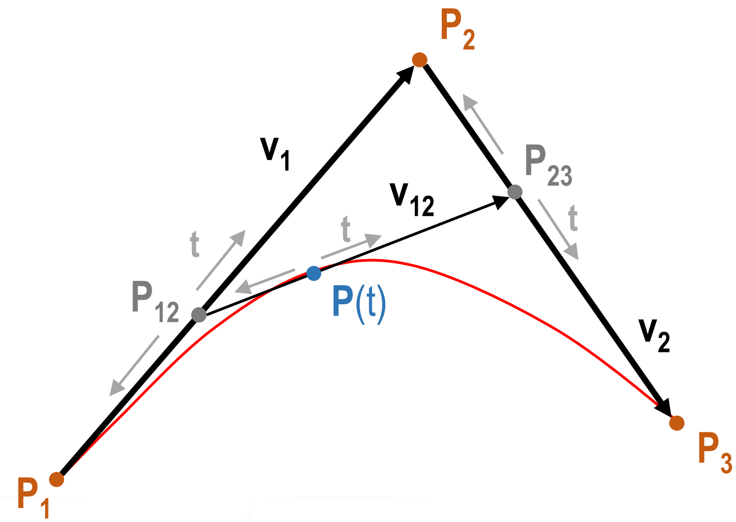
$$P(t) = P_{12} + t \cdot \mathbf{v_{12}}$$

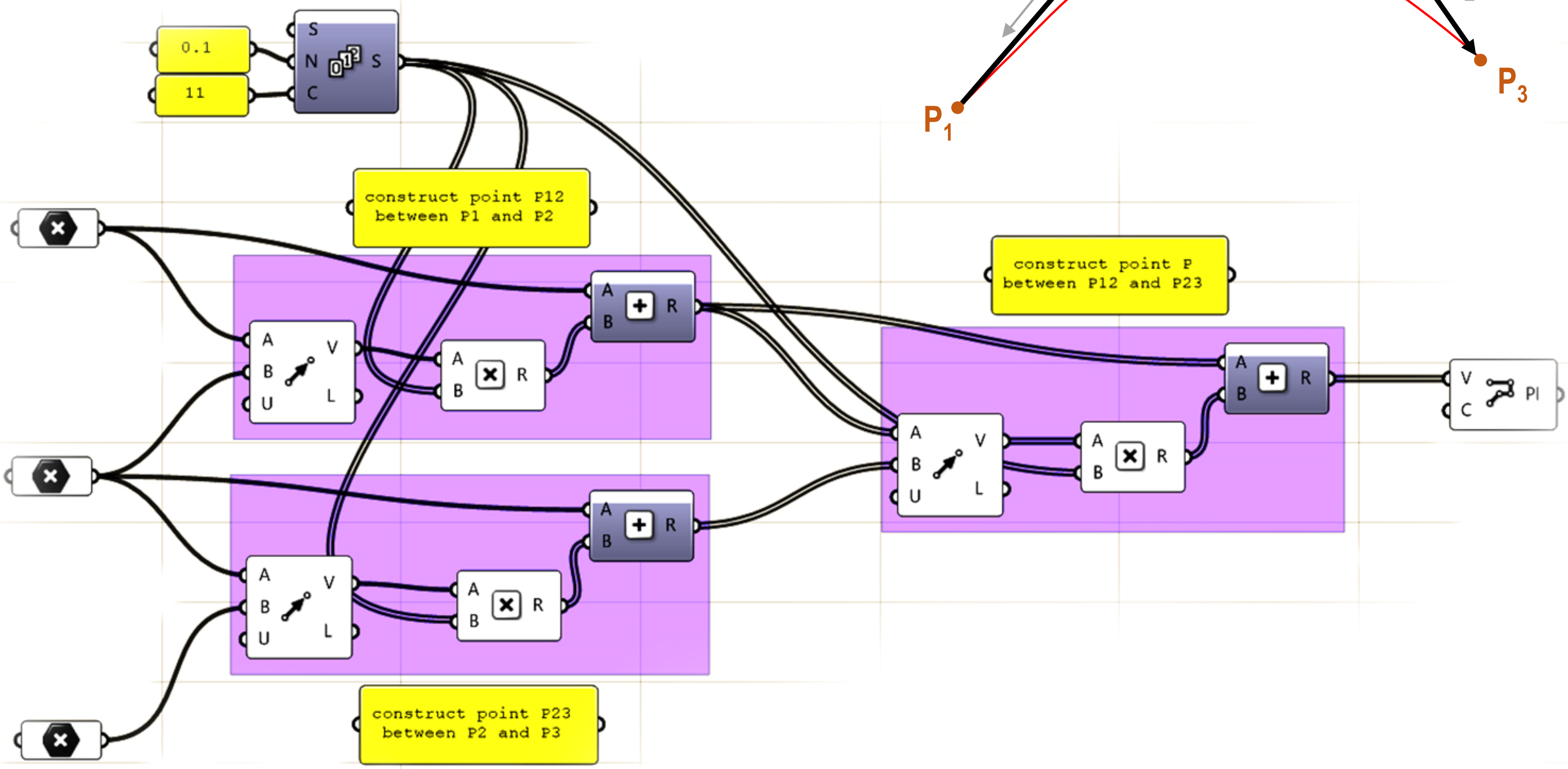
The resulting curve is called a Bézier-curve of degree 3

Check 2: construct the resulting Bézier-curve of degree 3.

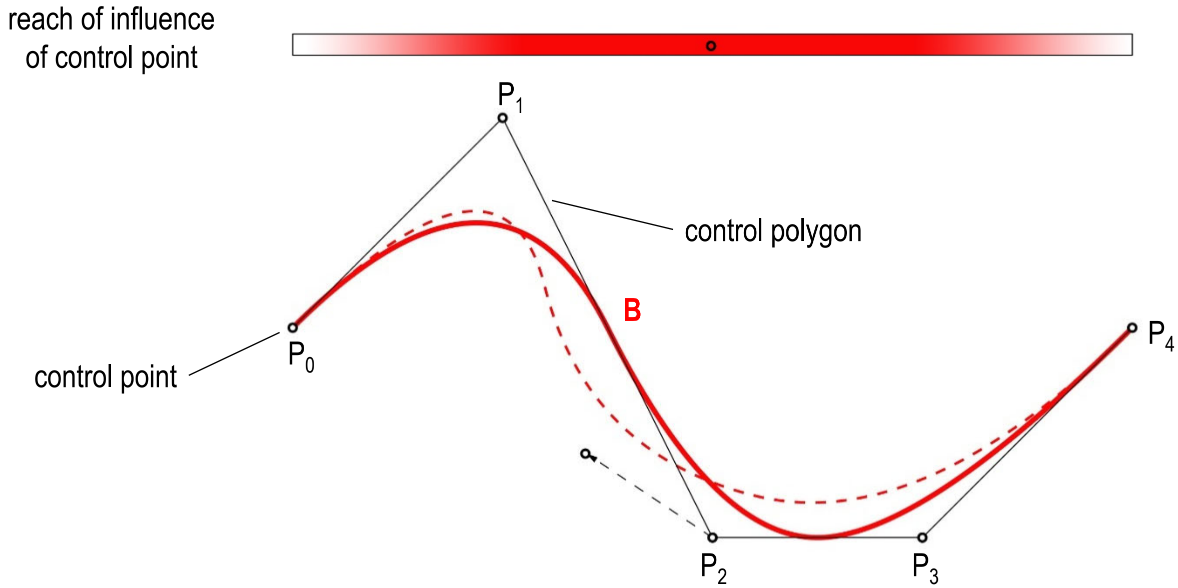
Bézier-Curve

$t \in [0,1]$





Bézier-Curve



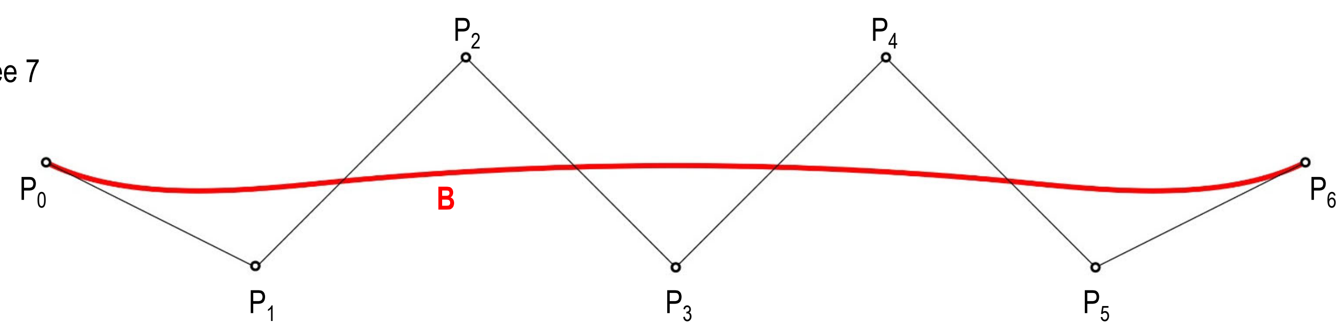
Bézier curve of degree 5

$$B(t) = \sum_{i=0}^{4} \binom{4}{i} t^i (1-t)^{4-i} P_i$$

problem 1: no localized control of form

problem 2: averaging out of curvature

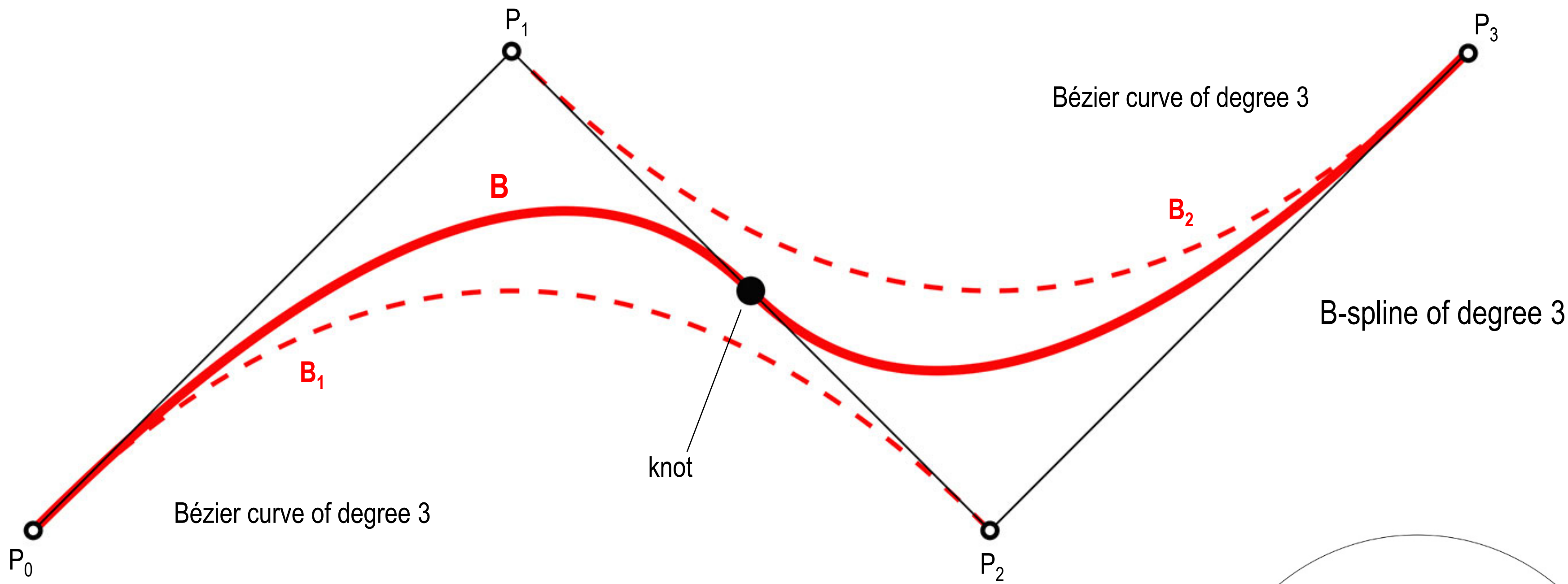
Bézier curve of degree 7



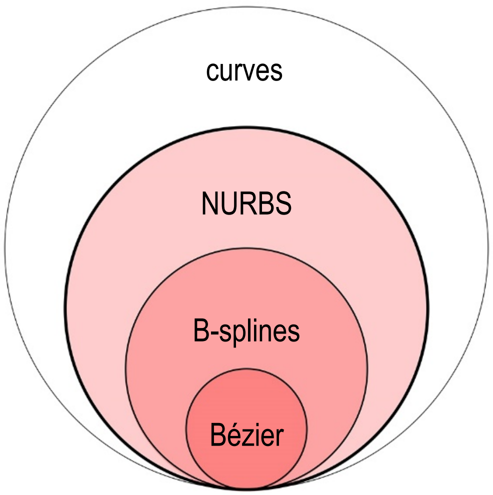
A"DS Aalto University
Design of Structures

Toni Kotnik, Professor of Design of Structures

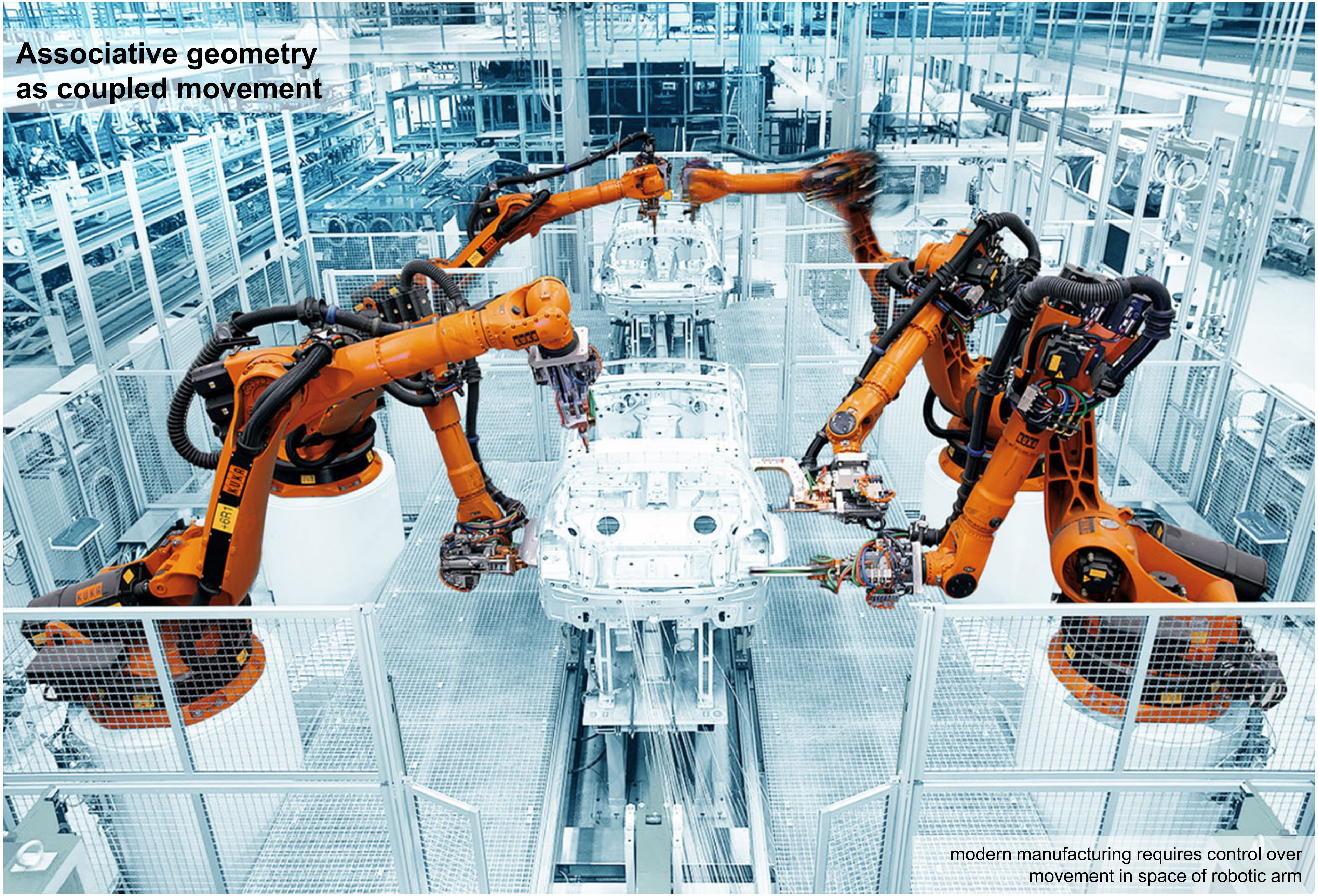
B-Spline



problem with B-spline: a B-spline is a polynomial curve, that means important curves like circles can only be approximated but not represented in a precise manner!

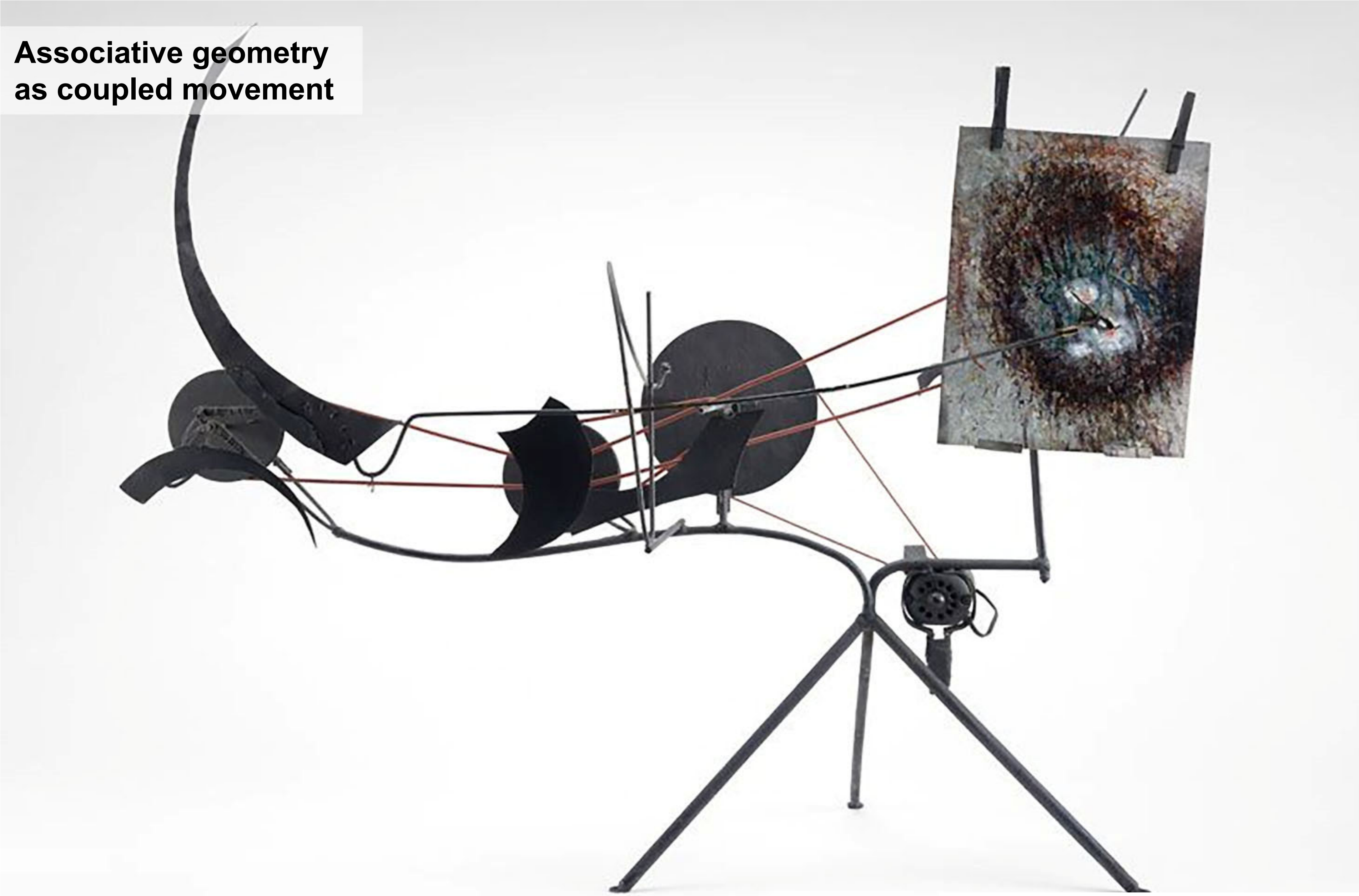


Associative geometry as coupled movement



modern manufacturing requires control over movement in space of robotic arm

Associative geometry as coupled movement



Jean Tinguely: Metamatic No.9, 1958

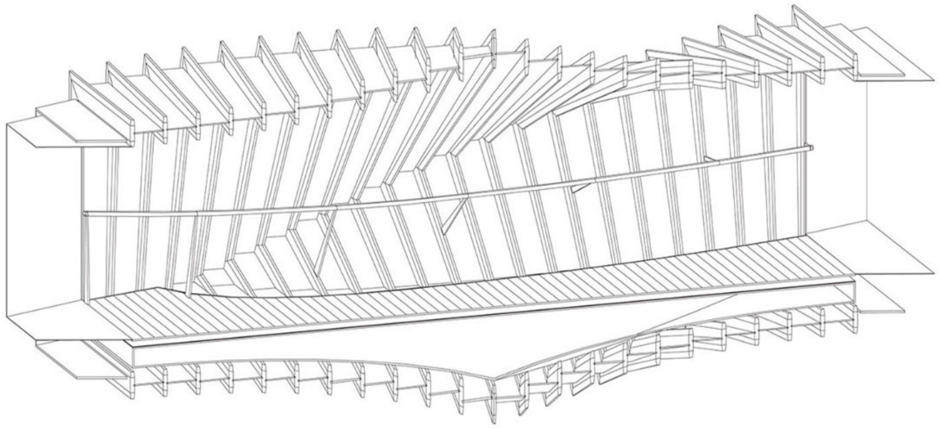
Associative geometry as coupled movement

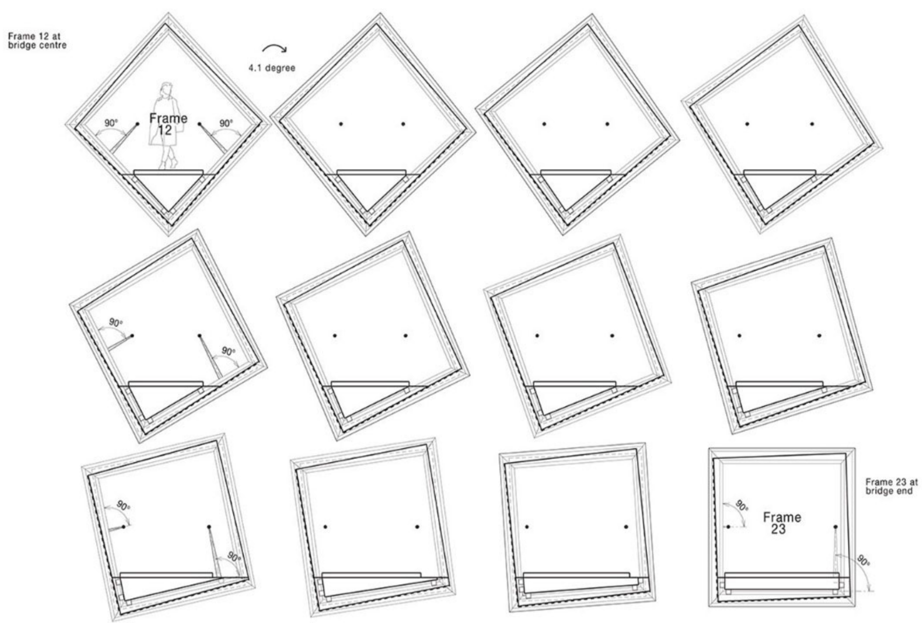
Wilkinson Eyre: Bridge of Aspiration
Royal Ballett School, London, UK, 2003

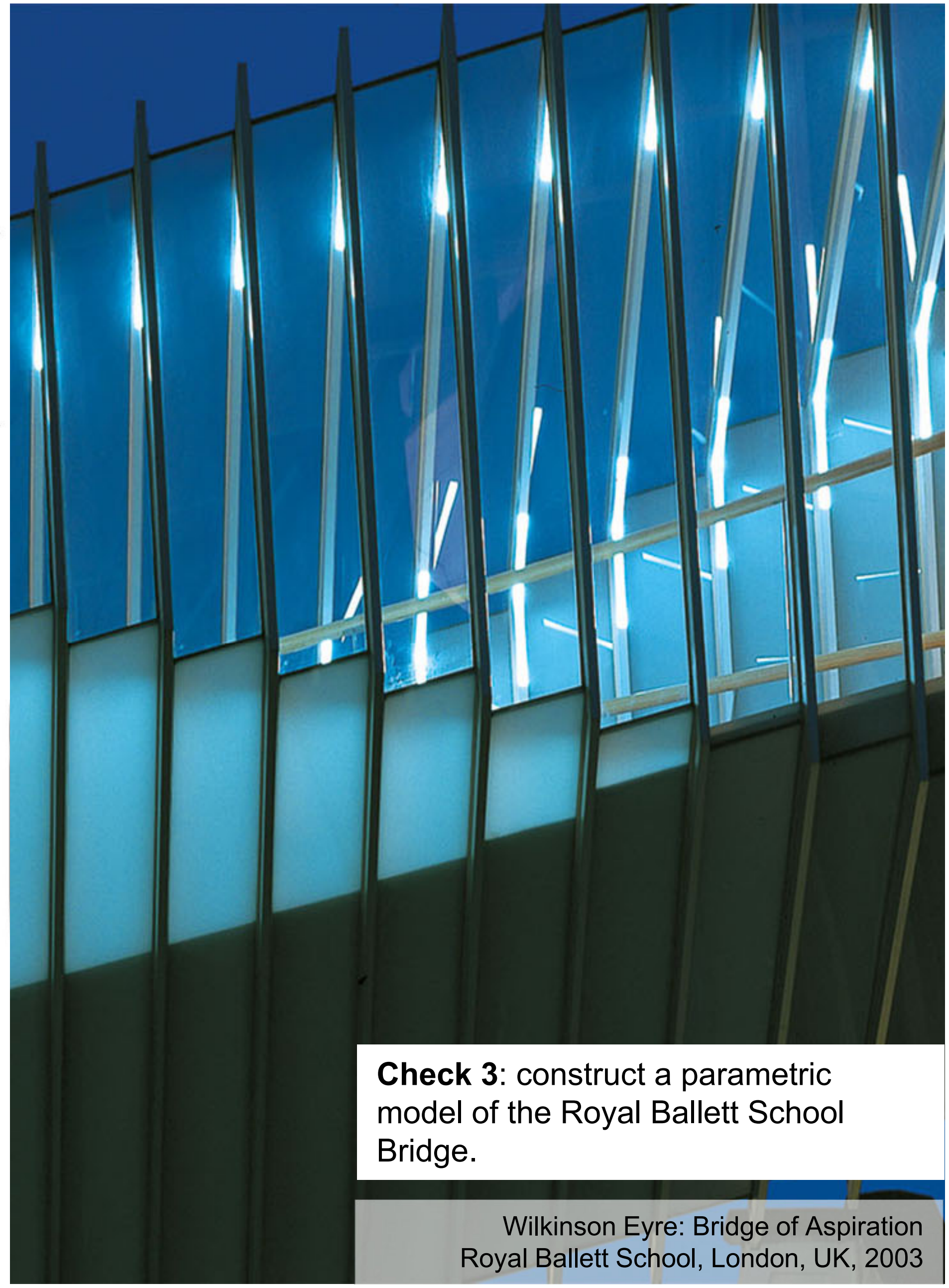
Associative geometry as coupled movement

Wilkinson Eyre: Bridge of Aspiration
Royal Ballett School, London, UK, 2003

Associative geometry as coupled movement



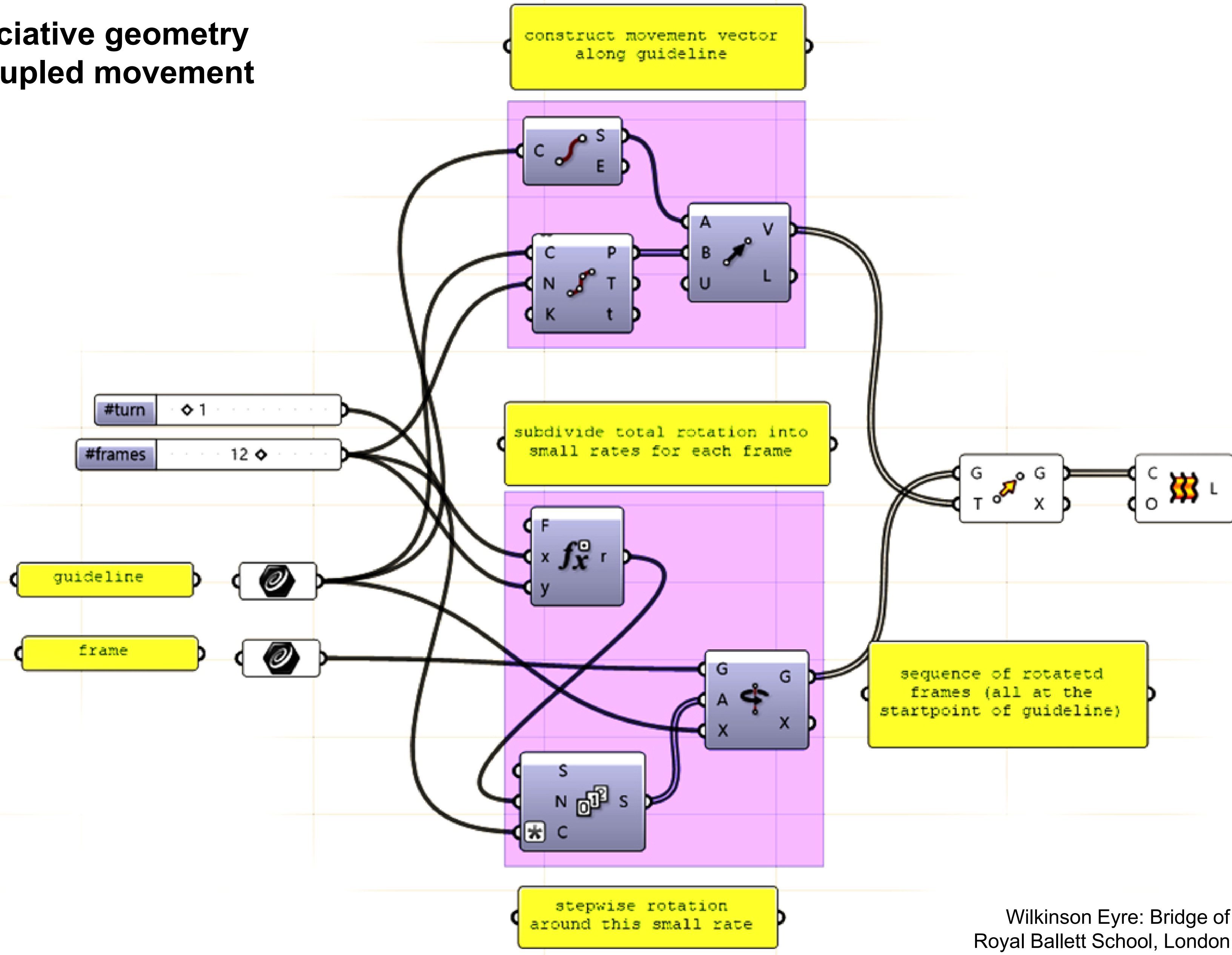




Check 3: construct a parametric model of the Royal Ballett School Bridge.

Wilkinson Eyre: Bridge of Aspiration
Royal Ballett School, London, UK, 2003

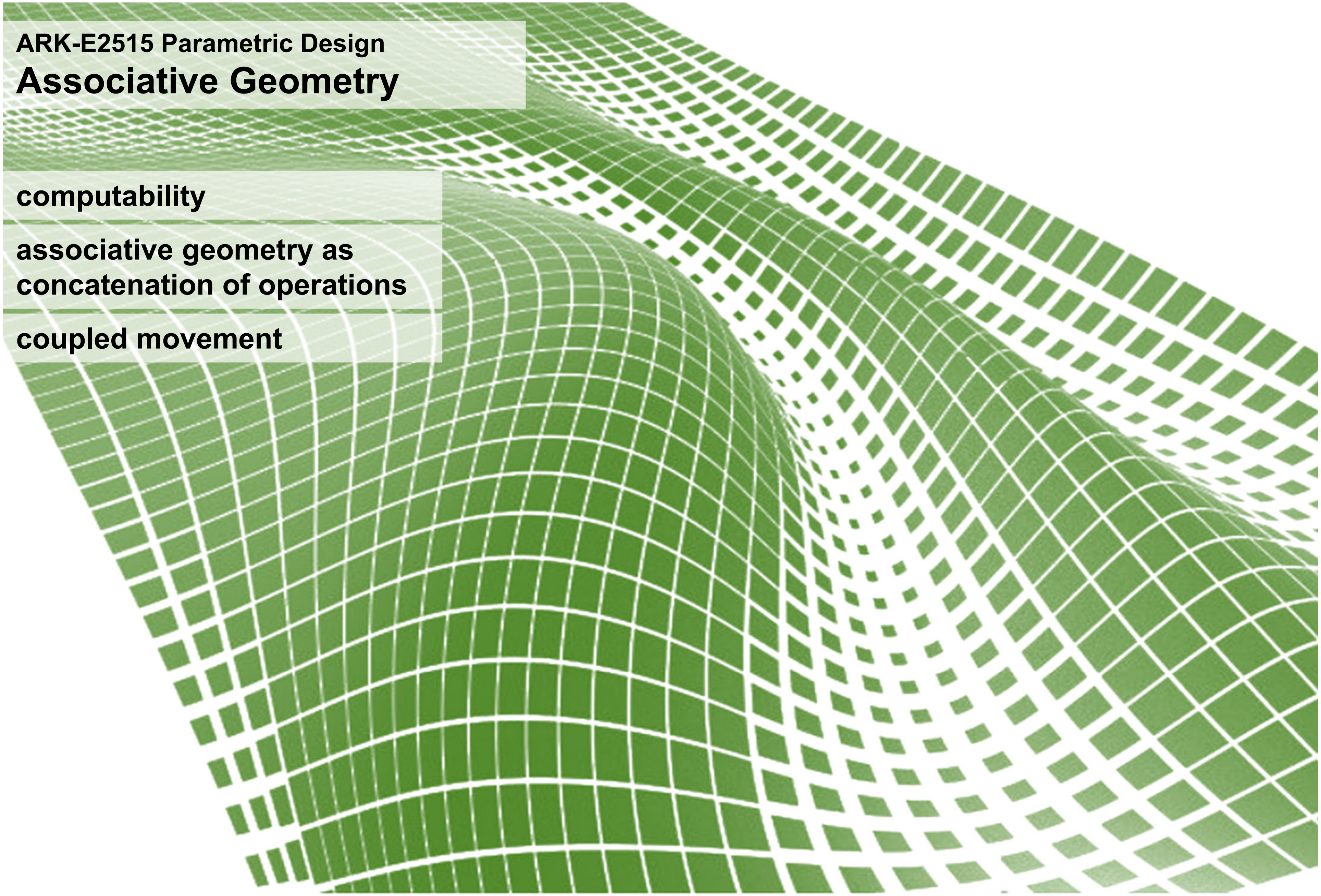
Associative geometry as coupled movement



Wilkinson Eyre: Bridge of Aspiration
Royal Ballett School, London, UK, 2003

ARK-E2515 Parametric Design

Associative Geometry



computability

associative geometry as concatenation of operations

coupled movement