Aalto University
School of Electrical Engineering

ELEC-C9610 Basics in Electronics

Lecture 2: Circuit transforms and theorems

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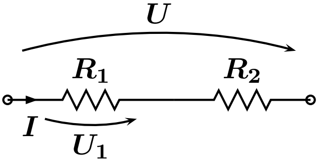
Voltage and current division

Voltage division rule:

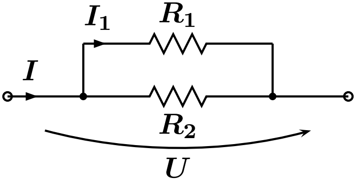


$$I = \frac{U}{R_1 + R_2} = \frac{U_1}{R_1}$$

$$U_1 = \frac{R_1}{R_1 + R_2} U$$

The **considered** resistance in numerator

Current division rule:



$$U = \underbrace{\frac{R_1 R_2}{R_1 + R_2}}_{\left[\frac{1}{R_1} + \frac{1}{R_2}\right]^{-1}} I = R_1 I_1$$

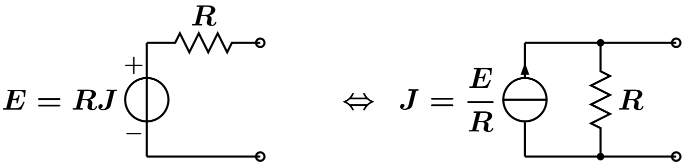
The **other** resistance in numerator

$$I_1 = \frac{R_2}{R_1 + R_2} I$$

Why do we need circuit transforms?

- Circuit transforms can be used to simplify the circuit: when there is less branches and nodes in the circuit, smaller number of Kirchhoff's equations are needed.
- Equivalent circuits behave electrically equally if studied from external nodes. Their voltage-current-equations are same.
- Important in circuit transforms: recognize the nodes

Source transforms



voltage source $\boldsymbol{E}$ and series resistance $\boldsymbol{R}$

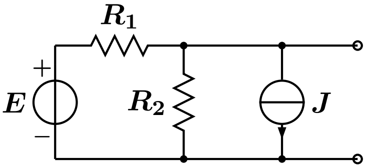
current source $\boldsymbol{J}$ and parallel resistance $\boldsymbol{R}$

Circuit transforms work to both directions.

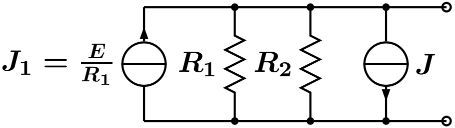
The circuits seems equal from the external nodes:

- Voltage between the external nodes is same.
- In the load connected between the external nodes, the current is same.

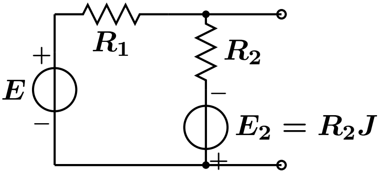
Example on source transform



Transform to voltage source:



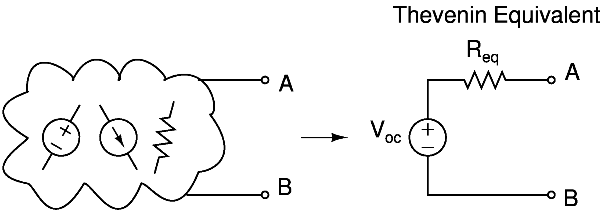
Transform to current source:



When transform is done, there appear new parallel and series connections and simplification of the circuit can proceed.

Thévenin's and Norton's method

To faciliate analysing a complex, we can create Thévenin or Norton equivalent circuit where we have only one source and one resistance.

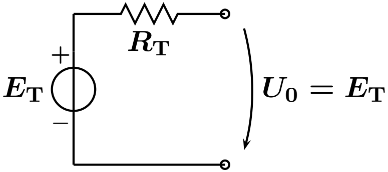


The value of source and resistance is determined such that by studying from external nodes A and B, the circuits above operate in the same manner.

Thévenin theorem: Any linear circuit consisting only of voltage sources, current sources and resistors can be replaced with an equivalent circuit, where there is a voltage source in series with a resistor.

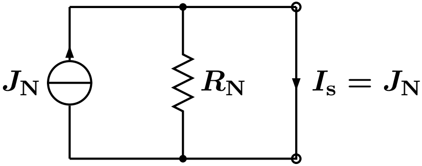
Thévenin and Norton source

Thévenin source



a voltage source in series with a resistor

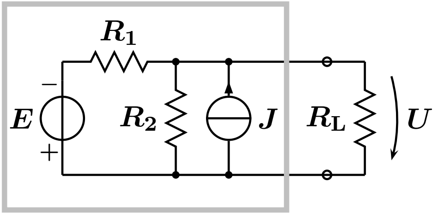
Norton source



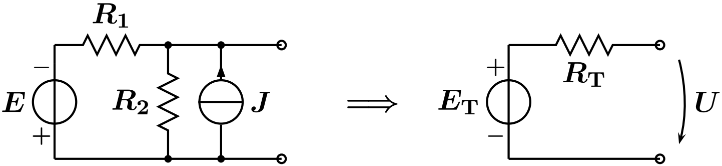
a current source in parallel with a resistor

Example on Thévenin's method

Find the Théveninin's equivalent circuit. Calculate $\boldsymbol{U}$, when load resistor $\boldsymbol{R}_{\mathbf{L}}$ is connected to circuit.

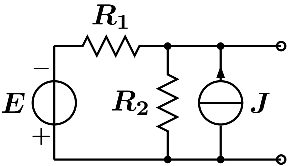


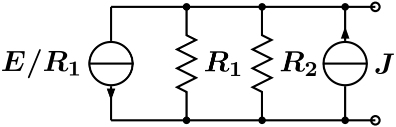
First, determine $\boldsymbol{E}_{\mathbf{T}}$ and $\boldsymbol{R}_{\mathbf{T}}$.



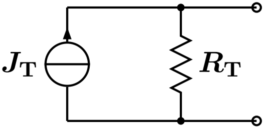
Example continues - finding Thévenin's equivalent

Let's use circuit transforms. First we do a source transform.





Next, we combine current sources and resistors in parallel:



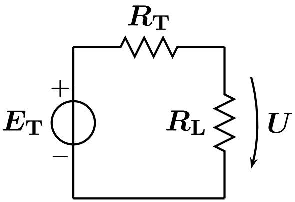
$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$J_T = J - \frac{E}{R_1}$$

Example continues - using Thévenin's equivalent

The second source transform and we have a Thévenin's source:

$$E_{\mathrm{T}} = R_{\mathrm{T}} J_{\mathrm{T}} = \frac{R_1 R_2}{R_1 + R_2} \cdot \left(J - \frac{E}{R_1} \right)$$



The voltage is calculated with voltage division rule:

$$U = \frac{R_{\mathrm{L}}}{R_{\mathrm{T}} + R_{\mathrm{L}}} E_{\mathrm{T}}$$

Linearity and superposition method

In a linear circuit, voltages and currents change with the same relation and the effect of sources can be calculated one by one.

Linearity:

$$f(i_1 + i_2) = f(i_1) + f(i_2)$$

$$f(ki_1) = kf(i_1)$$

Superposition method:

- The real currents and voltages in the circuit are obtained by summing up the currents and voltages due to the individual sources. Naturally, the directions of the currents and voltages have to be noted by signs.
- Power ($P = UI$) **is not** a linear quantity and it can be calculated only based on total currents and voltages.

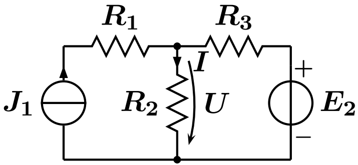
Superposition in practice

In a **linear** circuit, the effect of each source to currents and voltages in the circuit can be calculated separately.

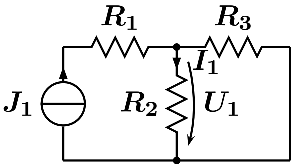
- We analyze the sub-circuits individually, which looks as if only **one** source is active and all others are switched off (set to zero).
- The voltage source is set to zero by replacing it with a short circuit ($\boldsymbol{U = 0}$) and the current source is set to zero by replacing it with an open circuit ($\boldsymbol{I = 0}$):

Draw a picture on each sub-problem what you analyze!

Example on superposition



We calculated the effect of sources one by one. Phase 1, currrent source:



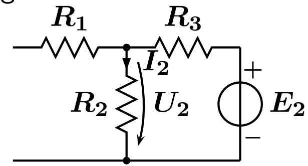
With current division:

$$I_1 = \frac{R_3}{R_2 + R_3} J_1$$

Voltage $U_1 = R_2 I_1$

Example on superposition continues

Phase 2, effect of voltage source:



Voltage division:

$$U_2 = \frac{R_2}{R_2 + R_3} E_2$$

Current $I_2 = G_2 U_2$.
Collecting the results:

$$I = I_1 + I_2 = \frac{R_3 J_1 + E_2}{R_2 + R_3}$$

$$U = U_1 + U_2 = \frac{R_2}{R_2 + R_3}(R_3 J_1 + E_2)$$