How to control the thermal power plant in order to ensure the stable operation of the plant?
In the assignment...

• Production of steam in a thermal power plant is analyzed
  – Burning process of fuel => Steam generation in a boiler => Production of high pressure steam => (Turbine) => Distribution of counter pressure steam => Steam battery => Consumption of steam
  – Steam used by, e.g., a paper mill

• Steam production affected by ”disturbance”: Steam consumption (also steam flow through the turbine)

• Steam production stabilized by
  – Controlling fuel injection
  – Controlling steam flow through the turbine
  – Charging and discharging the steam battery

• PID controller and state feedback controller are used

• Three sections of the power plant
  1. Steam production; upper section of the plant
  2. Steam distribution; turbine flow & counter pressure stock
  3. Steam battery

• Analysis and control of a large scale system (in principle)
Simulink model of the power plant

Steam production

Steam distribution

Steam battery
Steam production ("upper section")

1. Fuel injection
   - Stream generation in the boiler

2. Net flow into the boiler
   - Pressure of the boiler

3. Flow out of the boiler
   - Normal flow into the turbine
   - Net flow into the high pressure stock
   - Controlled flow into the turbine

4. Saturation of the control signal
   - Setting of valve z1

5. pk
6. z1
7. lp
8. fg
Steam battery

STEAM BATTERY

- Uncontrolled charging
- Uncontrolled discharging
- Net flow to the battery
- 1/mass
- Pressure factor
- Pressure of the battery
- 1/\text{mass}
- f(u)
- Uncontrolled discharging
- Sensation of the charging controller
- Setting of the discharging controller
- Setting of the discharging controller
- f(u)
- Uncontrolled charging
- Sensation of the charging controller
- Setting of the discharging controller
- Setting of the discharging controller

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On control theory
(e.g., Åström & Murray, Chapter 1)

• Feedback:
  – The control (= the input) of a system depends on the output / the state of the system

• Basic problems:
  – Tracking: The output should follow the external reference signal as accurate as possible (focus is to compensate the dynamics of the system)
  – Stabilization: The output should be constant (focus is to compensate disturbances)

• The assignment deals with stabilization
Idea of feedback

- $r(t)$ is the external reference signal
- Feedback from the output using the error quantity $e(t) = r(t) - y(t)$
  - $e(t) = 0 \Rightarrow$ OK! Otherwise: Adjust $u(t)$ until $e(t) = 0$
- Control problem: Construct the controller
  - Structure
  - Gains, i.e., parameters
PID-controller
(e.g., Åström & Murray, Chapter 10)

- P = Proportional, I = Integral, D = Derivative
  - P-term: The input depends on the current value of the error quantity
    - Insufficient for steady/constant disturbance
  - I-term: The input depends on the time integral of the error quantity
    - Destabilizes the system; relying on old information
  - D-term: The input depends on the time derivative of the error quantity
    - Stabilizes the system; issues on amplification of high frequency measurement or process noise

- Each term has a gain - tuning parameters $K_P$, $K_I$ and $K_D$ of the controller
  - Select the parameters in an appropriate way => Stability of the system

$$u(t) = K_P e(t) + K_I \int_{0}^{t} e(\tau) d\tau + K_D \frac{d}{dt} e(t)$$
State feedback controller
(e.g., Åström & Murray, Chapter 6)

• Linear dynamic system \( \frac{dx}{dt} = Ax + Bu \)
  – External reference signal = 0
  – Control the system such that \( x = 0 \)
• Controls = Linear combinations of states: \( u = -Kx \)
  – Closed loop system: \( \frac{dx}{dt} = (A-BK)x \)
• System matrix of the closed loop system: \( A-BK \)
• The open loop system is controllable (see, e.g., Åström & Murray, Chapter 6) => Arbitrary dynamics for the closed loop system
• The problem: Select gain \( K \)
• The state feedback controller does not have necessarily integrating feature
  – Integration should be augmented if needed (step disturbances)
Optimal state feedback controller
(e.g., Kirk, Chapter 5.2)

• Select $u$ such that the functional

$$J[u] = \frac{1}{2} \int_0^T x(t)^T Rx(t) + u(t)^T Qu(t) dt + \frac{1}{2} x(T)^T Px(T)$$

is minimized (linear-quadratic (LQ) problem)

• Weights $R \Rightarrow$ penalty related to large states, weights $Q \Rightarrow$ penalty related to large controls, weights $P \Rightarrow$ penalty related to large terminal states

• Feedback solution obtained by deriving and solving the necessary conditions for the optimal control

  – State equation, co-state equation, optimal control (see the material of the MS-E2148 course)
• Assume that the co-state is of form $S(t)x(t)$
  => Riccati equation for $S$
    – The optimal control is the time variant linear combination of the states: $u^* = -K^*(t)x(t)$

⇒ Solution: Integrate Riccati equation backward => $S(t)$ => the optimal feedback gain $K^*(t)$ => Employ the control $u(t) = -K^*(t)x(t)$

• $S$ typically stabilized quickly => Time invariant (but suboptimal) gain $K^*$ obtained by solving algebraic Riccati equation (derivatives of $S$ are set to be zeros)
  • (Matlab: lqr/lqr2)
Feedbacks in the assignment
**Linearization** (e.g., Åström & Murray, Chapter 5.4)

- Controllers can be used (cautiously) with nonlinear systems
  - PID-controller can be tuned by using a real-life system – a system model is not needed but can be used
- Tuning of a State feedback controller – also PID if a model is used - requires a linear system model => Nonlinear systems must be linearized
- Nonlinear system \( \frac{dx}{dt} = f(x(t), u(t)), \ y(t) = g(x(t), u(t)) \)
  - Analyze the stationary/equilibrium point \((x_0, u_0)\) (and the corresponding \(y_0)\) and small differences \(\Delta x = x - x_0, \ \Delta u = u - u_0, \ \Delta y = y - y_0\)
  - It holds
    \[
    \frac{d\Delta x(t)}{dt} = \frac{\partial f}{\partial x}\Delta x(t) + \frac{\partial f}{\partial u}\Delta u(t) \\
    \Delta y(t) = \frac{\partial g}{\partial x}\Delta x(t) + \frac{\partial g}{\partial u}\Delta u(t)
    \]
    Jacobians evaluated at \((x_0, u_0, y_0)\)
- Note: Valid domain of linearization?
On large scale systems

• Large-scale system = System consists of several subsystems connected each other loosely
  – thousands of variables
  – analysis and synthesis using direct methods challenging or impossible

• "Theory" of large-scale systems: Approaches, methods and techniques for tackling such systems
  – "divide and conquer"

• Typical applications areas: optimization and simulation

• More esoteric themes dealing with large-scale systems:
  – decentralized control, coordination, autonomous agents, agent simulation, self-organization, artificial life,...
• Basic idea: subsystems treated separately by taking into account interactions and dependences between the subsystems
• Interactions and dependences treated in an iterative way
  – Subsystems treated with wrong (but hopefully converging) assumptions on interactions and dependences
• Typically two level algorithms
  – Upper level: Updating interactions and dependences with fixed subsystems
  – Lower level: Updating subsystems with fixed interactions and dependences
Structural versus mathematical large-scale system

- Often subsystems interacting identifiable wholes
  - For example, multi-part mechanical systems: Parts and subsystems interact through different articulations; interactions due to supporting forces

- ”Large-scale system” can also be originated from mathematical analysis
  - For example, discretization of a continuous time dynamic optimization problem: each discretization point depends only on proximate points – points far away from each other loosely coupled

- Regardless of origin of a large-scale system, mathematical description of the system has utilizable structure
  - For example, the Jacobian of the constraints of a discretized dynamic optimization almost block diagonal
Important solution paradigms

- Analysis of large-scale systems using decentralizing methods enables parallel and distributed computation
  - A single processor for each single subsystem (lower level)
  - One processor coordinates computation (upper level)

- Algorithms and data structures for sparse matrices
  - Decentralization on algorithm level
How do large-scale systems relate to the assignment?

- The thermal power plant is a large-scale system
  - Three subsystems connected each other loosely: Steam production, Steam distribution, Steam battery
- First: Each subsystem is tuned separately
- Second: Interactions between the subsystems are taken into account and the tuning is updated
- Iterative process
Comments / hints, Exercises 1-2

• Exercise 1:
  – Read the work instructions!
  – Familiarize yourself with the Simulink model of a power plant
  – The system is initially in a steady/equilibrium state – the steady state values of the variables are given in the work instructions

• Exercise 2:
  – Control variables of the upper section: fuel injection $u$, setting of the valve of the turbine flow $z_1$
  – Turbine flow increases (="disturbance") => How much the setting of the valve change? => What happens to the pressure of the high pressure stock?
Exercise 3:
- Operation of the upper section of the plant is only analyzed!
- Offset of the pressure of the high pressure stock from the steady state pressure must be below 2%
- Rapid changes in the fuel injection is not preferable (such "control" fuels are expensive)
  => Multiple objective optimization problem (stable pressure of the high pressure stock is more important!)
- *Control of the fuel injection (u) using feedback from the pressure of the high pressure stock (pkp)*
- Modify the Simulink model
- Tune P-controller with the step response experiment (u=+1kg/s) => How the controller works when turbine flow is changed? => No good! => Tune PI => How it works? => Tune PID => How it works?
- Tuning of PID (e.g., Åström & Murray, Chapter 10)
- Maximum value of the derivative of the response using, e.g., difference approximation
Comments / hints, Exercise 4

• Exercise 4:
  – State feedback controller for the upper section
  – *Control, i.e., the fuel injection* \((u)\) *is the linear combination of the steam generation in the boiler\((fp)\), the pressure of the boiler\((pk)\) and the pressure of the high pressure stock\((pkp)\)*
  – The model of the upper section must be linearized! A linear open loop system is controllable \(\Rightarrow\) arbitrary dynamics for the closed loop system \(\Rightarrow\) stabilization possible
  – Modify the simulink model
  – Tune the gains of the controller such that the eigenvalues of the system matrix of the closed loop system are on the left-half complex plane
  – Linear quadratic dynamic optimization problem \(\Rightarrow\) Riccati differential equation \(\Rightarrow\) algebraic Riccati equation (Matlab’s functions lqr, lqr2)
  – Select the weight matrices of the criterion appropriately; compare different matrices; study eigenvalues of A-BK (should be on the left-half complex plane)
  – Other means for defining appropriate eigenvalues laborious!!!
Comments / hints, Exercises 5-6

• Exercise 5:
  – Issue on the state feedback controller – fixed offset in the output from the steady state output, cf. P-controller
  – Extend the system by taking into account a new state variable
  – Time derivative of the new state variable is
    \[ \dot{x} = \int_0^t (r_k - p_k) \, dt \]
    \[ x = \int_0^t (p_k r_0 - p_k r) \, dt \]
    the error quantity, i.e., the variable is
    the integral of the error quantity
  – Tune the controller using the solution of the LQ problem – selection of the weight matrices of the criterion – comparisons
  – How the controller works?

• Exercise 6:
  – Compare and discuss the application of the PID-controller and the state feedback controller – Which one is better?
  – Go with PID
Comments / hints, Exercise 7

• Exercise 7:
  – Reduction of the upper section model into a first order system
  – Linearize the upper section; write the linearized model in the form of a transfer function (Laplace-transformation; frequency space)
  – Construct the Pade approximation for the transfer function
    • See, Norton pp. 225-227
    • Derive the Taylor series of the transfer function
    • Set the series equal to the first order Pade approximation (rational function approximation) => Parameters of Pade
  – State space representation into transfer function: Matlab’s function ss2tf
  – Taylor series using, e.g., Mathematica’s function ”series”
  – Create a simple Simulink model containing only the transfer functions of the Pade approximation (1-3 functions)
  – Compare the step responses provided by the original system and the approximation
  – The approximation is not used in the following exercises
Comments / hints, Exercises 8-9

- **Exercise 8:**
  - The upper section of the plant works fantastically (after exercise 6)
  - Consumption flow of steam (=”disturbance”) suddenly changes => How does this affect the pressure of the counter pressure stock? (Should be within 10% of the equilibrium value)

- **Exercise 9:**
  - *Control of the turbine flow (z1) using feedback from the pressure of the counter pressure stock (pvp)*
  - Modify the Simulink model
  - P, PI, I, ID, PID controllers – tune such that the counter pressure within acceptable limits, i.e., close to the equilibrium value
  - How different controllers work? Only experiments - e.g., tuning of the controllers based on the step response is not required!
  - Controls of the turbine flow and the fuel injection take care of low frequency and large amplitude disturbances in steam consumption
Exercise 10:

- The steam battery is included in the analysis
- Control of the input and output flow of the steam battery (charge and discharge valves z2 and z3) using feedback from the pressure of the counter pressure stock (pvp)
- Ready made P-controllers in the Simulink model; saturation limits for Z2 and Z3 have so far been zero => modify them to appropriate values given in the work instructions
- Find appropriate gains (Kin ja Kout) such that the controllers of the steam battery and the turbine flow provide jointly the good behaviour of the plant – try & test alternative disturbances in steam consumption
- The steam battery takes care of large frequency disturbances in steam consumption => compensate sudden consumption changes => no variations in the fuel injection
Comments / hints, Exercise 11-12

• Exercise 11:
  – Adjust the gains of all the controllers such that
    • The counter pressure stays within the given limits
    • Fluctuations in the pressure of the high pressure stock as small as possible
    • Use of expensive ”control” fuel in the heating of the boiler minimized
  – Test the operation of the plant as a whole
  – Different step and ramp disturbances in the steam consumption – also frequency responses (high frequencies, low frequencies)
  – Give a general recommendation to the consumer of steam regarding
    • The amplitudes of disturbances allowed at high and low frequencies
    • The size of step disturbances allowed with the given limits of the counter pressure

• Exercise 12:
  – Write the report – see the work instructions
References

• An Introduction to Identification; Norton J.P., Academic Press, 2009
• Optimal Control Theory; Kirk D.E., Prentice-Hall, 2004