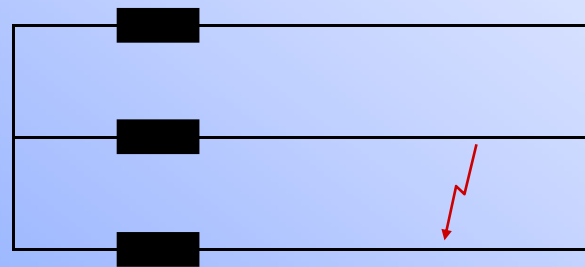


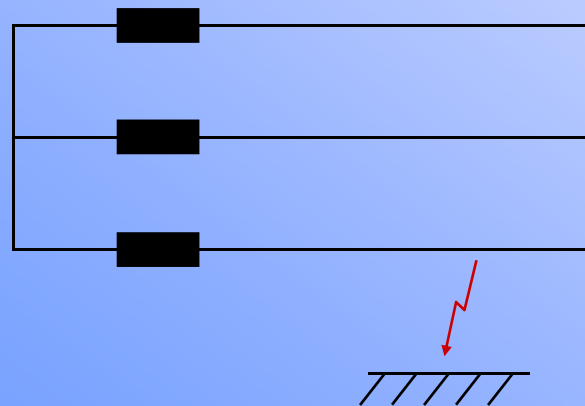
Power system faults



3 - phase
short circuit

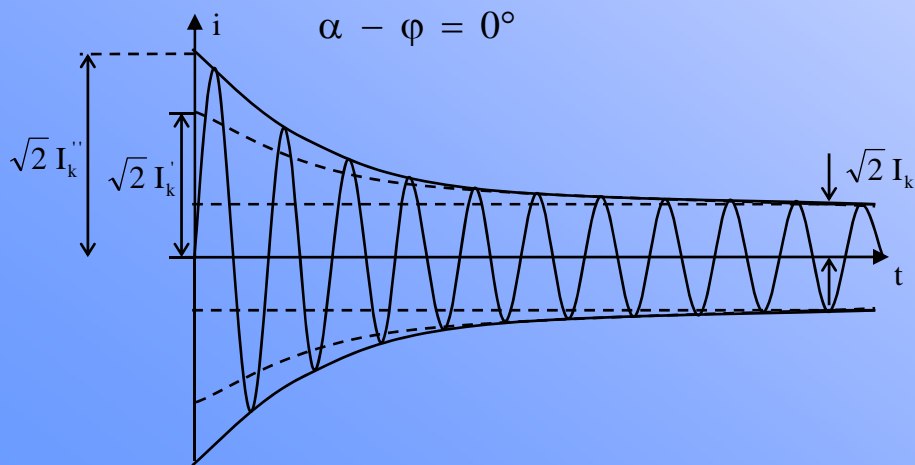
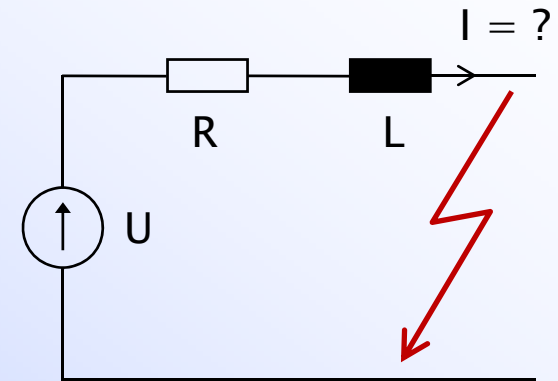
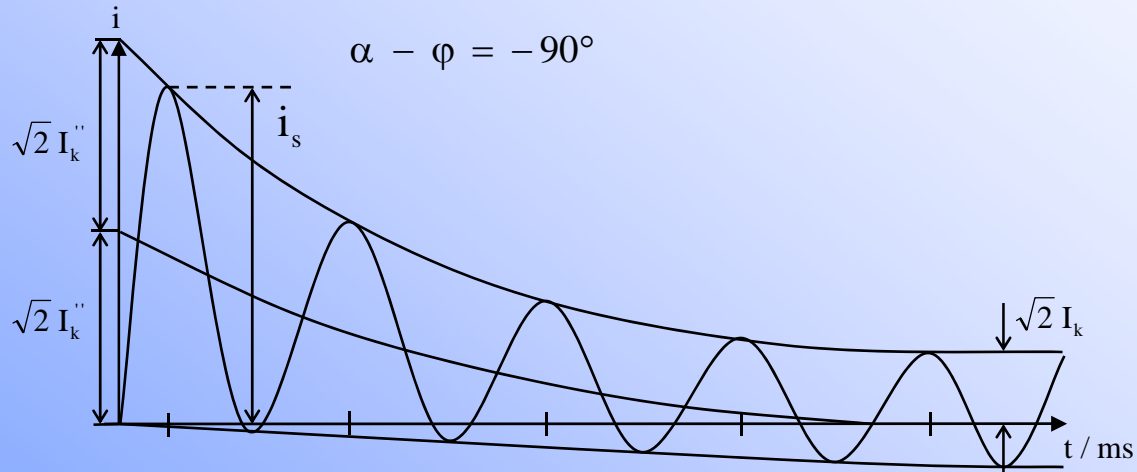


2 - phase
short circuit



1 - phase
earth fault

The nature of short circuit current



$$X_d'' < X_d' < X_d$$

$$\Rightarrow I_k'' > I_k' > I_k$$

The nature of short circuit current

$$u = \sqrt{2} U \sin(\omega t + \alpha)$$

α = switching time after the voltage zero crossing,
the time t is computed since this moment

$$\begin{aligned} Ri + L \frac{di}{dt} &= \sqrt{2} U \sin(\omega t + \alpha) \\ \Rightarrow \frac{\sqrt{2} U}{Z} \left[\sin(\omega t + \alpha - \varphi) - e^{-\frac{R}{L}t} \sin(\alpha - \varphi) \right] \\ Z &= \sqrt{R^2 + (\omega L)^2} \quad \because \quad \tan \varphi = \frac{\omega L}{R} \end{aligned}$$

Peak short circuit current:

$$\begin{aligned} I_s &= H \sqrt{2} I_K'' \\ H &= f\left(\frac{R}{X}\right) = \text{attenuation factor} \\ I_K'' &= \text{subtransient short circuit current} \end{aligned}$$

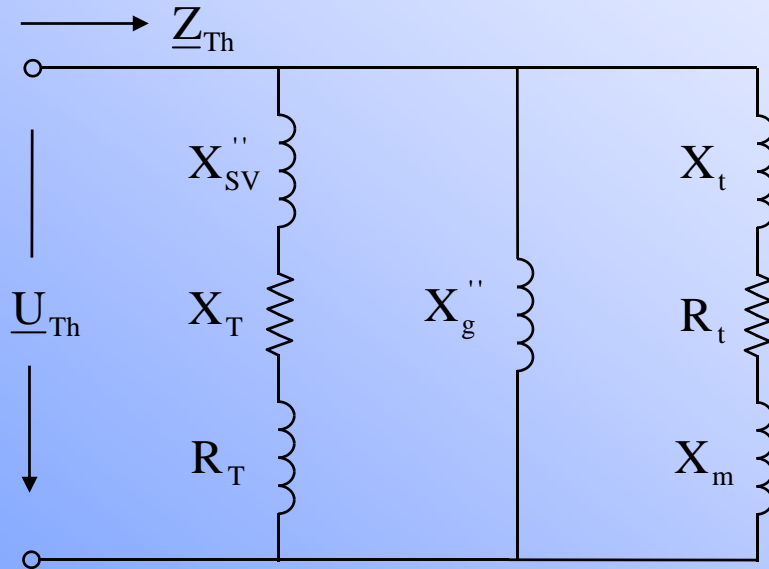
At transmission voltages:

$$\begin{aligned} H &\approx 1,8 \quad \left(\frac{R}{X} \approx 0,07 \right) \\ \Rightarrow I_s &= 2,5 I_K'' \end{aligned}$$

*Note: I_k'' is rms value
 I_s max instantaneous value*

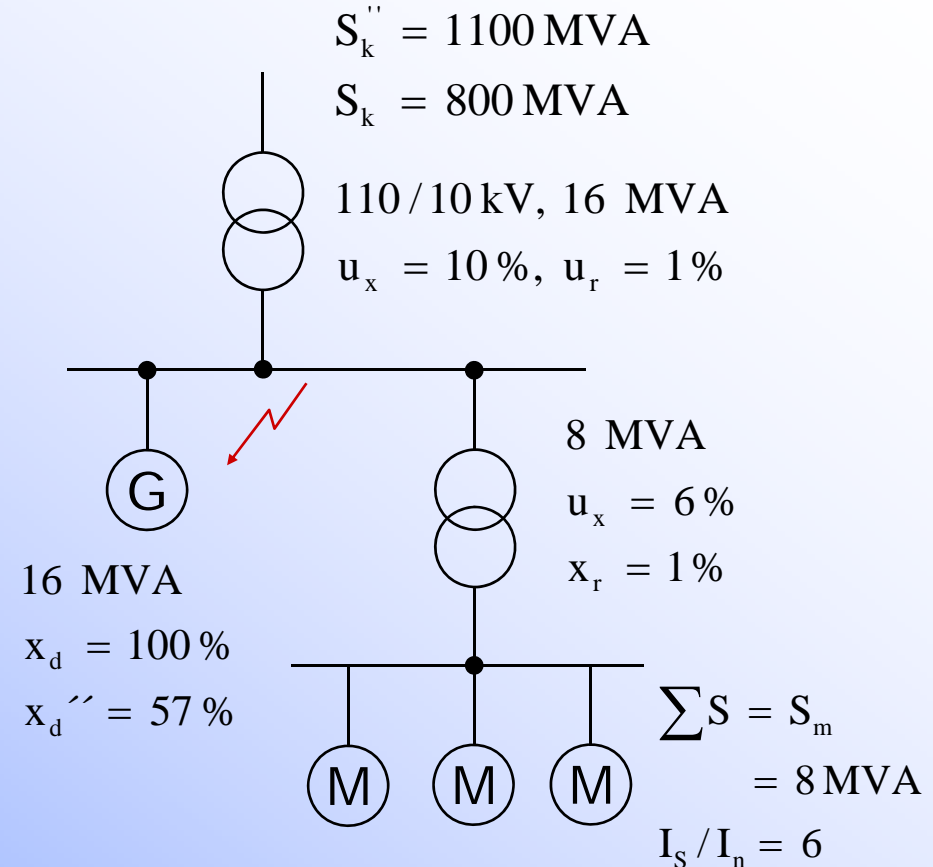
Example: 3-phase short circuit in a power plant

One line diagram:



Subtransient current $I_k'' = ?$

Computation at 10 kV voltage level



1. Power transmission grid: \underline{z}_v

$$\underline{z}_v = R_T + j(X_T + X_{SV}'')$$

$$= u_r \frac{U^2}{S_T} + j \left(u_x \frac{U^2}{S_T} + \frac{U^2}{S_k''} \right)$$

$$= 0,0625 + j0,716 \Omega = 0,719 \angle 85^\circ \Omega$$

2. Generator influence \underline{z}_g

$$\underline{z}_g = j x_d'' \frac{U^2}{S_k} = j 0,57 \frac{10^2}{16} = 3,563 \angle 90^\circ \Omega$$



Example: 3-phase short circuit

3. The motors \underline{Z}_m

$$\begin{aligned}\underline{Z}_m &= \underline{R}_t + j(x_t + x_m) \\ &= u_r \frac{U^2}{S_t} + j \left(u_x \frac{U^2}{S_t} + \frac{U^2}{I_s / I_n \cdot S_m} \right) \\ &= 0,125 + j2,833 \Omega = 2,836 \angle 87,5^\circ \Omega\end{aligned}$$

4. The Thevenin impedance

$$\underline{Z}_{Th} = \left(\frac{1}{\underline{Z}_v} + \frac{1}{\underline{Z}_g} + \frac{1}{\underline{Z}_m} \right)^{-1} = 0,494 \angle 86,1^\circ \Omega$$

$$\underline{u}_{Th} = \frac{10}{\sqrt{3}} \angle 0^\circ \text{ kV}$$

$$\Rightarrow \underline{I}_k'' = \frac{\underline{u}_{Th}}{\underline{Z}_{Th}} = \underline{\underline{11,69 \angle -86,1^\circ \text{ kA}}}$$

Example: 3-phase short circuit

A simple way : $R \ll x$, ol. $R \sim 0$

\Rightarrow computation using powers S

Subtransient short circuit power in busbar :

$$S_k'' = S_{k,sv}'' + S_{k,g}'' + S_{k,m}''$$

1. Transmission grid :

$$S_{k,sv}'' = \frac{S_k'' \cdot S_{kT}}{S_k'' + S_{kT}} \quad ; \quad S_{kT} = \frac{1}{u_x} \cdot S = 160 \text{ MVA}$$

$$= \frac{1100 \cdot 160}{1100 + 160} \text{ MVA} = 139,7 \text{ MVA}$$

2. Generator :

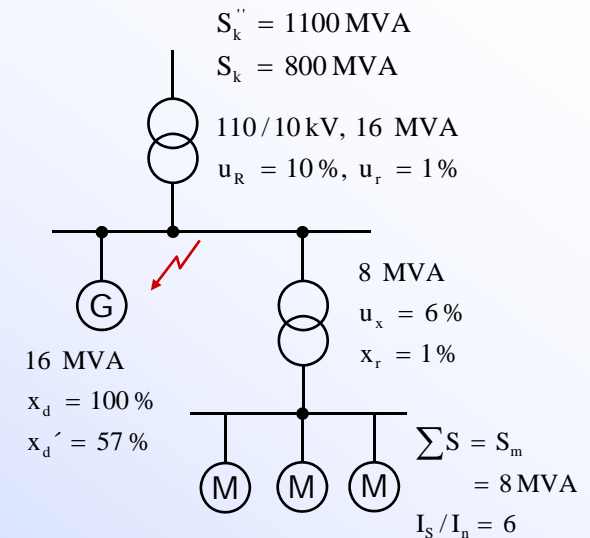
$$S_{k,g}'' = \frac{1}{x_d''} S_g = 28,1 \text{ MVA}$$

3. Motors :

$$S_{k,m}'' = \frac{S_{kT} \cdot S_{km}}{S_{kT} + S_{km}} \quad ; \quad S_{km} = I_s / I_n \cdot S_m = 48 \text{ MVA}$$

$$S_{k,t} = \frac{1}{u_x} S = \frac{8}{0,06} = 133,3 \text{ MVA}$$

$$= \frac{133,3 \cdot 48}{133,3 + 48} \text{ MVA} = 35,3 \text{ MVA}$$



Example: 3-phase short circuit

$$\begin{aligned} S_k'' &= S_{k,sv}'' + S_{k,g}'' + S_{k,m}'' \\ &= 139,7 + 28,1 + 35,3 \text{ MVA} \\ &= 203,1 \text{ MVA} \end{aligned}$$

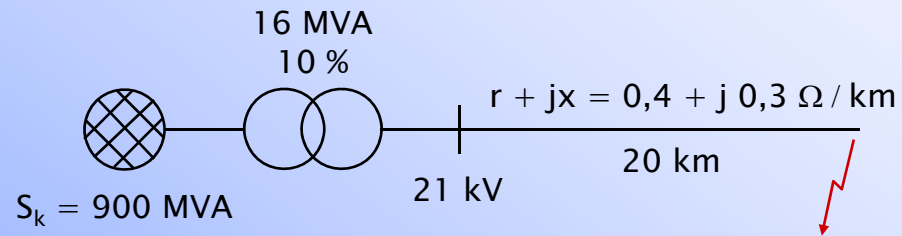
$$\begin{aligned} S_k'' &= \sqrt{3} U I_k'' \\ \Rightarrow I_k'' &= \frac{S_k''}{\sqrt{3} U} = \underline{11,73 \text{ kA}} \end{aligned}$$

Steady state I_k ? (motors do not affect)

$$\begin{aligned} S_k &= S_{k,sv} + S_{kg} \\ &= \frac{S_k \cdot S_{kT}}{S_k + S_{kT}} + \frac{1}{x_d} S_g \\ &= \frac{800 \cdot 160}{800 + 160} + \frac{1}{1,0} \cdot 16 \text{ MVA} = 149,3 \text{ MVA} \end{aligned}$$

$$\Rightarrow I_k = \frac{S_k}{\sqrt{3} U} = 8,62 \text{ kA}$$

Example : MV-distribution line & 3-ph vs. 2-ph faults



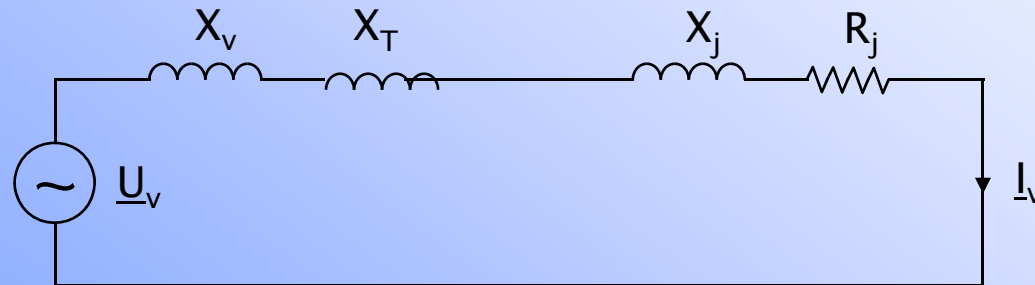
*X_v&X_t grid&transformer reactance
X_j&R_j line reactance&resistance*

$$\underline{z}_j = R_j + jX_j = 8 + j6 \Omega$$

$$X_T = u_x \frac{21^2}{16} \Omega = 2,76 \Omega$$

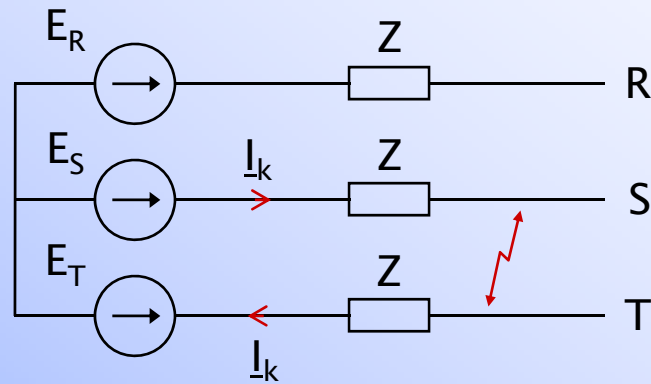
$$X_v = \frac{U^2}{S} = \frac{21^2}{900} \Omega = 0,49 \Omega$$

3-phase fault current



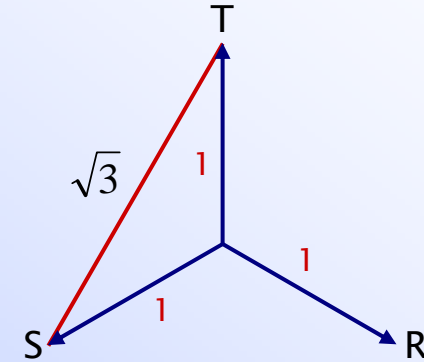
$$\begin{aligned} \underline{I}_k &= \frac{\underline{U}_v}{\underline{z}} = \frac{\underline{U}_v}{R_j + j(x_v + x_T + x_j)} ; U_v = \frac{U_p}{\sqrt{3}} = \frac{21}{\sqrt{3}} \\ &= \frac{21}{\sqrt{3} \cdot 12,23 \angle 49^\circ} \text{ kA} \\ &= \underline{\underline{0,99 \angle -49^\circ \text{ kA}}} \end{aligned}$$

2-phase fault current



$$\begin{cases} \underline{U}_{Th} = \underline{E}_S - \underline{E}_T \\ \underline{Z}_{Th} = 2\underline{Z} \end{cases}$$

$$\underline{I}_k = \frac{\underline{E}_S - \underline{E}_T}{2\underline{Z}} = \frac{\sqrt{3} E_R \angle -90^\circ}{2\underline{Z}}$$



With the numbers:

$$\underline{I}_k = \frac{\sqrt{3}}{2} \frac{E_R}{Z} = \frac{\sqrt{3}}{2} \cdot 0,99 \text{ kA} (= 0,86 \text{ kA})$$

⇒

$$I_{k2v} = \frac{\sqrt{3}}{2} \cdot I_{k3v}$$

(holds, when $\underline{Z}_1 = \underline{Z}_2$)

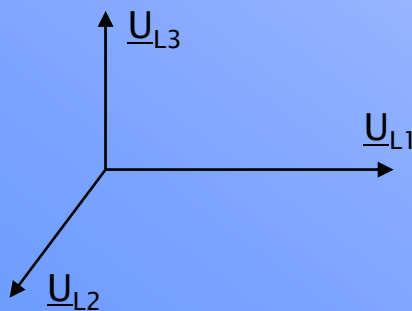
Symmetrical components

The relation between phase quantities and symmetrical components:

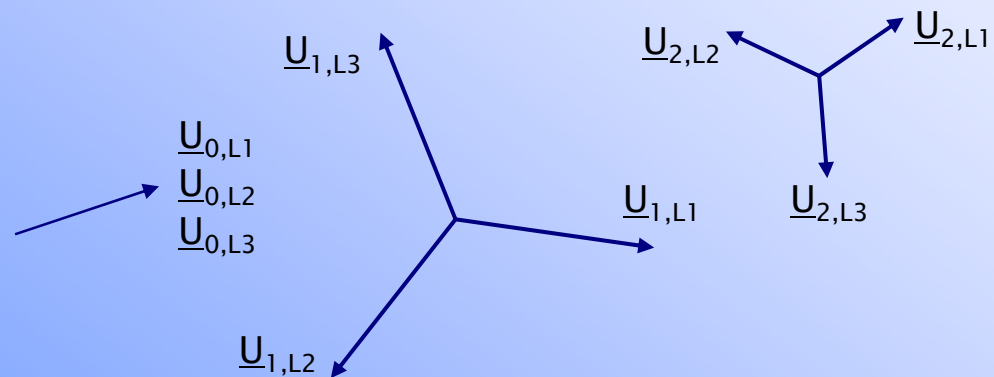
$$\begin{bmatrix} \underline{U}_{L1} \\ \underline{U}_{L2} \\ \underline{U}_{L3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} \begin{bmatrix} \underline{U}_0 \\ \underline{U}_1 \\ \underline{U}_2 \end{bmatrix}$$

The left hand vector includes the phase voltages, the right hand vector zero-, positive- and negative sequence voltages. The transformation matrix is defined using the phase shift operator $\underline{a} = 1 \angle 120^\circ$. The inverse transformation is:

$$\begin{bmatrix} \underline{U}_0 \\ \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \begin{bmatrix} \underline{U}_{L1} \\ \underline{U}_{L2} \\ \underline{U}_{L3} \end{bmatrix}$$



An example case of phase voltages



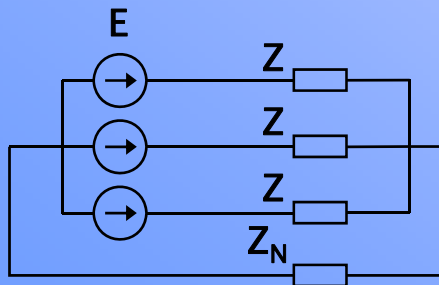
And the corresponding symmetrical components

Symmetrical components

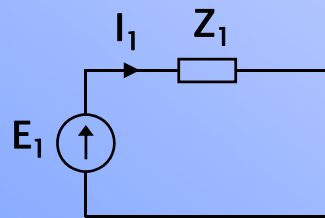
Positive sequence system includes the normal symmetric three phase system. Computations can be done using a one line diagram.

The negative sequence components are similar to positive ones, but they rotate in an opposite order. For passive components, the impedance Z_1 and Z_2 are equal, but not for rotating machines.

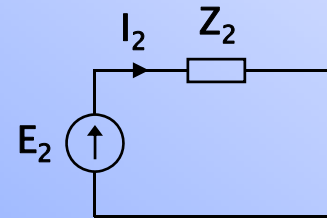
The zero sequence component is similar in all the three phases. It causes a current, which has to return through neutral wire, where the current I_0 is hence three-fold. For this reason the impedance of the neutral wire is also taken three fold.



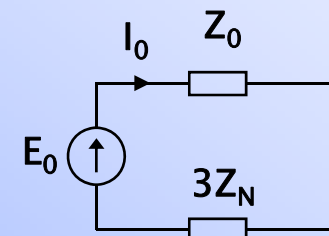
Network example



Positive sequence network

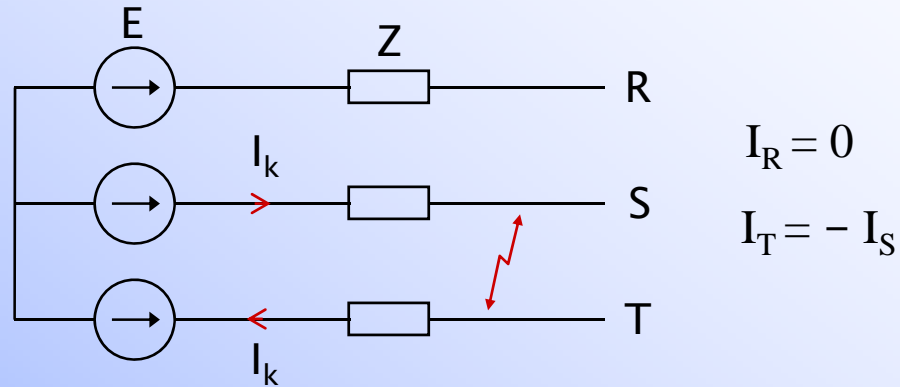


Negative sequence



Zero sequence

2-phase short circuit using symmetric components



$$\begin{cases} I_0 \\ I_1 \\ I_2 \end{cases} = \frac{1}{3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{vmatrix} \begin{cases} I_R \\ I_S \\ I_T \end{cases} \quad \begin{matrix} 1. \\ 2. \\ 3. \end{matrix}$$

$$\underline{a} = 1 \angle 120^\circ$$

$$1. \Rightarrow I_0 = 0$$

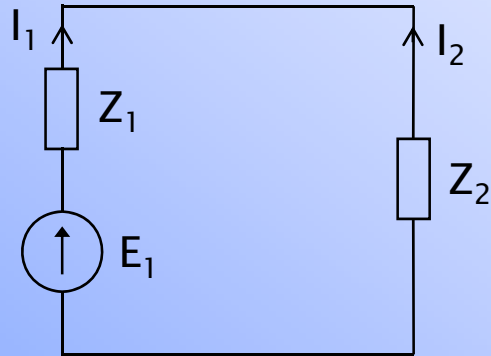
$$2. \ \& \ 3. \ \& \ I_T = -I_S \Rightarrow I_2 = -I_1$$

Voltage source is symmetric:

$$\Rightarrow E_1 = E_R \ ; \ E_2 = 0 \ ; \ E_0 = 0$$

2-phase short circuit using symmetric components

One line diagram:



$$\underline{I}_0 = 0$$

$$\underline{I}_1 = \frac{\underline{E}_R}{\underline{Z}_1 + \underline{Z}_2}$$

$$\underline{I}_2 = -\underline{I}_1$$

$$\begin{vmatrix} \underline{I}_R \\ \underline{I}_S \\ \underline{I}_T \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{vmatrix} \begin{vmatrix} \underline{I}_0 \\ \underline{I}_1 \\ \underline{I}_2 \end{vmatrix}$$

Let us solve the \underline{I}_S :

$$\begin{aligned} \underline{I}_S &= \underline{a}^2 \underline{I}_1 + \underline{a} \underline{I}_2 \\ &= \frac{\underline{a}^2 \underline{E}_R - \underline{a} \underline{E}_R}{\underline{Z}_1 + \underline{Z}_2} = \frac{\sqrt{3} \underline{E}_R \angle -90^\circ}{\underline{Z}_1 + \underline{Z}_2} \end{aligned}$$

IF $\underline{Z}_1 = \underline{Z}_2 = \underline{Z}$, then

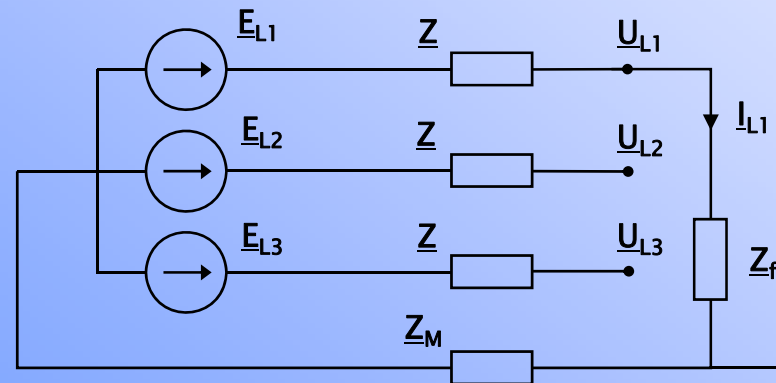
$$\underline{I}_S = \frac{\sqrt{3} \underline{E}_R \angle -90^\circ}{2\underline{Z}}$$

Single phase earth fault

One phase conductor is connected into the ground. The fault is unsymmetric. *The zero sequence system strongly depends on how Neutral is grounded.*

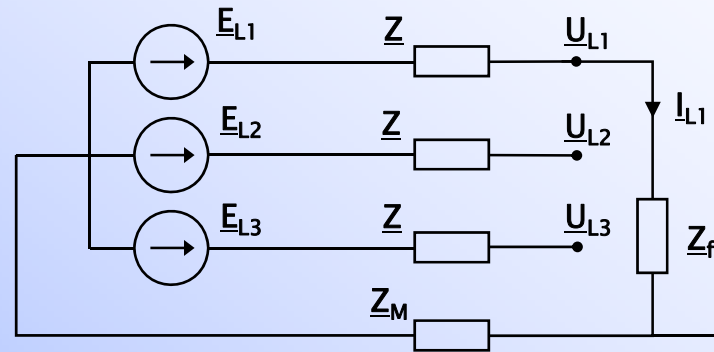
The earth fault differs clearly from the short circuit faults. In ungrounded or compensated neutral systems, the fault current is small. The voltage between sound phases and ground rises (up to the line voltage).

In Finland the 10 kV and 20 kV systems usually are ungrounded or compensated neutral systems.



One phase earth fault

One phase earth fault



Solution using symmetric components

During the earth fault: $\underline{U}_{L1} = \underline{Z}_f \underline{I}_{L1}$

$$\underline{I}_{L2} = 0$$

$$\underline{I}_{L3} = 0$$

$$\begin{bmatrix} \underline{U}_{L1} \\ \underline{U}_{L2} \\ \underline{U}_{L3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} \begin{bmatrix} \underline{U}_0 \\ \underline{U}_1 \\ \underline{U}_2 \end{bmatrix}$$

It follows: $\underline{U}_0 + \underline{U}_1 + \underline{U}_2 = \underline{Z}_f \underline{I}_{L1} *$

$$\underline{a} = 1 \angle 120^\circ.$$

$$\left. \begin{array}{l} \underline{I}_0 + \underline{a}^2 \underline{I}_1 + \underline{a} \underline{I}_2 = 0 \\ \underline{I}_0 + \underline{a} \underline{I}_1 + \underline{a}^2 \underline{I}_2 = 0 \end{array} \right\} \underline{I}_0 = \underline{I}_1 = \underline{I}_2$$

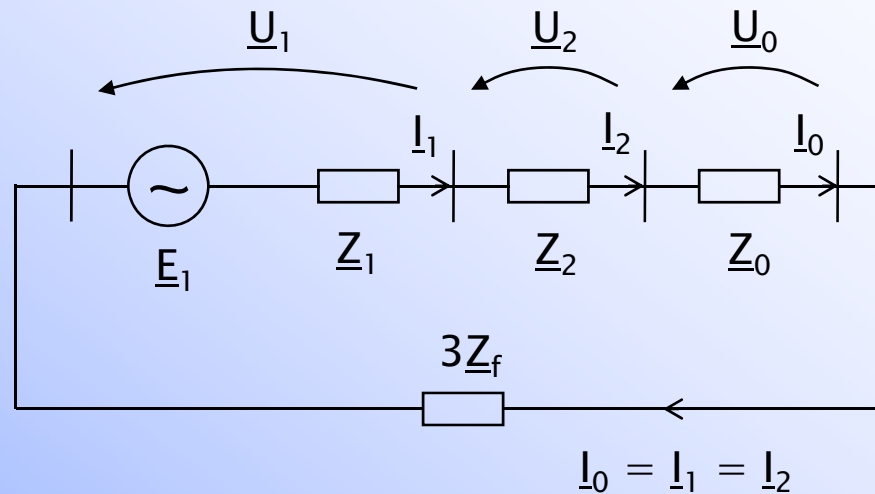
$$\Rightarrow \underline{U}_0 = -\underline{Z}_0 \underline{I}_0$$

$$\underline{U}_1 = \underline{E}_1 - \underline{Z}_1 \underline{I}_1$$

$$\underline{U}_2 = -\underline{Z}_2 \underline{I}_2$$

$$* \quad -\underline{Z}_0 \underline{I}_0 + \underline{E}_1 - \underline{Z}_1 \underline{I}_1 - \underline{Z}_2 \underline{I}_2 = 3 \underline{Z}_f \underline{I}_0 \quad \rightarrow$$

One phase earth fault



Component networks are in series connection

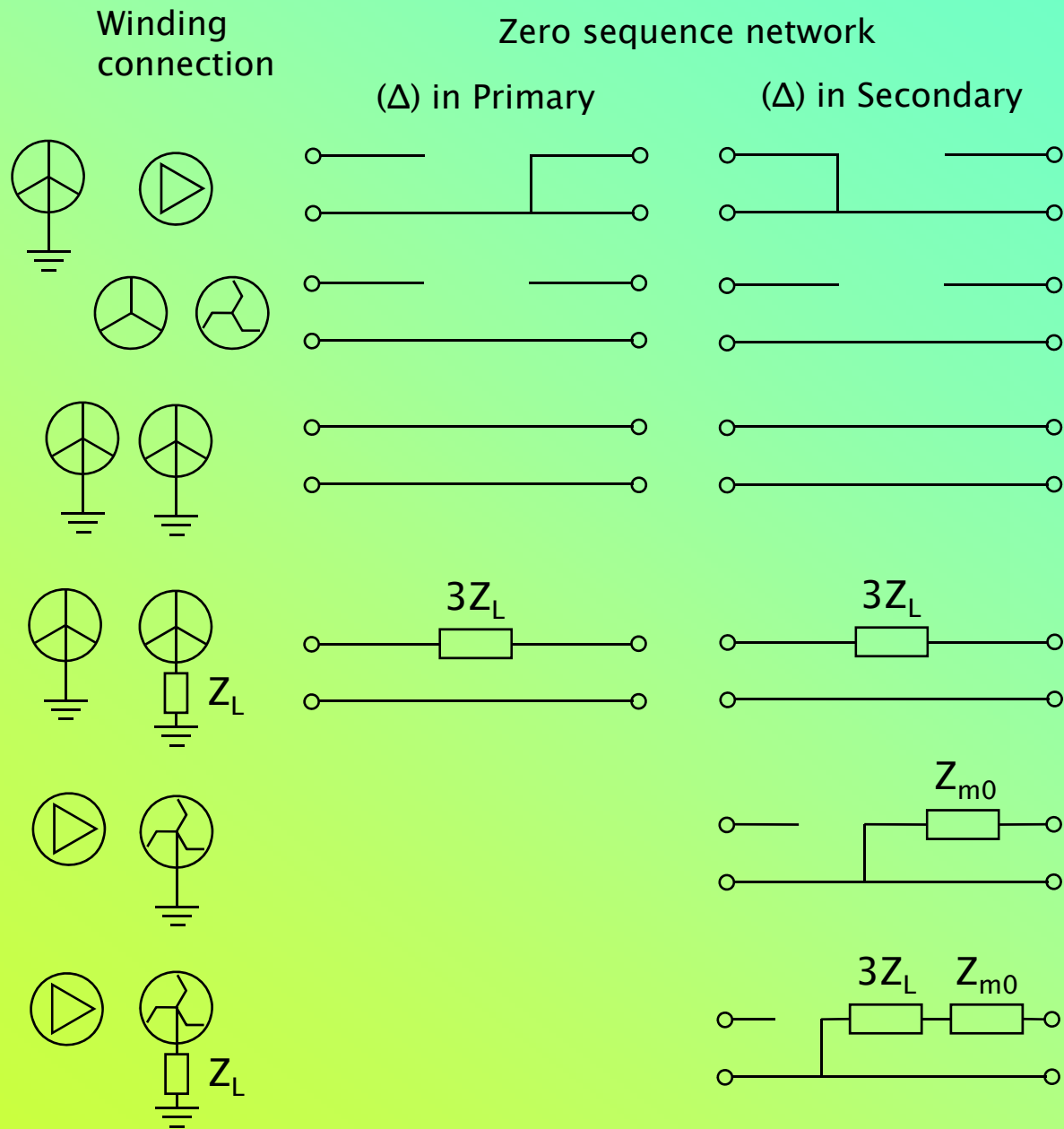
Which gives for the zero sequence current:

$$\underline{I}_0 = \frac{\underline{E}_1}{\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2 + 3\underline{Z}_f} = \frac{\underline{E}_1}{3\underline{Z} + 3\underline{Z}_M + 3\underline{Z}_f}$$

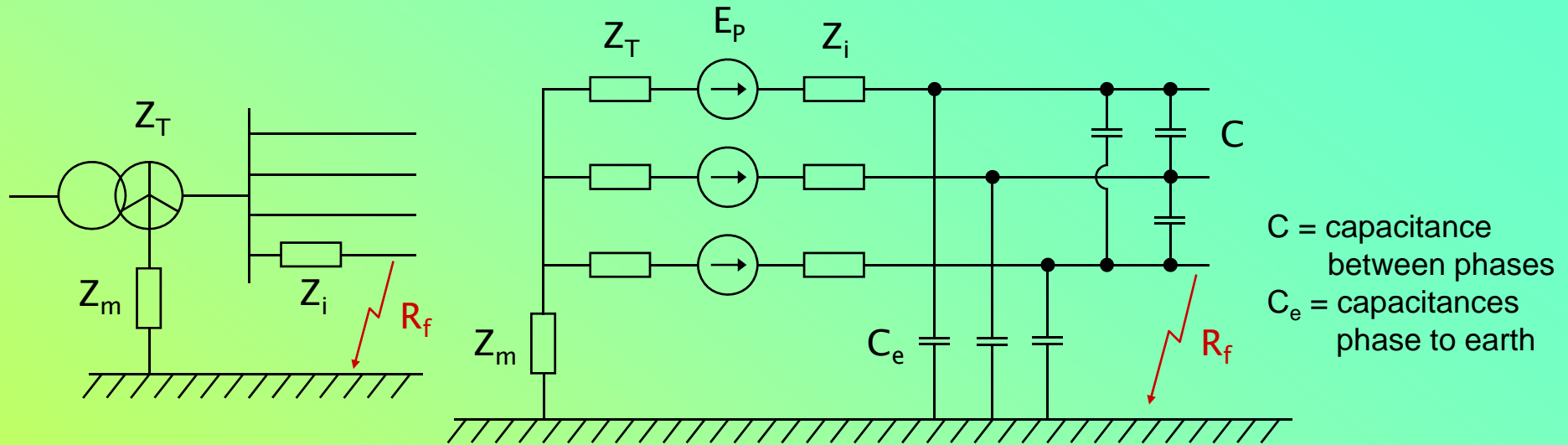
The total fault current is three times the zero sequence current:

$$\underline{I}_f = 3\underline{I}_0 = \frac{3\underline{E}_1}{\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2 + 3\underline{Z}_f} = \frac{\underline{E}_1}{\underline{Z} + \underline{Z}_M + \underline{Z}_f}$$

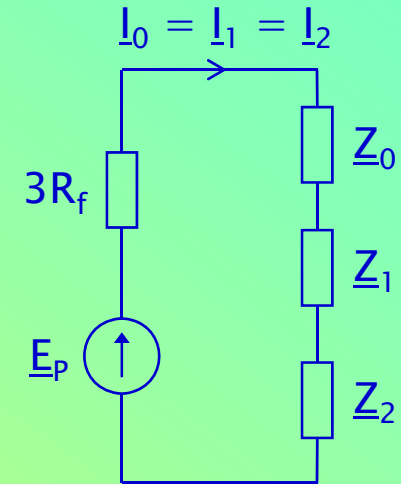
Transformer windings in zero sequence networks



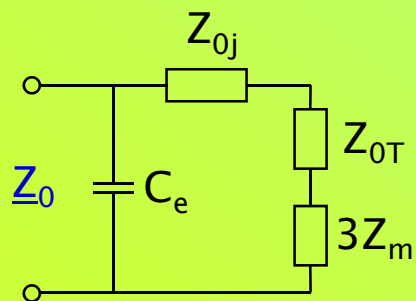
Solution for 1-phase earth fault :



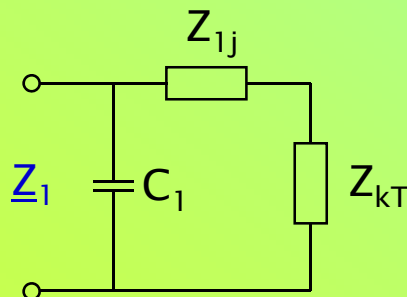
$$\underline{I}_f = 3\underline{I}_0 = \frac{3\underline{E}_p}{\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2 + 3R_f} = \frac{\underline{E}_p}{\frac{1}{3}(\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2) + R_f}$$



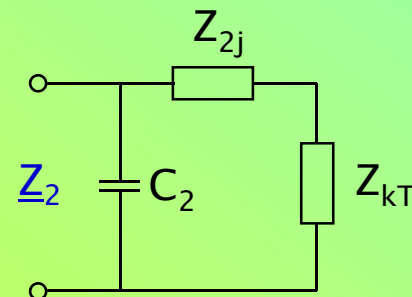
0-network



Positive seq. /



negative seq. ntwk



$$C_1 = C_e + 3C$$

$$C_1 = C_2$$

Comparison of 1-ph, 2-ph, 3-ph faults for fault currents (Thevenin's method) :

1-ph:
$$\underline{I}_f = \frac{\underline{E}_P}{\frac{1}{3}(\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2) + R_f}$$

2-ph:
$$\underline{I}_f = \frac{\sqrt{3}\underline{E}_P}{\underline{Z}_1 + \underline{Z}_2 + R_f}$$

3-ph:
$$\underline{I}_f = \frac{\underline{E}_P}{\underline{Z}_1 + R_f}$$

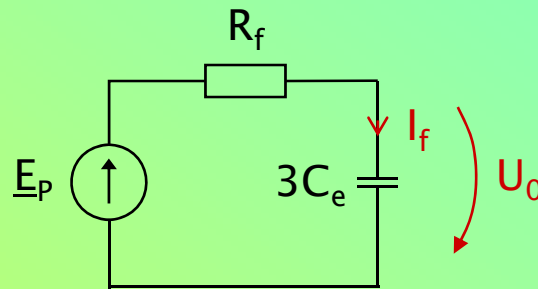
*Ep is phase voltage; Rf is fault resistance
Z1,Z2,Z0 are sequence impedances*

Earth faults in ungrounded systems :

$$\underline{Z}_0 \approx \frac{1}{j\omega C_e}$$

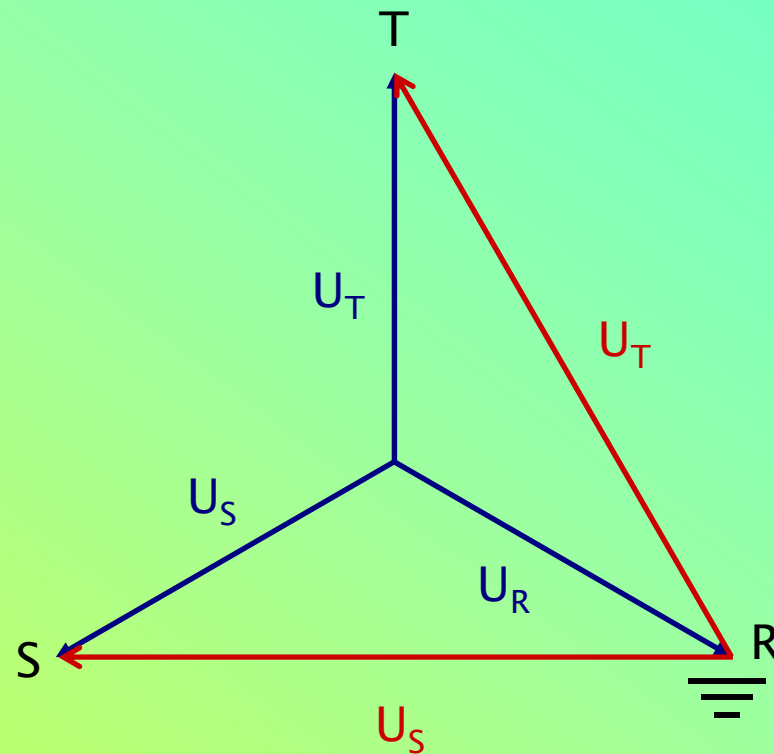
$$\underline{Z}_0 \gg \underline{Z}_1 \quad \& \quad \underline{Z}_0 \gg \underline{Z}_2$$

⇒ one line diagram:



$$\left\{ \begin{array}{l} \underline{I}_f = \frac{\underline{E}_p}{R_f + \frac{1}{j\omega 3C_e}} \\ \frac{U_0}{E_p} = \frac{1}{\sqrt{1 + (3\omega C_e R_f)^2}} \end{array} \right.$$

Voltages during an earth fault :



In sound network

$$U_R = U_S = U_T = U_P$$

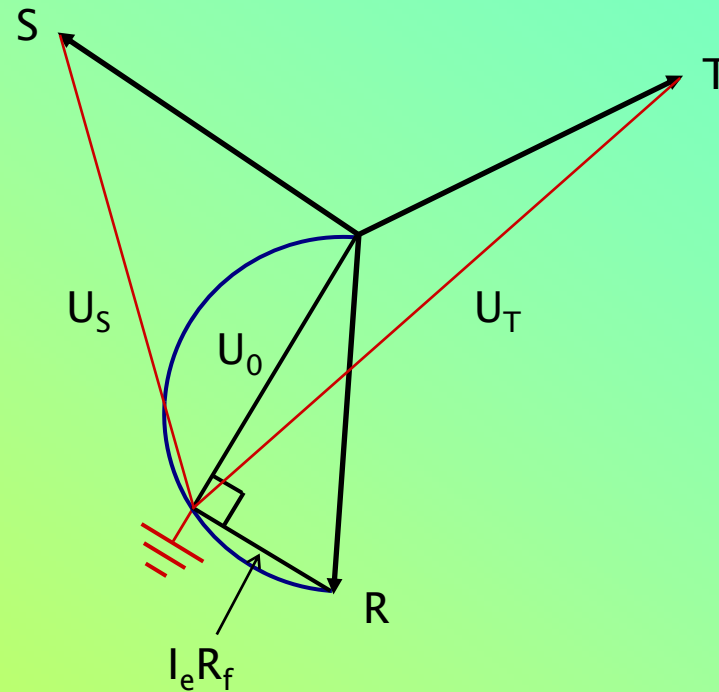
During earth fault, if $R_f = 0$

$$U_R = 0 \quad ; \quad U_T = \sqrt{3} U_P \quad ; \quad U_S = \sqrt{3} U_P$$

Voltages during an earth fault :

$$R_f > 0$$

$$\underline{U}_0 + \underline{I}_e R_f = -\underline{U}_P$$



$$U_T > \sqrt{3} U_P ?$$

Max. voltage for U_T , when

$$R_f = 0,37 \frac{1}{j\omega C_0}$$

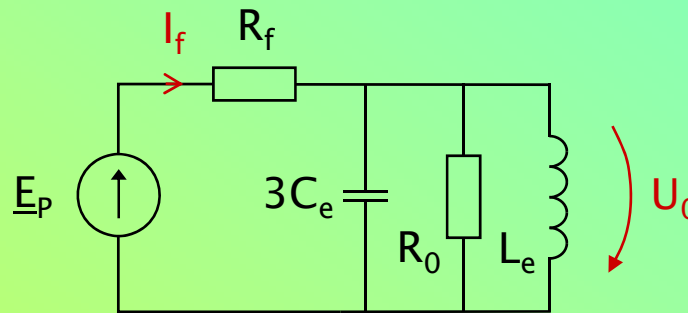
And $U_T = 1,05 \cdot \sqrt{3} \cdot U_P$

Compensated neutral systems : $Z_m = \omega L_e$

Inductance L_e is selected such that the current of C_e is cancelled

$$\underline{Y}_0 \approx j\omega C_e - \frac{1}{j\omega 3L_e} \approx 0$$

One line diagram:



R_0 is the network leakage resistance

If 100 % compensation :

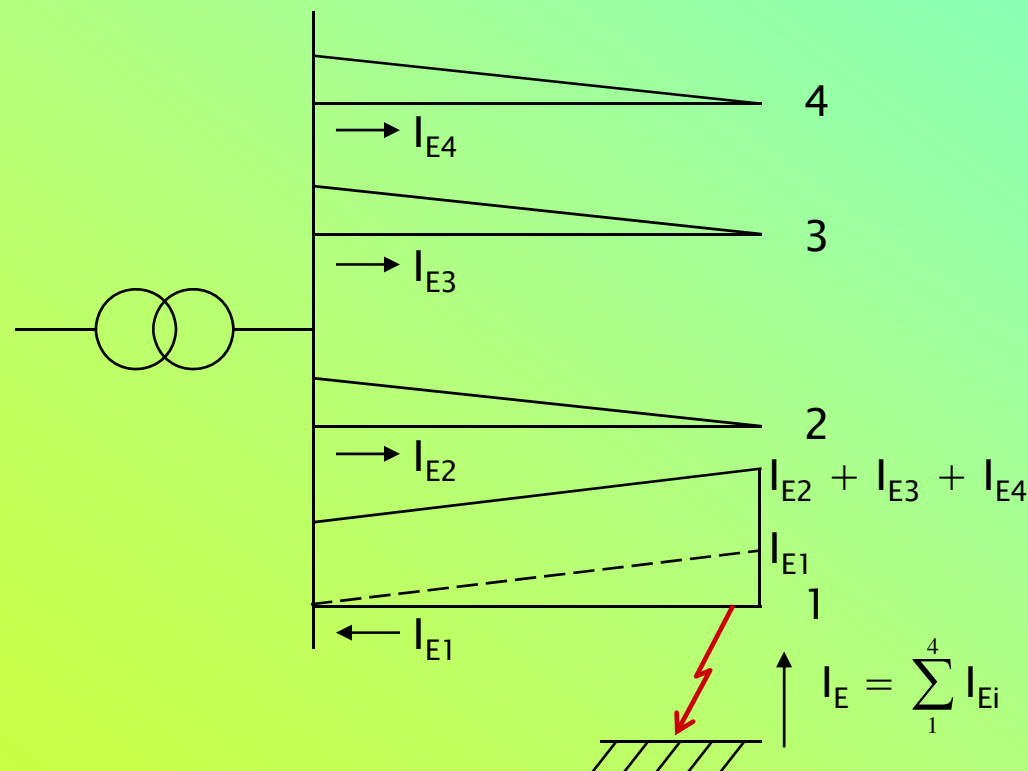
$$\begin{cases} I_f = \frac{E}{R_f + R_0} \\ \frac{U_0}{E} = \frac{R_0}{R_0 + R_f} \end{cases}$$

Earth fault currents in an ungrounded system

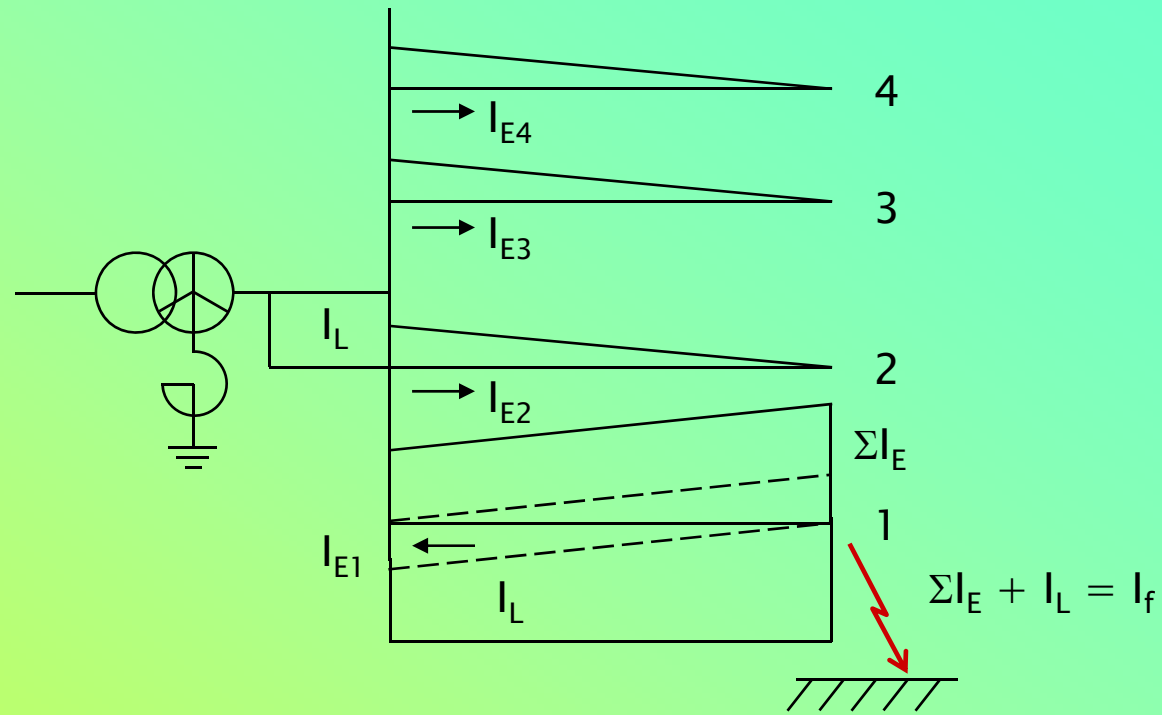
The current measured at the substation ($\sum I_0$) includes the current in the fault location less the current which flows through the earth capacitances of the faulty line:

$$\sum I_0 = \frac{C_0 - C_{01}}{C_0} I_{ef}$$

where C_0 is total earth capacitance of the network, C_{01} is earth capacitance of the line concerned and I_{ef} is the total earth fault current.



Earth fault currents in a compensated system



Current in the fault location in case of 100% compensation

