Power consumption

- Power system loads need for proper operation
  - real power
  - (inductive) reactive power
- Real power is used in
  - load resistances
  - network component series resistances
  - network component shunt admittances

Power production

- Real power is produced by
  - generators in power plants
- Reactive power is produced by
  - synchronous machines (~ exitation)
  - capacitors (thyristor controlled ?)
  - line shunt capacitances
- Real power must be transmitted from power plant to the site of consumption

Generators

- Mostly produce real power
- Reactive power generation / absorption in disturbance situations
- VAr absorption ability is limited
- VAr control = voltage control = exitation current control

Parallel capacitors

- Overcompensated if VAr produced > load VAr demand

Series capacitors

- Series capacitor reduces the line reactance
- Main goal is to improve the stability (X reduction)

Dependence of (P, Q) ~ (δ, V)

In transmission systems δ is usually small and R << X

Power angle equation :

$$P = \frac{U_1 U_2}{X_2} \sin\delta$$

$$Q = \frac{U_1^2}{X} - \frac{U_1 U_2}{X} \cos\delta$$

$$\frac{\partial P}{\partial \delta} = \frac{U_1 U_2}{X} \cos\delta \qquad \cos\delta \approx 1$$

Real power mostly depends on angle δ

$$\frac{\partial P}{\partial U_1} = \frac{U_2}{X} \sin\delta \qquad \sin\delta \approx 0$$

$$\frac{\partial Q}{\partial \delta} = \frac{U_1 U_2}{X} \sin\delta$$

$$\frac{\partial Q}{\partial U_1} = \frac{2U_1}{X} - \frac{U_2}{X} \cos\delta$$

Reactive power mostly depends on voltages

Large generation deficiencies

- Frequency starts to fall (rate depends on stiffness)
- Automatic power control tries to increase production
- If frequency still falls, loads are disconnected
  – two 10 % steps
- Power system is divided into parts
  – large cities may be isolated networks
- Turbogenerators disconnected, when
  $f = 47...48$ Hz (risk of mechanical resonance)

Frequency control of a power system

- Frequency is the same for the whole power system
- When the load exceeds production, the frequency starts to fall
- All the frequency controllers of generators react simultaneously
- The share of power plants in power control depends on
  - plant size
  - existing reserve capacity
  - droop settings

Power system real power control

The kinetic energy in rotating masses :

$$1.\quad W_k = \frac{1}{2} J\omega^2$$

Nordic system: $W_k \approx 300\,000$ MWs

Change of load balance & frequency :

$$2.\quad \Delta P = \frac{\partial W_k}{\partial t} = \omega J \frac{d\omega}{dt}$$

$$\text{solving } 1.\quad \omega J = \frac{2W_k}{\omega} \quad \text{and substituting in 2.}$$

$$\Rightarrow \Delta P = \frac{2W_k}{\omega} \frac{d\omega}{dt}$$

$$\Rightarrow \frac{d\omega}{dt} = \frac{\omega \Delta P}{2W_k} \qquad ; \ \omega = 2\pi f$$

$$\Rightarrow \frac{df}{dt} = \frac{f \Delta P}{2W_k}$$

Example

$$\Delta P = 200 \text{ MW}$$

$$W_k = 300\,000 \text{ MWs}$$

$$\Rightarrow \frac{df}{dt} = 0{,}0166 \text{ Hz/s}$$

Frequency characteristics of load

– load is decreased with frequency

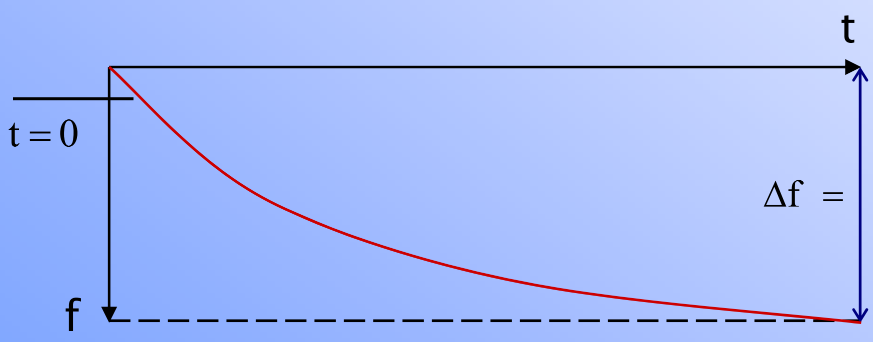
$$3.\ P_k = P_{k0} + K_V \Delta f$$

In Finland K_V max ≈ 150 MW/Hz (K_V is stiffness of the system)
In Nordic system 1000 – 2000 MW/Hz

Example *IF* $K_V = 1000\ \text{MW/Hz}$

$$\Delta P = 200\ \text{MW and } \frac{df}{dt} = 0{,}0166\ \text{Hz/s}$$

$$\text{then } \frac{dP_k}{dt} = 0{,}0166 \cdot 1000 = \underline{16{,}6\ \text{MW/s}}$$



$$\frac{df}{dt},\ t = 0$$

$$\Delta f = \frac{\Delta P}{K_V} = -0{,}2\ \text{Hz}$$

In final state the change of frequency is

$$(3.) \Rightarrow \Delta f = \frac{\Delta P}{K_V} = \frac{-200}{1000}\ \text{Hz} = -0{,}2\ \text{Hz}$$

For the frequency change Δf(t), the following applies:

$$\Delta f\,(t) = -\frac{\Delta \mathrm{P}}{\mathrm{K_V}}(1-\mathrm{e}^{-t/T})$$

T is the time constant $\dfrac{2\mathrm{W_k}}{\mathrm{f_0 K_V}}$

In the example case $\mathrm{T} = \dfrac{2\cdot 300\,000}{50\cdot 1000}\ \mathrm{s} = 12\ \mathrm{s}$

Example 2: 1300 MW plant disconnected in Nordic system

$$\begin{cases} \Delta \mathrm{P} = 1300\ \mathrm{MW} \\ \mathrm{K_V} = 1000\ \mathrm{MW/Hz} \\ \mathrm{W_k} = 300\,000\ \mathrm{MWs} \end{cases}$$

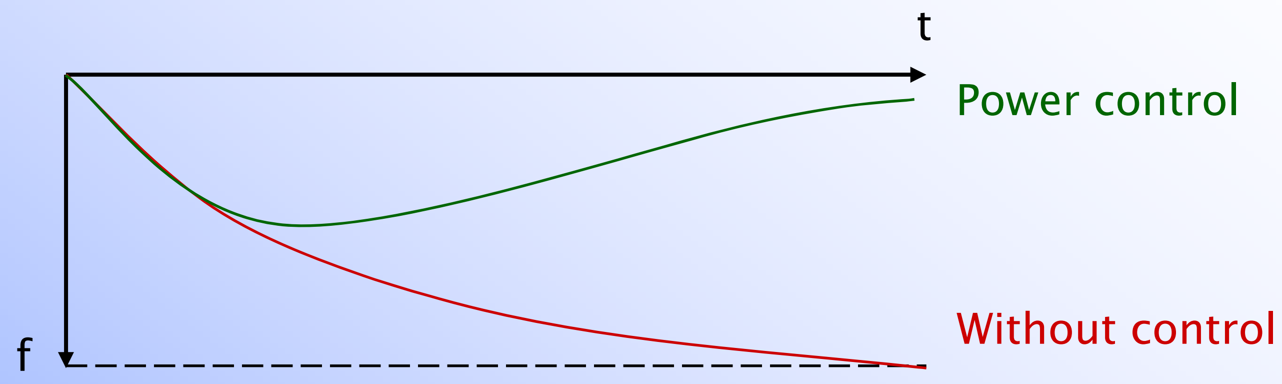
$$\frac{\partial f}{\partial t} = \frac{1}{2}\, f\, \frac{\Delta P}{W_k} = \frac{1}{2}\ 50\ \frac{1300}{300\,000}\ \mathrm{Hz/s} = 0{,}11\ \mathrm{Hz/s}$$

Frequency change if no frequency control:

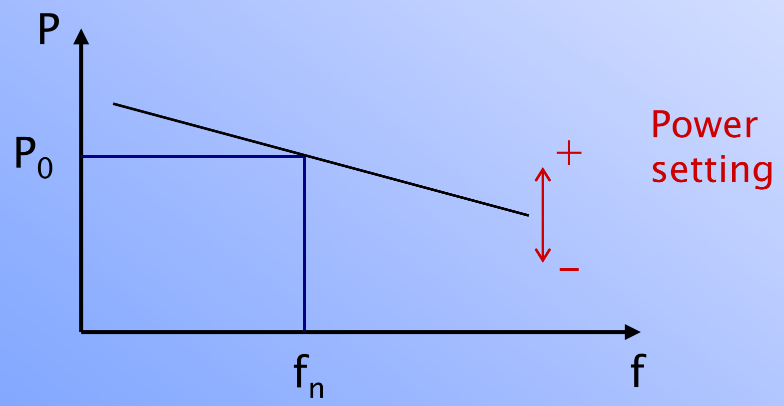
$$\Delta \mathrm{P} = \mathrm{K_V}\Delta f$$

$$\Rightarrow \Delta \mathrm{f} = \frac{+\Delta \mathrm{P}}{\mathrm{K_V}} = \frac{-1300}{1000}\ \mathrm{Hz} = -1{,}3\ \mathrm{Hz}$$

The effect of power (frequency) control:



The characteristics of the power controller (droop):



Voltage control of power systems

The goals of voltage control

In transmission system:

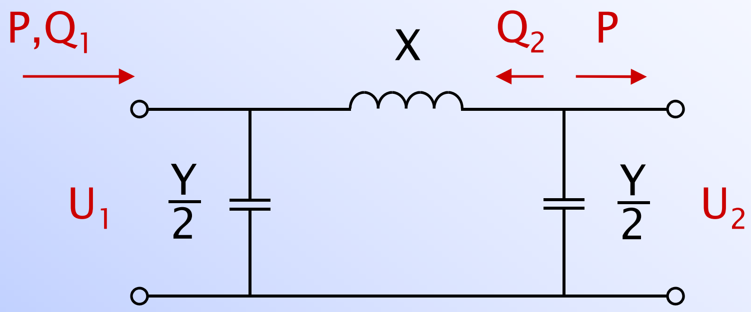
- reduction of power losses
  - reactive power balance
  - corona losses
- higher limit max operation voltages
- lower limit due to turns ratios
- reactive power reserves

In distribution systems:

- reduction of power losses

  $S = \sqrt{3}\, UI^{*} \quad P_h = 3I^2R$
- satisfying the reactive power demand of loads
- line capacity increase
- load shedding ?

Reactive power balance of a power line



Reactive power flowing into the line:

$$U_1 = U_2 = U = \text{constant}$$

$$Q_1 = \frac{U_1^2}{X} - \frac{U_1 U_2}{X}\cos\delta = \frac{U^2}{X}(1-\cos\delta)$$

$$Q_2 = \frac{U_2^2}{X} - \frac{U_1 U_2}{X}\cos\delta = \frac{U^2}{X}(1-\cos\delta)$$

Reactive power produced by the shunt capacitance:

$$Q_C = YU^2$$

The reactive power balance in the line is:

$$Q_h = Q_1 + Q_2 - Q_C$$

$$= \frac{2U^2}{X}(1-\cos\delta) - YU^2$$

The natural load of the line

= power angle δ such that the reactive power consumed in X equals the reactive power produced in C

$$\Leftrightarrow \quad \frac{2U^2}{X}(1-\cos\delta) = YU^2$$

$$\Rightarrow \quad 2(1-\cos\delta) = XY$$

$$\text{for small angles} \quad \cos\delta \approx 1 - \frac{\delta^2}{2}$$

$$\Rightarrow \quad \delta \approx \sqrt{XY}$$

Corresponding P : $\quad P = \dfrac{U_1 U_2}{X} \sin\delta$

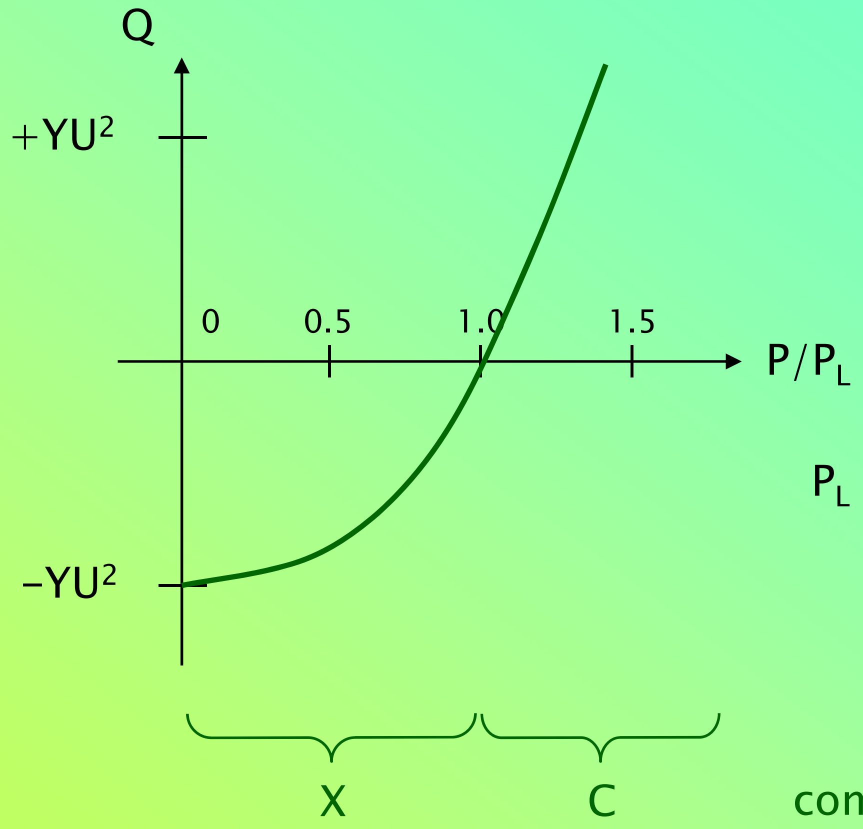
$$P = \frac{U_1 U_2}{X}\sin\delta \approx \frac{U^2}{X}\delta \qquad (\sin\delta \approx \delta \; ; \; \delta \text{ is small})$$

$$\Rightarrow \quad \boxed{P \approx \frac{U^2}{\sqrt{\frac{X}{Y}}} = \frac{U^2}{Z_0}}$$

P is the natural load power of the line

Z_0 is the surge impedance of the line

Reactive power balance versus load



P_L = line natural power

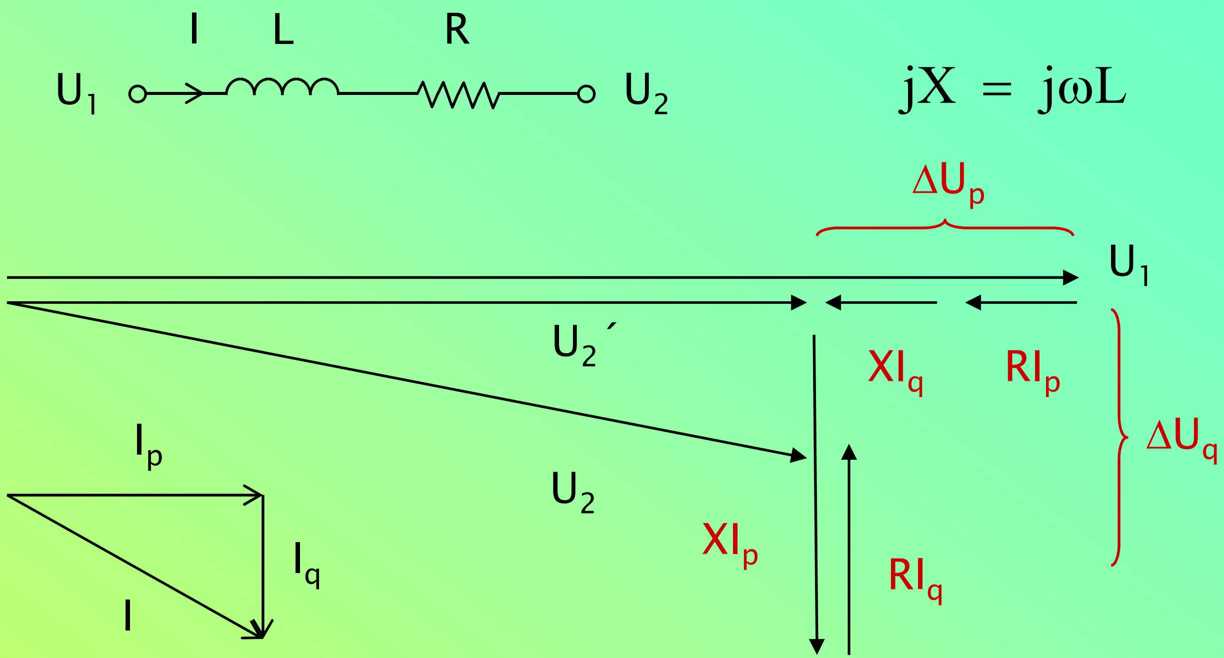
compensation

- for small load, the voltage in the line end tends to rise
- for large load, the voltage in the line end tends to fall

Natural power of some lines

Rated voltage kV	OH-line 3-ph (MW)	Cables 3-ph (MW)
10	0,26	2,6
20	1,0	10
45	5,4	54
110	32	320
110 (2 conductors)	43	
220	130	1300
380	390	
380 (2 conductors)	475	
380 (3 conductors)	540	

Voltage drop of a distribution line



Longitudinal component :

$\Delta U_p = RI_p + XI_q$

Transverse component :

$\Delta U_q = RI_q + XI_p$

For small small ΔU (< 10%)
it holds approximately :

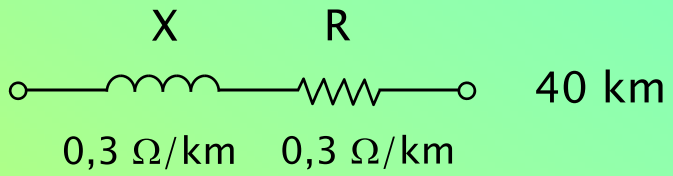
$|U1| - |U2| \approx \Delta U_p = RI_p + XI_q$

Distribution network and reactive power compensation

3 motivations:

- reduce losses
- increase load capacity
- reduce ΔU

Example, long distribution line :



$$\begin{cases} \underline{Z} = 12 + j12\ \Omega \\ U = 20\ \text{kV} \\ I = 50\ \text{A} \\ \cos\varphi = 0{,}9 \end{cases}$$

$$\Rightarrow \quad I_p = 45\,\text{A}\ ;\ I_q = 21{,}8\text{A}$$

Compensation capacitor (shunt):

$$Q_C = 600\,\text{kVAr}$$

$$\Rightarrow \quad I_C = \frac{Q}{\sqrt{3}\,U} = 17{,}32\ \text{A}$$

Distribution network and reactive power compensation

No compensation	With shunt capacitor
I_p 45 A	I_p 45 A
I_q 21,8 A	I_q 21,8 A – 17,3 A = 4,48 A
I 50 A	I 45,22 A

Losses : $P_h = 3RI^2$

$P_L = 90$ kW	$P_L = 73{,}6$ kW

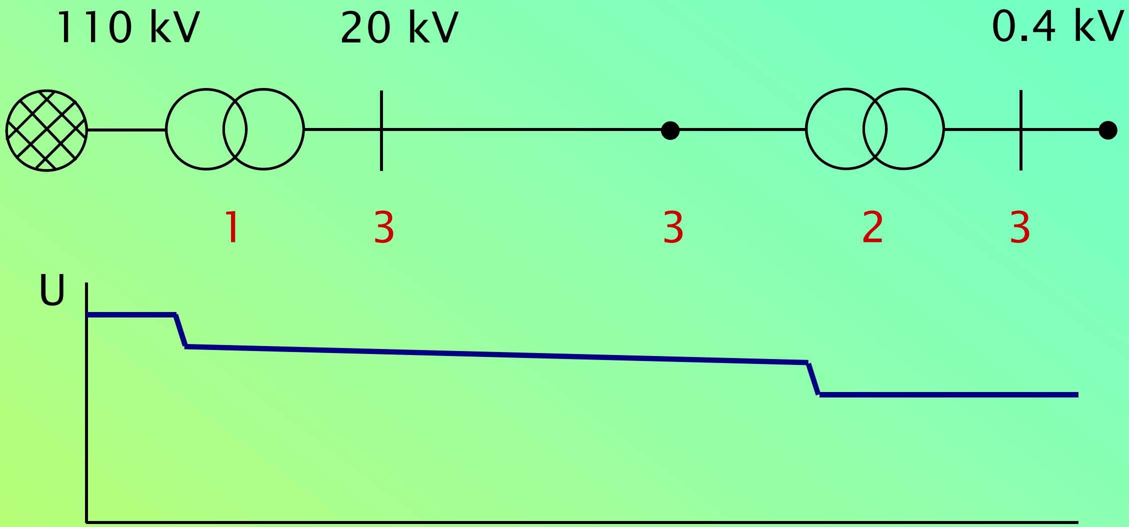
Load current :

50 A	45,22 A (–10%)

Voltage drop : $\Delta U \approx RI_p + XI_q$

$\Delta U = 12 \cdot 45 + 12 \cdot 21{,}8$ V	$\Delta U = 12 \cdot 45 + 12 \cdot 4{,}48$ V
$= 801{,}6$ V	$= 593{,}8$ V
$\widehat{=}$ 6,94 %	$\widehat{=}$ 5,14 %

Distribution network and voltage control



Voltage control:

1. On-load tap-changer

   Eg. $110 \pm 9 \cdot 1{,}67\ \% \ / \ 21$ kV

2. Off-load tap-changer

   Eg. $20 \pm 2 \cdot 2{,}5\ \% \ / \ 0{,}4$ kV

3. Compensation (capacitor)

   $\Delta U_p = RI_p + XI_q$

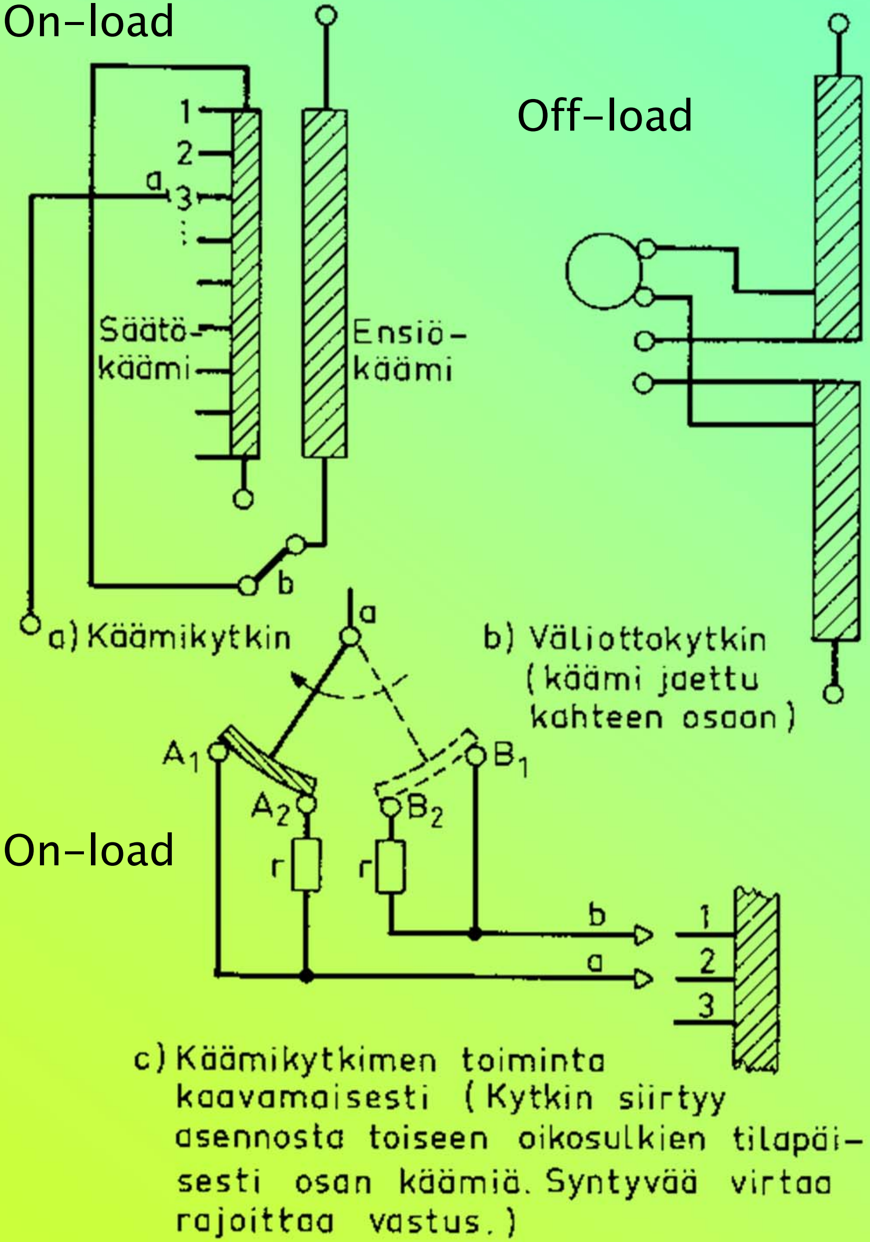
$\Leftrightarrow$ reduces the reactive current and thus voltage drop

The control of on-load tap-changer

On-load

Off-load

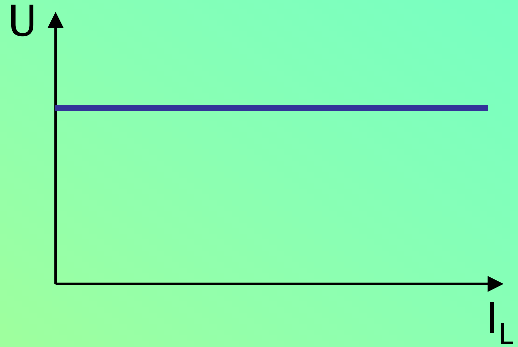
On-load



Characteristics

U = constant

Ex. 20,4 – 20,8 kV



Current compounded