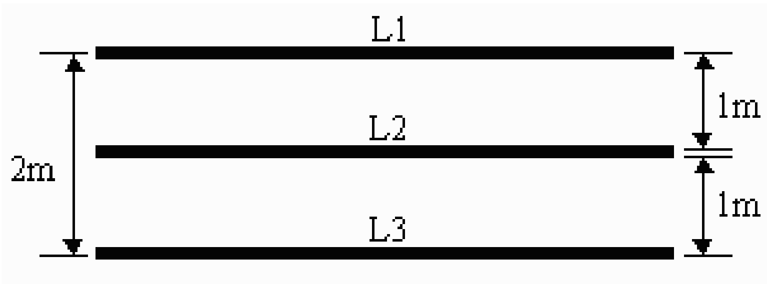
Aalto-yliopisto
Teknillinen korkeakoulu

Exercise 10

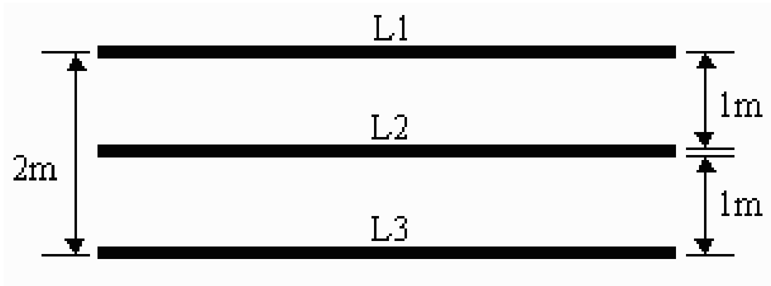
Power systems

Question 1



A three-phase power line consists of three parallel conductors in the same horizontal plane. The two outer conductors are each 1 m from the center conductor. If the conductor diameter is 6 mm, **calculate the average inductance per phase of a 1 km length of the line**. Assume the expression for the inductance per meter of length.

Question 1



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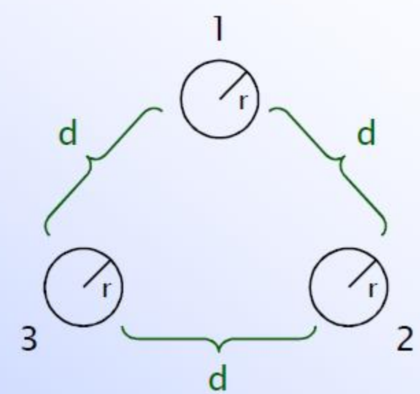
$$L = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{d}{r}\right]$$

$$\mu_0 = 4\pi \cdot 10^{-7}\ \text{H/m}$$

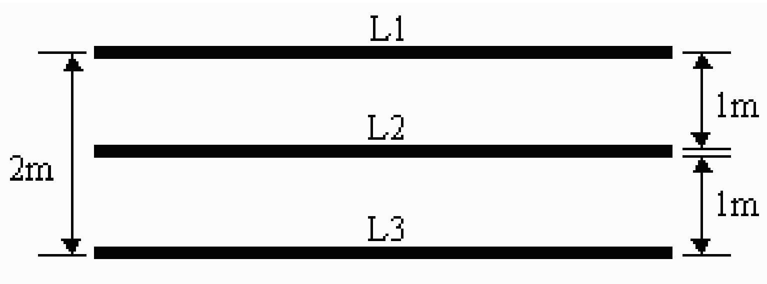
If the distances between the phases are not equal:

$$d_{eq} = \sqrt[3]{d_{12} \cdot d_{23} \cdot d_{31}}$$



Question 1

Inductance, when conductor diameter is 6 mm



$$d_{eq} = d = \sqrt[3]{1\times1\times2m} = \sqrt[3]{2}m$$

$$r = \frac{d_c}{2} = \frac{6\text{mm}}{2} = 3\text{mm} = 3\times10^{-3}\text{m}$$

$$L = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{d}{r}\right]$$

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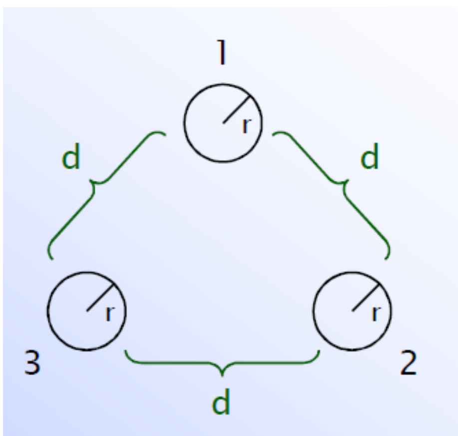


$$L = 2\times10^{-7}\left[\frac{1}{4} + \ln\left(\frac{d}{r}\right)\right]\frac{\text{H}}{\text{m}}$$

1km :

$$L = 2\times10^{-7}\left[\frac{1}{4} + \ln\left(\frac{\sqrt[3]{2}}{3\times10^{-3}}\right)\right]\frac{\text{H}}{\text{m}}\times1000\text{m}$$

$$= \underline{\underline{1.26\text{mH}}}$$

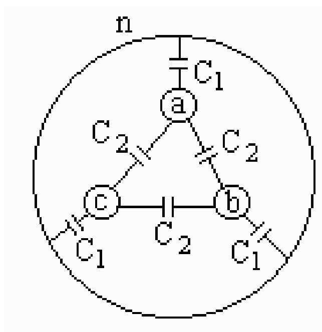
Question 2

In a three-core cable, the capacitance between the three cores short-circuited together and the sheath is 0.87 μF/km, and that between two cores connected together to with the sheath and the third core is 0.84 μF/km. **Determine the MVA required to keep 16 km of this cable charged when the supply is 33 kV**, three phase, 50 Hz.

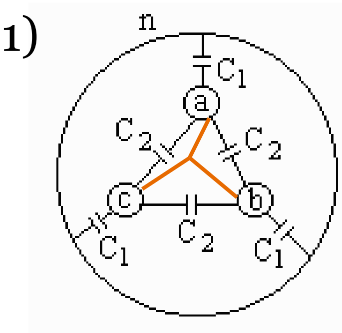
Question 2

MVA for charging 16 km of cable with 33 kV supply

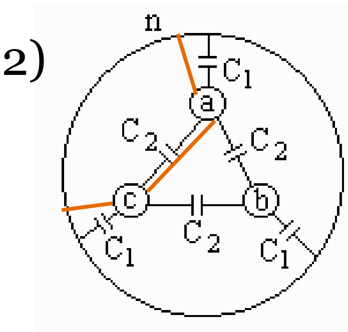
Three-core cable:



1)

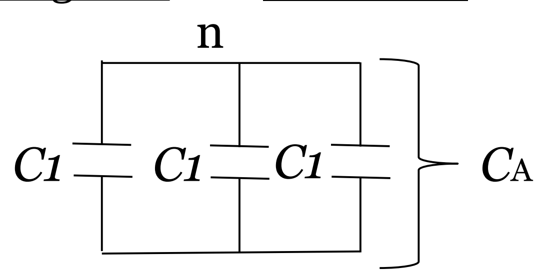


2)



1) Capacitance between <u>the three cores short-circuited together</u> and <u>the sheath</u> is 0.87 μF/km:

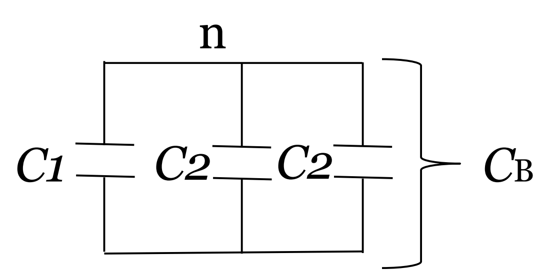
$$C_A = 0.87\frac{\mu F}{km} = \underbrace{3\cdot C_1}_{parallel} \quad \Rightarrow C_1 = \frac{1}{3}C_A = 0.29\frac{\mu F}{km}$$



2) Between <u>two cores connected together with the sheath</u> and <u>the third core</u> is 0.84 μF/km:

$$C_B = 0.84\frac{\mu F}{km} = 2\cdot C_2 + C_1$$

$$\Rightarrow C_2 = \frac{1}{2}(C_B - C_1) = \frac{1}{2}(0.84 - 0.29)\frac{\mu F}{km} = 0.275\frac{\mu F}{km}$$



Question 2

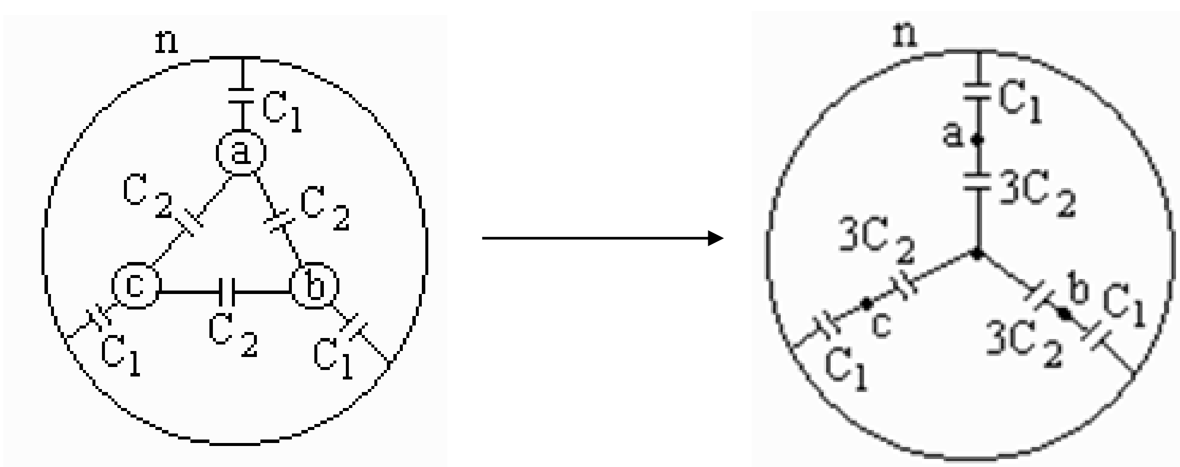
MVA for charging 16 km of cable with 33 kV supply

We can make a delta-star transformation for easier solution:

$$Q_\Delta = 3UI_\Delta \underbrace{\sin\varphi}_{=-1(cap.)} = -3U\frac{U}{Z_\Delta} = -3U^2Y_\Delta$$

$$Q_\lambda = 3\frac{U}{\sqrt{3}}I_\lambda \underbrace{\sin\varphi}_{=-1(cap.)} = -3\left(\frac{U}{\sqrt{3}}\right)^2\frac{1}{Z_\lambda} = -U^2Y_\lambda \overset{\text{power equilibrium}}{=} Q_\Delta$$

$$\Rightarrow Y_\lambda = 3Y_\Delta$$



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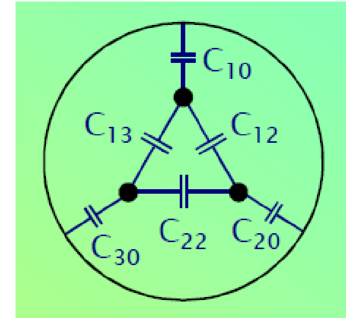


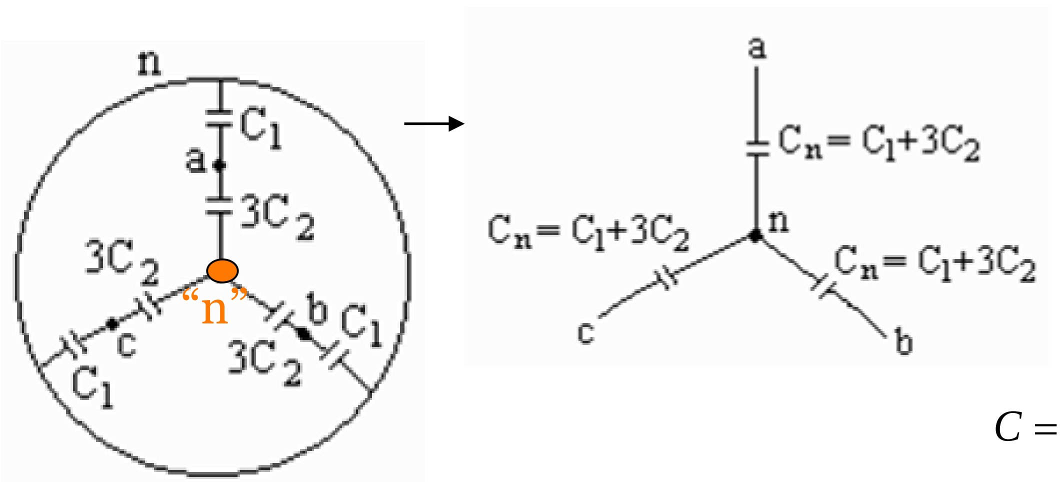
Something to go after...

Positive sequence C :

$$C = C_{10} + 3C_{12}$$

$$C_1 = 0.29\frac{\mu\text{F}}{\text{km}} \quad C_2 = 0.275\frac{\mu\text{F}}{\text{km}}$$

From the sheats/neutrals perspective:



Cap. per phase :

$$C = C_1 + 3C_2 = (0.29 + 3\times 0.275)\frac{\mu\text{F}}{\text{km}} = 1.115\frac{\mu\text{F}}{\text{km}}$$

For a 16 - km cable : $C = 1.115\frac{\mu\text{F}}{\text{km}} \times 16\text{km} = 17.84\mu\text{F}$

For a three-phase system the apparent power is:

$$S = 3\left(\frac{U}{\sqrt{3}}\right)^2 \frac{1}{Z} = YU^2 = \omega CU^2 = 2\pi fCU^2 = 2\pi \times 50 \times 17.84 \times 10^{-6} \times \left(33\times 10^3\right)^2 \text{VA}$$

$$= \underline{\underline{6.1\text{MVA}}}$$

Question 3

An AAC is composed of 37 strands, each having a diameter of 0.333 cm. **Compute the dc resistance in ohms per kilometer at 75°C**. Assume that the increase in resistance due to spiraling is 2%.

Use
resistivity for aluminum:
  0.0283 Ωmm^2/m at 20°C
temperature dependence:
  0.00403 /°C

Question 3
dc resistance

An AAC is composed of 37 strands, each having a diameter of 0.333 cm. Compute the dc resistance in ohms per kilometer at 75°C. Assume that the increase in resistance due to spiraling is 2%.

$$R = \frac{\rho}{A} \cdot l$$

AAC is an all-aluminum conductor

Resistivity → $\rho = 2.83 \times 10^{-8}\ \Omega\text{m}$ at 20°C

and $\alpha = 0.00403$ per °C

Diameter of a strand is $d = 0.333$ cm $= 0.00333$ m

Total area of the conducting material→ $A = 37 \times \frac{\pi}{4}(0.00333)^2 = 3.222 \times 10^{-4}\, m^2$

$$R_{20} = \rho \frac{l}{A}$$

$$\Rightarrow$$

$$\frac{R_{20}}{l} = \frac{\rho}{A} = \frac{2.83 \times 10^{-8}\ \Omega\text{m}}{3.222 \times 10^{-4}\ \text{m}^2} \times 1000$$

$$= 0.0878 \frac{\Omega}{\text{km}}$$

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$$\frac{R_{20}}{l} = 0.0878 \frac{\Omega}{km}$$

Spiraling effect :

$$\frac{R'_{20}}{l} = 1.02 \cdot \frac{R_{20}}{l} = 1.02 \cdot 0.0878 \frac{\Omega}{\text{km}} = 0.0896 \frac{\Omega}{\text{km}}$$

Zero at DC

Total resistance :

$$R = [1 + \alpha\,(\vartheta - 20°C)]\,(R_{20} + \Delta R)$$

$$\frac{R_{75}}{l} = \frac{R'_{20}}{l}[1 + \alpha(75 - 20)]$$

$$= 0.0896 \times [1 + 0.00403 \times (75 - 20)] = 0.109 \frac{\Omega}{km}$$

Question 4

A three-phase 60-Hz line has flat horizontal spacing. The conductors have an outside diameter of 3.28 cm with 12 m between conductors. **Determine the capacitive reactance to neutral in ohm-meters and the capacitive reactance of the line in ohms if its length is 200 km.** Presume that the distance to ground is much larger than the distance between conductors.

Question 4

Capacitances of an OH–line

r = conductor radius

h = geometric mean height

$$h = \sqrt[3]{h_1 \cdot h_2 \cdot h_3}$$

a, A : geometric mean distances

$$a = \sqrt[3]{a_{12} \cdot a_{23} \cdot a_{13}} \quad ; \quad A = \sqrt[3]{A_{12} \cdot A_{23} \cdot A_{13}}$$

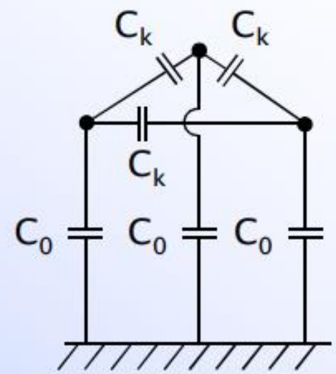
Positive sequence capacitance $C = C_0 + 3C_k$

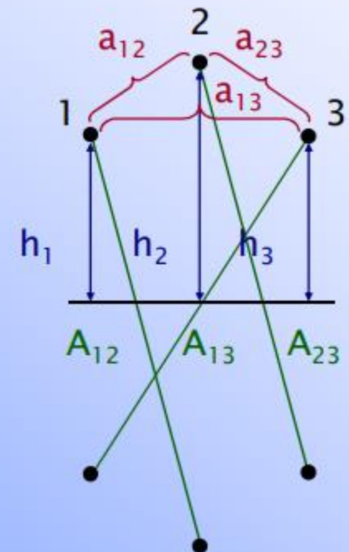
$$c = \frac{2\pi\varepsilon_0}{\ln \frac{2ha}{rA}}$$

Zero sequence capacitance C_0

$$c = \frac{2\pi\varepsilon_0}{\ln \frac{2h}{r}\left(\frac{A}{r}\right)^2}$$

ε_0 = vacuum permittivity $8{,}84 \cdot 10^{-12}$ F/m





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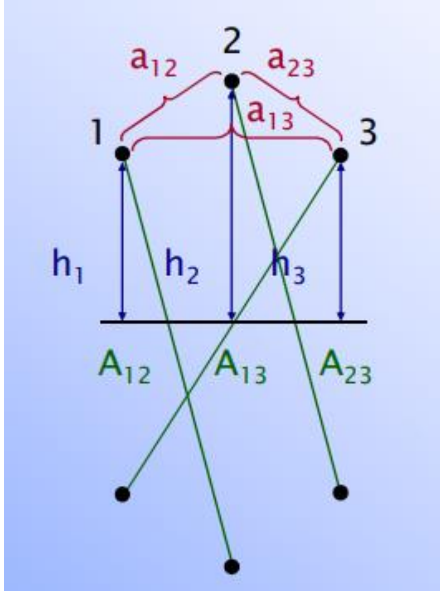

$$C\left[\frac{\mathrm{F}}{\mathrm{m}}\right] = \frac{2\pi\varepsilon_0}{\ln\left(\frac{2ha}{rA}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{a}{r}\right) - \ln\left(\frac{A}{2h}\right)}$$

When the distance to ground (h) is much larger than the distance between the conductors, the total distances of $2h$ and A are nearly equal. That is to say:

$$C\left[\frac{\mathrm{F}}{\mathrm{m}}\right] = \frac{2\pi\varepsilon_0}{\ln\left(\frac{a}{r}\right) - \ln\left(\frac{A}{2h}\right)} \approx \frac{2\pi\varepsilon_0}{\ln\left(\frac{a}{r}\right) - \ln(1)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{a}{r}\right)}$$

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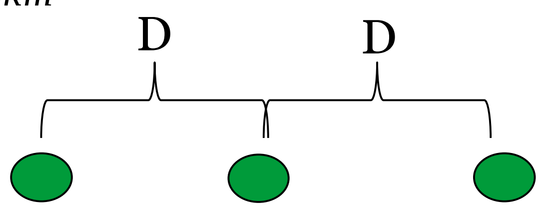
A three-phase 60-Hz line has flat horizontal spacing. The conductors have an outside diameter of 3.28 cm with 12 m between conductors. Determine the capacitive reactance to neutral in ohm-meters and the capacitive reactance of the line in ohms if its length is 200 km. Presume that the distance to ground is much larger than the distance between conductors.

$d = 3.28cm = 0.0328m \quad \Rightarrow \quad r = 0.0164m$

$D = 12m$

$l = 200km$



$$D_{eq} = \sqrt[3]{D_{ab} \times D_{bc} \times D_{ac}} = \sqrt[3]{12 \times 12 \times 24}\text{m} = 15.12\text{m}$$

The line-to-neutral capacitance per meter and capacitive reactance in ohm-meters:

$$C\left[\frac{\text{F}}{\text{m}}\right] = \frac{2\pi\varepsilon_0}{\ln\left(\frac{D_{eq}}{r}\right)} = \frac{10^{-9}}{18 \cdot \ln\left(\frac{15.12}{0.0164}\right)} = 8.138 \cdot 10^{-12}\ \frac{\text{F}}{\text{m}}$$

$$\varepsilon_0 = \frac{10^{-9}}{36\pi}\left[\frac{\text{F}}{\text{m}}\right] = 8.842 \cdot 10^{-12}\ \frac{\text{F}}{\text{m}}$$

$$\Rightarrow X_C = \frac{1}{\omega C} = \frac{\ln\left(\frac{D_{eq}}{r}\right)}{2\pi \times 60 \times 2\pi \times \frac{10^{-9}}{36\pi}} = 4.77 \times 10^7 \times \ln\left(\frac{D_{eq}}{r}\right) = \underline{\underline{3.256 \times 10^8\ \Omega \cdot \text{m}}}$$

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$$d = 3.28cm = 0.0328m \quad \Rightarrow \quad r = 0.0164m$$

$$D = 12m$$

$$l = 200km$$

Line capacitance and reactance for the 200 km line:

$$C = 8.138 \cdot 10^{-12}\,\frac{\text{F}}{\text{m}} \times \frac{1000\,\text{m}}{\text{km}} \times 200\text{km} = 1.63\,\mu\text{F}$$

$$\Rightarrow X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 60 \times 1.63 \times 10^{-6}} = \underline{\underline{1627\Omega}}$$