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Teknillinen korkeakoulu

Exercise Session 11

Power systems

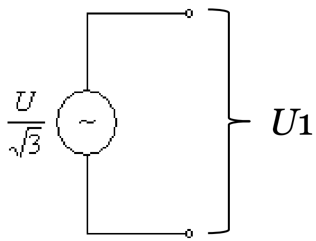
Question 1

In an isolated neutral network there is a high-resistance 1-phase earth fault. The network's earth capacitance is C_0, voltage U and the fault resistance is R_f. Derive expressions for

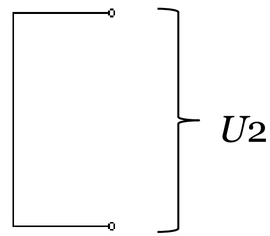
a) earth fault current
b) neutral point voltage
c) faulty phase's voltage
d) healthy phases' voltage

Question 1

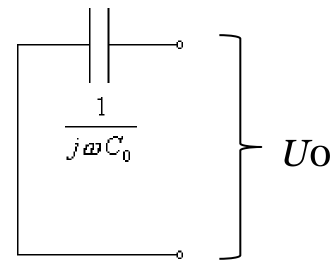
Positive sequence network (positive sequence impedances small compared to the zero sequence network impedances):



Negative sequence network (negative sequence impedances small compared to the zero sequence network impedances):



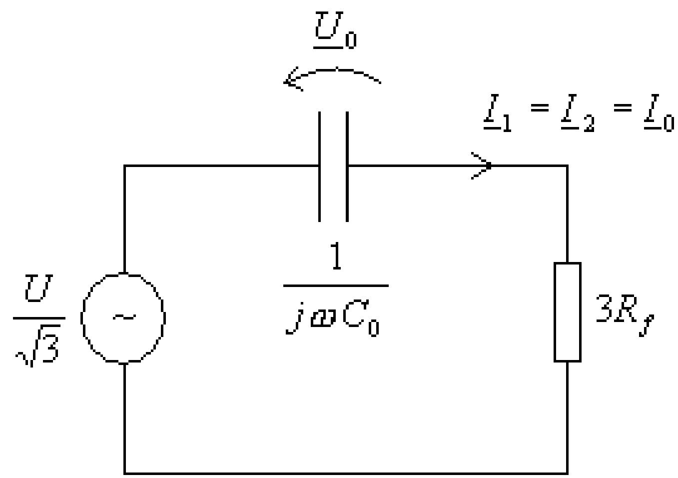
Zero sequence network:



As we have learned previously (see E5Q3), in a single phase-to-earth fault, the sequence networks are in series and the circuit is closed by an impedance 3 times the fault impedance →

Question 1

Single phase-to-earth fault:

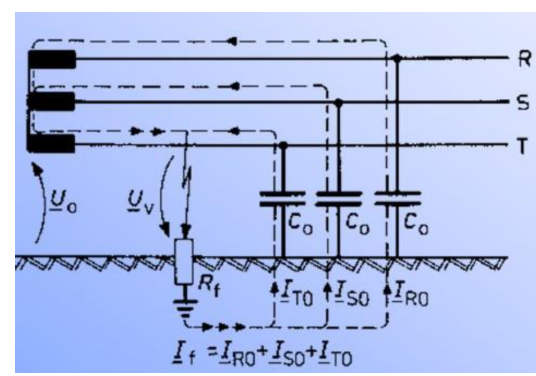


a) earth fault current

$$\underline{\underline{I}}_e = \underline{I}_1 + \underline{I}_2 + \underline{I}_0 = 3\underline{I}_0 = 3\cdot\frac{\dfrac{U}{\sqrt{3}}}{\dfrac{1}{j\omega C_0}+3R_f} = \underline{\underline{\frac{j3\omega C_0}{1+j3\omega C_0 R_f}\cdot\frac{U}{\sqrt{3}}}}$$

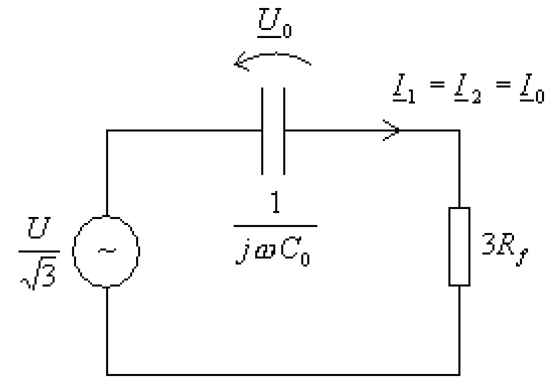
b) neutral point voltage (zero sequence voltage is the voltage between neutral and ground)

$$\underline{\underline{U}}_O = -\underline{Z}_0\underline{I}_0 = -\frac{1}{j\omega C_0}\cdot\frac{\dfrac{U}{\sqrt{3}}}{\dfrac{1}{j\omega C_0}+3R_f} = \underline{\underline{-\frac{1}{1+j3\omega C_0 R_f}\cdot\frac{U}{\sqrt{3}}}}$$



Question 1

Single phase-to-earth fault:



c) faulty phase's voltage

$$\underline{U}_R = \underline{U}_1 + \underline{U}_2 + \underline{U}_0 = \frac{U}{\sqrt{3}} + 0 + \left(-\frac{1}{1 + j3\omega C_0 R_f} \cdot \frac{U}{\sqrt{3}} \right) = \frac{j3\omega C_0 R_f}{1 + j3\omega C_0 R_f} \cdot \frac{U}{\sqrt{3}}$$

$$\begin{bmatrix} \underline{U}_{L1} \\ \underline{U}_{L2} \\ \underline{U}_{L3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} \begin{bmatrix} \underline{U}_0 \\ \underline{U}_1 \\ \underline{U}_2 \end{bmatrix}$$

d) healthy phases' voltage

$$\underline{U}_S = \underline{a}^2 \underline{U}_1 + \underline{a}\underline{U}_2 + \underline{U}_0 = \underline{a}^2 \cdot \frac{U}{\sqrt{3}} + \underline{a} \cdot 0 + \left(-\frac{1}{1 + j3\omega C_0 R_f} \cdot \frac{U}{\sqrt{3}} \right)$$

$$\underline{U}_S = \left(\underline{a}^2 - \frac{1}{1 + j3\omega C_0 R_f} \right) \cdot \frac{U}{\sqrt{3}}$$

Correspondingly:

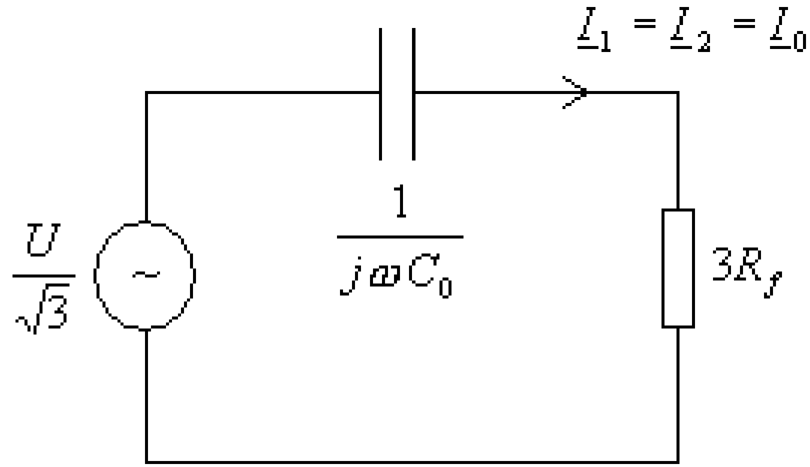
$$\underline{U}_T = \left(\underline{a} - \frac{1}{1 + j3\omega C_0 R_f} \right) \cdot \frac{U}{\sqrt{3}}$$

Question 2

Show that in an isolated neutral network, that has an earth capacitance $c_0 = 6.13$ nF/km, the zero resistance earth fault current can be approximately expressed as:

$I_e = \frac{U \times l}{300}$, where $[U] = 1$ kV, $[I_e] = 1$ A, $[l] = 1$ km

Question 2



Single phase-to-earth fault:

If $R_f = 0\,\Omega$, then

$$\underline{I}_e = \underline{I}_1 + \underline{I}_2 + \underline{I}_0 = 3\underline{I}_0 = 3 \cdot \frac{\frac{U}{\sqrt{3}}}{\frac{1}{j\omega C_0}} = j\omega C_0 \cdot U\sqrt{3}$$

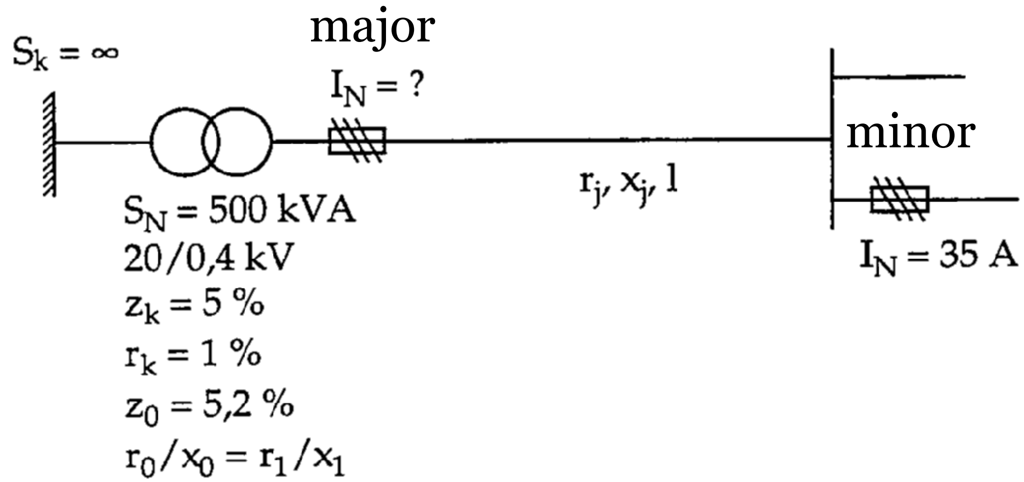
$$I_\mathrm{e} = \omega C_0 U\sqrt{3} = 2\pi \cdot 50\,\tfrac{1}{\mathrm{s}} \cdot 6.13\,\frac{\mathrm{nF}}{\mathrm{km}} \cdot l \cdot U \cdot \sqrt{3}$$

$$\Longrightarrow \quad I_e = 3.34 \cdot 10^{-6}\, U \cdot l\, \frac{1}{\Omega\,\mathrm{km}} = \frac{U \cdot l}{300\,000\,\Omega\,\mathrm{km}}$$

If $[U]$ = 1 kV and $[l]$ = 1 km, then

$$I_e \approx \frac{U \cdot l}{300}\,\mathrm{A}$$

Question 3

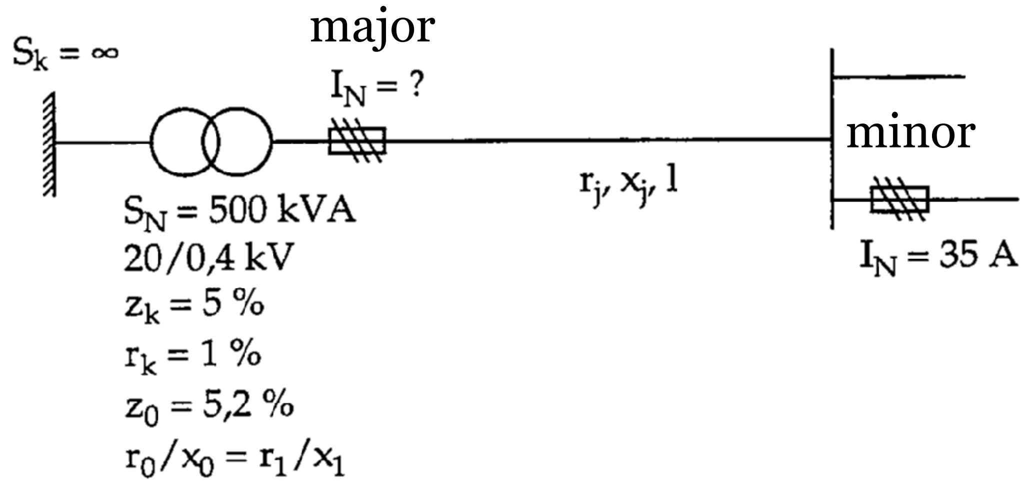


Choose the smallest possible fuse to the transformer's low-voltage side so that the protection is selective considering the I_N = 35 A fuse. The two cases are:

1) l = 150 m, x_j = 0.075 Ω/km, r_j = 0.103 Ω/km
2) l = 600 m, x_j = 0.104 Ω/km, r_j = 0.868 Ω/km

Selectivity can be considered sufficient when the major fuse's melting time is at least ten times the melting time of the minor fuse plus the maximum arcing time (10 ms). When we have short melting times (t<1 ms), fusing is selective enough when the major fuse's melting energy I^2t_S is at least three times the minor fuse's operation energy I^2t_a. The melting times and energies are presented in the following pictures.

Question 3



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2) $l = 600$ m, $x_j = 0.104\ \Omega/\text{km}$, $r_j = 0.868\ \Omega/\text{km}$

Selectivity can be considered sufficient when the major fuse's melting time is at least ten times the melting time of the minor fuse plus the maximum arcing time (10 ms). When we have short melting times (t<1 ms), fusing is selective enough when the major fuse's melting energy I^2t_S is at least three times the minor fuse's operation energy I^2t_a. The melting times and energies are presented in the following pictures.

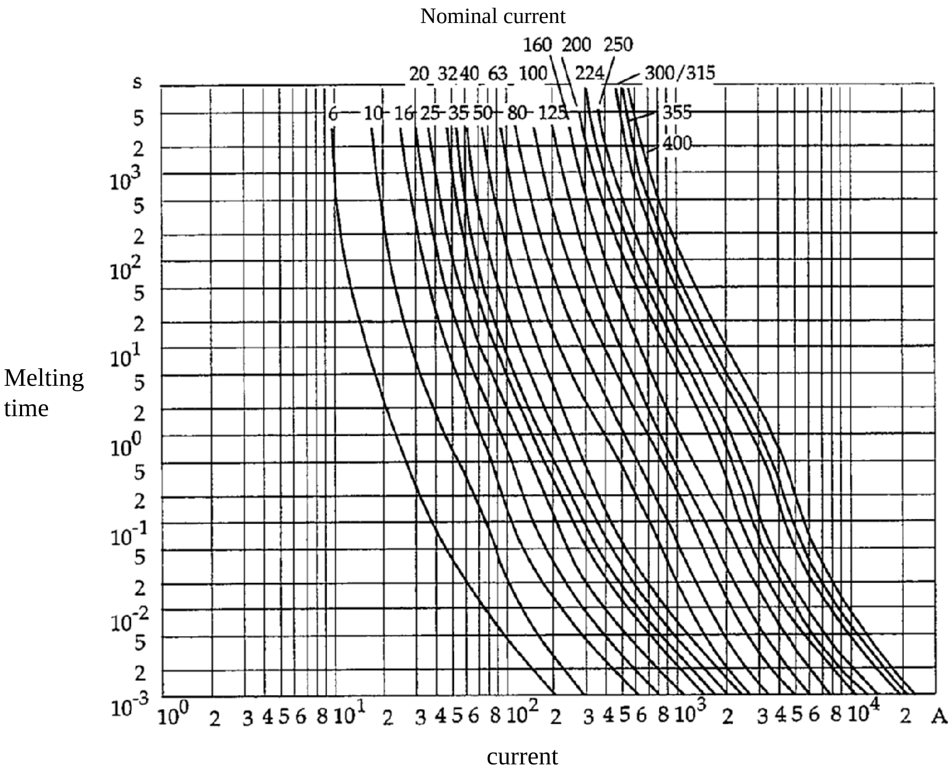
If t>10ms:

$$t_{\text{major}} \geq 10t_{\text{minor}} + t_{\text{minor arcing}}$$

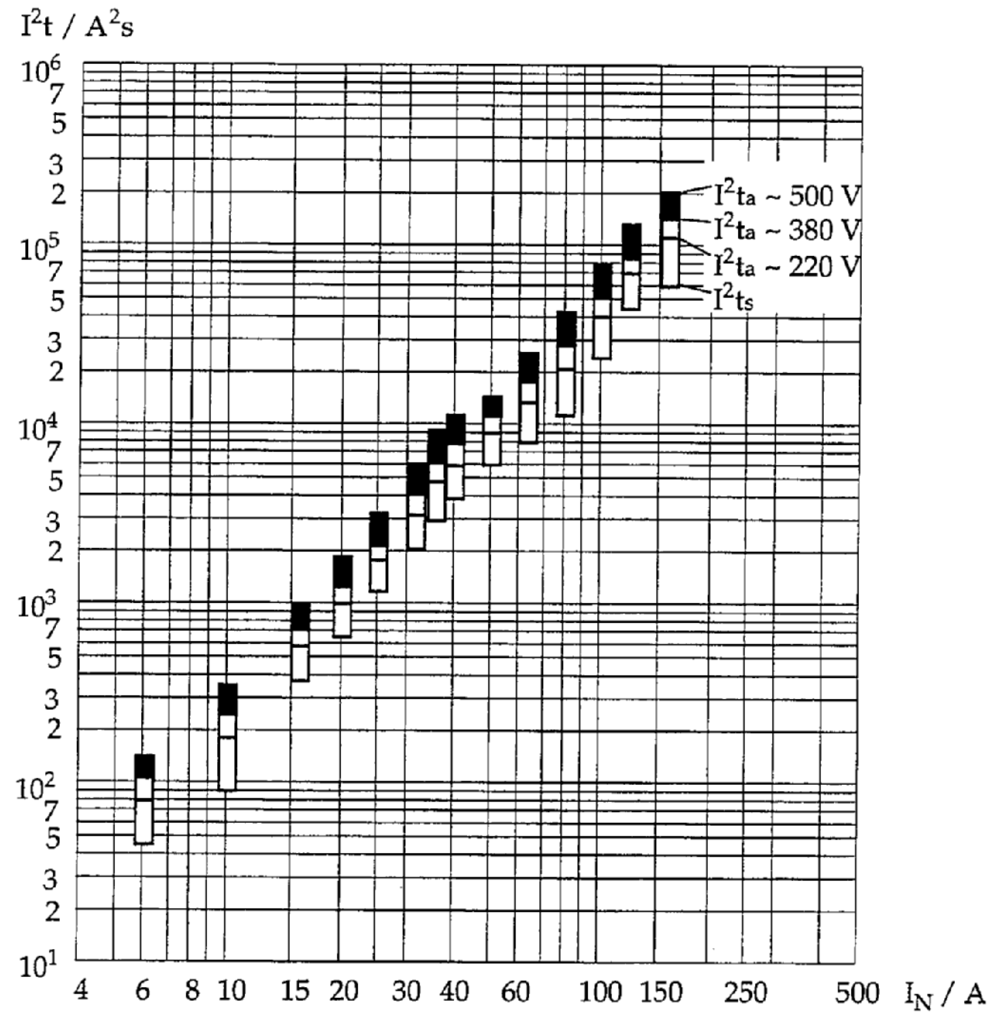
If t<1 ms:

$$I^2t_{\text{s,major}} \geq 3I^2t_{\text{a,minor}}$$

Question 3



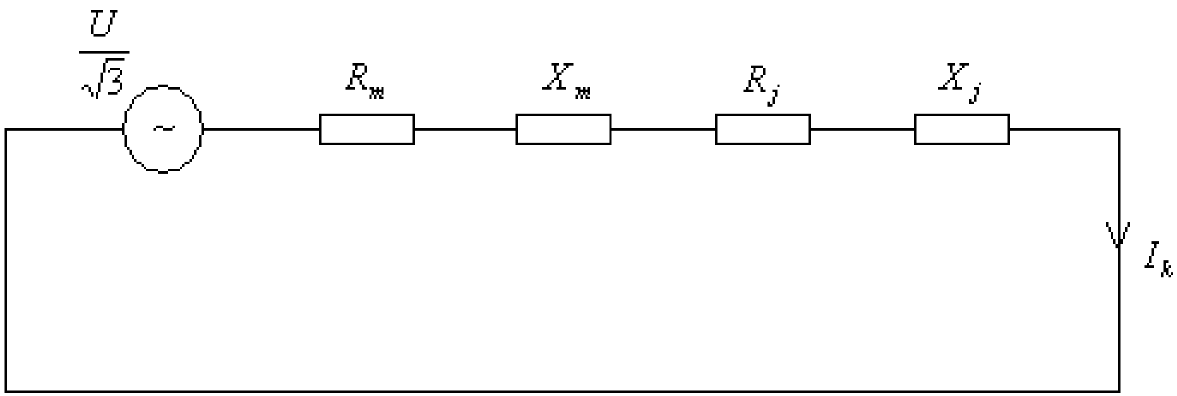
Melting times of fuses with different nominal current as a function of current.



I^2t_S melting energy, I^2t_a operation energy, melting time is less than 1 ms.

Question 3

The worst case from the selectivity point of view is when a short circuit happens at the 35-A fuse terminals. Let us compute the 3-phase short circuit current in this point:
Assuming that the voltage is at its rated value.



In 400-V voltage level, transformer parameters:

$$U = 20\ \text{kV} \cdot \left(\frac{0.4}{20}\right) = 400\ \text{V} \qquad R_m = r_k \cdot \frac{U_N^2}{S_N} = 0.01 \cdot \frac{(400\ \text{V})^2}{500\ \text{kVA}} \approx 3.20\ \text{m}\Omega$$

$$X_m = \sqrt{Z_k^2 - r_k^2} \cdot \frac{U_N^2}{S_N} = \sqrt{(0.05)^2 - (0.01)^2} \cdot \frac{(400\ \text{V})}{500\ \text{kVA}} \approx 15.68\ \text{m}\Omega$$

Question 3: Part 1

Line characteristics:

$$R_j = r_j \cdot l = 0.103 \frac{\Omega}{\text{km}} \cdot 0.150 \text{ km} \approx 15.45 \text{ m}\Omega \qquad X_j = x_j \cdot l = 0.075 \frac{\Omega}{\text{km}} \cdot 0.150 \text{ km} \approx 11.25 \text{ m}\Omega$$

Short circuit current:

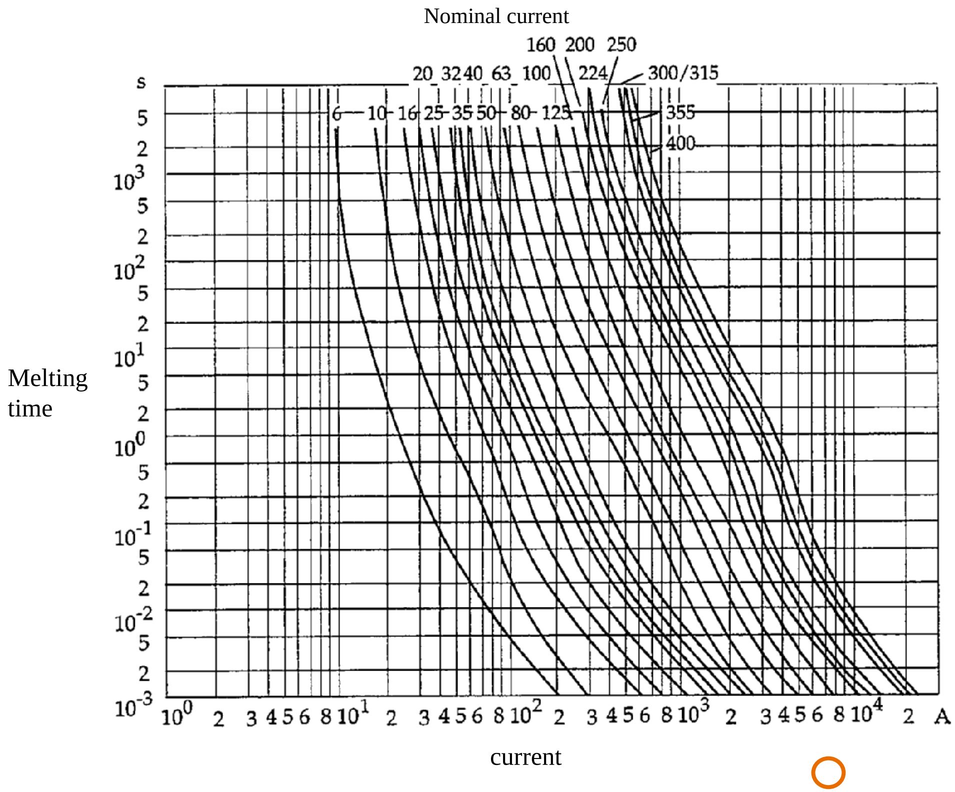
$$I_k = \frac{\frac{U}{\sqrt{3}}}{\sqrt{(R_m + R_j)^2 + (X_m + X_j)^2}} = \frac{\frac{400 \text{ V}}{\sqrt{3}}}{\sqrt{(3.20 + 15.45)^2 + (15.68 + 11.25)^2} \text{ m}\Omega} = 7.05\text{kA}$$

From figures shown on the previous slide: the operation time of a 35-A fuse with the short circuit current $I_\text{k} = 7.05$ kA, fuse melting time corresponds to a value below 1 ms, therefore, we use the melting/operating energy diagram. From the second diagram the 35-A fuse corresponds to a operating energy value of $I^2 t_a = 5 * 10^3 \text{A}^2\text{s}$ at 220 V. According to our initial selectivity criteria (if t<1ms, then $I^2 t_{s,major} \geq 3 I^2 t_{a,minor}$),
the $I^2 t_{s,major} \geq 3 * 5 * 10^3 = 15 * 10^3 \text{A}^2 \text{ s}$.

On the diagram the next fuse that satisfies this condition is a 100 A fuse.

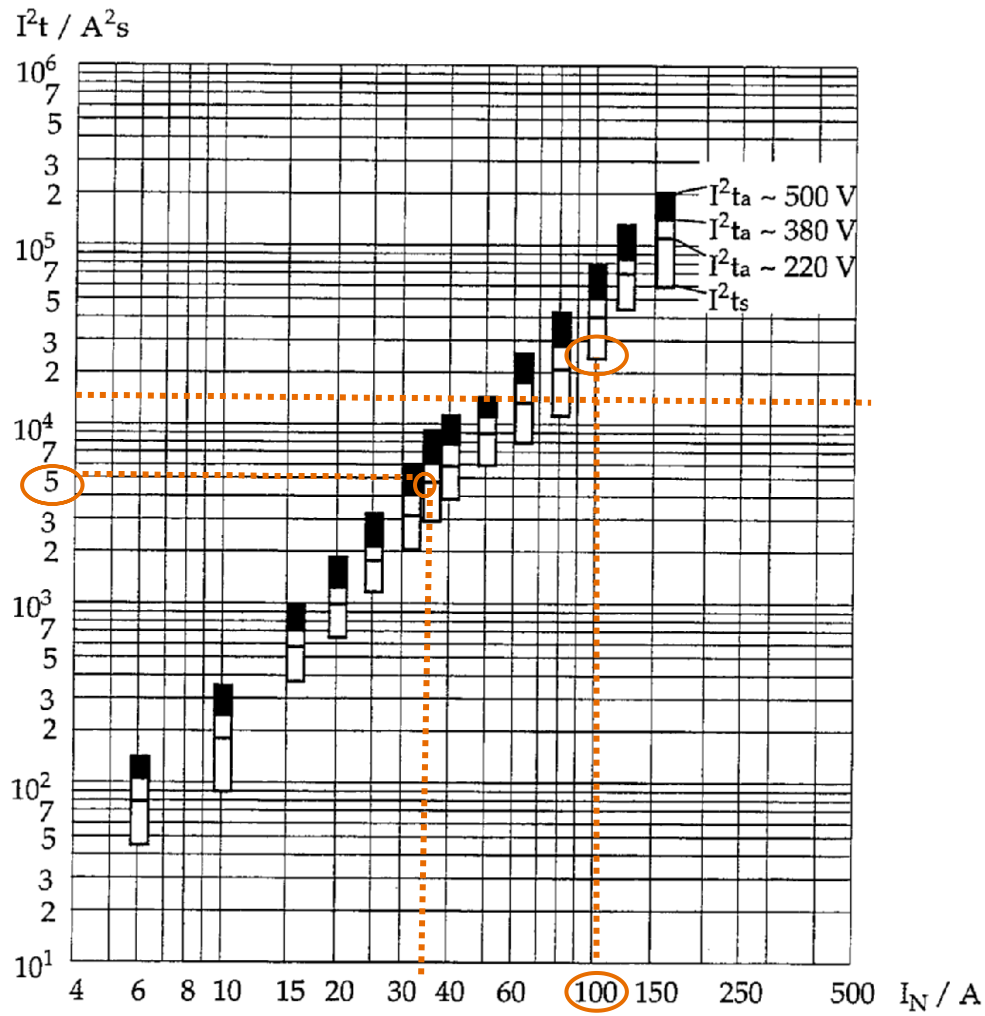
The major fuse size $\underline{\underline{I_N = 100 \text{ A}}}$

Question 3



Melting times of fuses with different nominal current as a function of current.

$I_k = 7.05\ \text{kA}, I_N = 35\ \text{A}$



I^2t_S melting energy, I^2t_a operation energy, melting time is less than 1 ms.

Question 3: Part 2

Line characteristics:

$$R_j = r_j \cdot l = 0.868 \frac{\Omega}{\text{km}} \cdot 0.600 \text{ km} = 520.80 \text{ m}\Omega$$

$$X_j = x_j \cdot l = 0.104 \frac{\Omega}{\text{km}} \cdot 0.600 \text{ km} = 62.40 \text{ m}\Omega$$

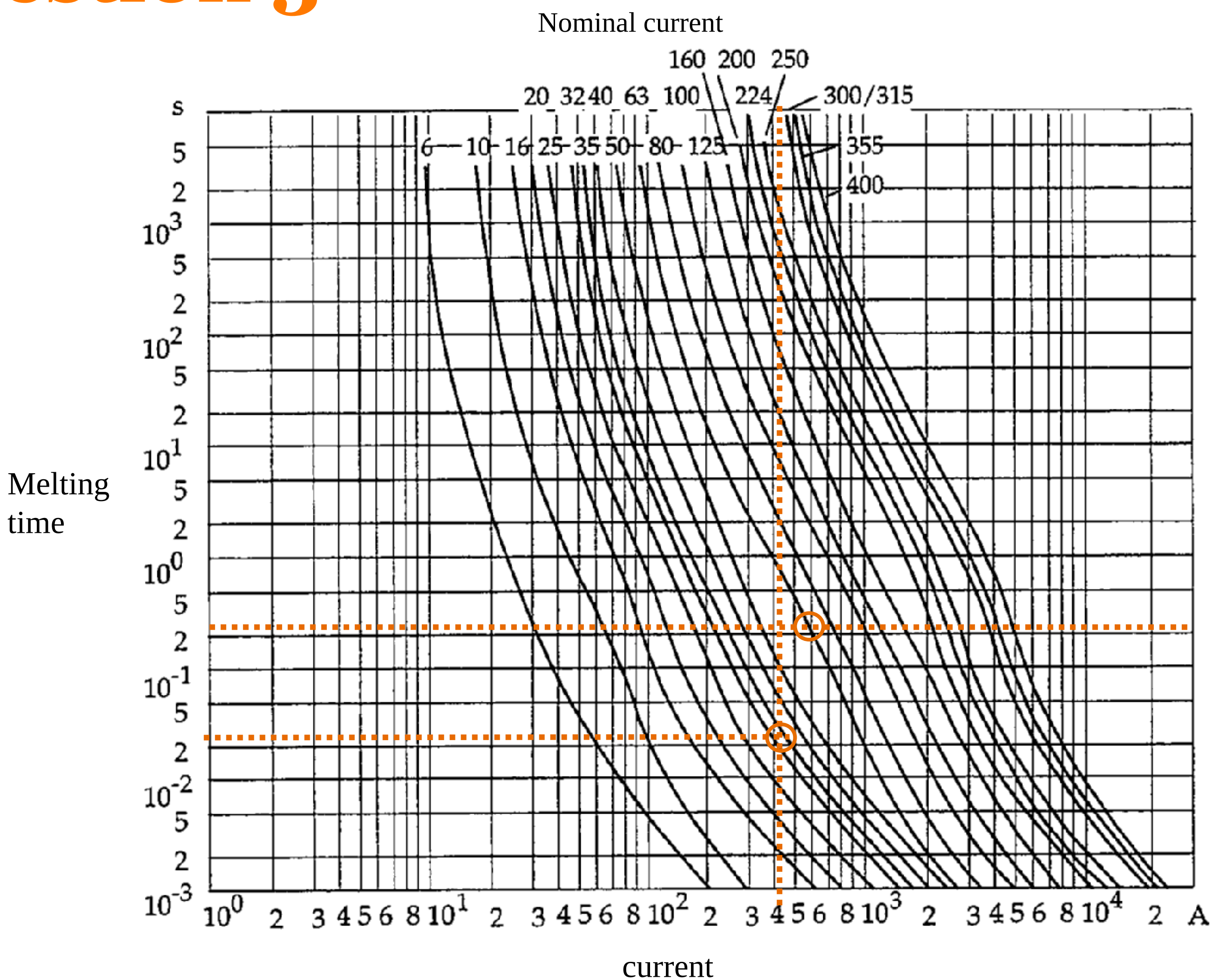
Short circuit current:

$$I_k = \frac{\frac{U}{\sqrt{3}}}{\sqrt{(R_m + R_j)^2 + (X_m + X_j)^2}} = \frac{\frac{400 \text{ V}}{\sqrt{3}}}{\sqrt{(3.20 + 520.80)^2 + (15.68 + 62.40)^2} \text{ m}\Omega} = 435.9\text{A}$$

From figures shown on the previous slide: the operation time of a 35-A fuse with the short circuit current $I_k = 436$ A, fuse melting time corresponds to a value 20 – 30 ms. Therefore, we use the same diagram. According to our initial selectivity criteria (if $t >$10 ms, then $t_{\text{major}} \geq 10t_{\text{minor}} + 10$ ms), the $t_{\text{major}} \geq 10 * 20 \text{ ms} + 10 \text{ ms} = 210 \text{ ms}$. On the diagram, the next fuse that satisfies this condition is a 63-A fuse.

The major fuse size $\underline{\underline{I_N = 63 \text{ A}}}$

Question 3



Melting times of fuses with different nominal current as a function of current.

$$I_k = 436\ \text{A}, I_N = 35\ \text{A}$$

Question 4

In a 20 kV overhead line network the zero sequence capacitance is 6 nF/km per phase. At a secondary substation occurs a single phase to earth fault. Grounding resistance is 20 Ω. Total length of lines is 300 km.

a) Calculate the earth fault current and the voltage in the grounded parts.

b) How quick should the relay trip to meet the safety requirements

c) If ground fault current is reduced be a compensation coil, how big a coil is needed

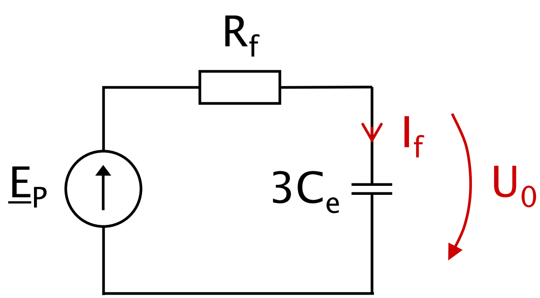
Question 4

a) Calculate the earth fault current and the voltage in the grounded parts.

Network ground capacitance Ce = 300 km * 6 nF/km = 1.8 μF

$$\underline{I}_f = \frac{\underline{E}_P}{R_f + \frac{1}{j\omega 3C_e}}$$

$$= \frac{20kV/\sqrt{3}}{20\Omega + \frac{1}{j\omega * 3 * 1.8\mu F}}$$



= 20A (earth fault current)

Voltage in grounded parts (like transformer tank)

Ue = RI = 20 Ω * 20 A = 400 V

Question 4

b)How quick should the relay trip to meet the safety requirements

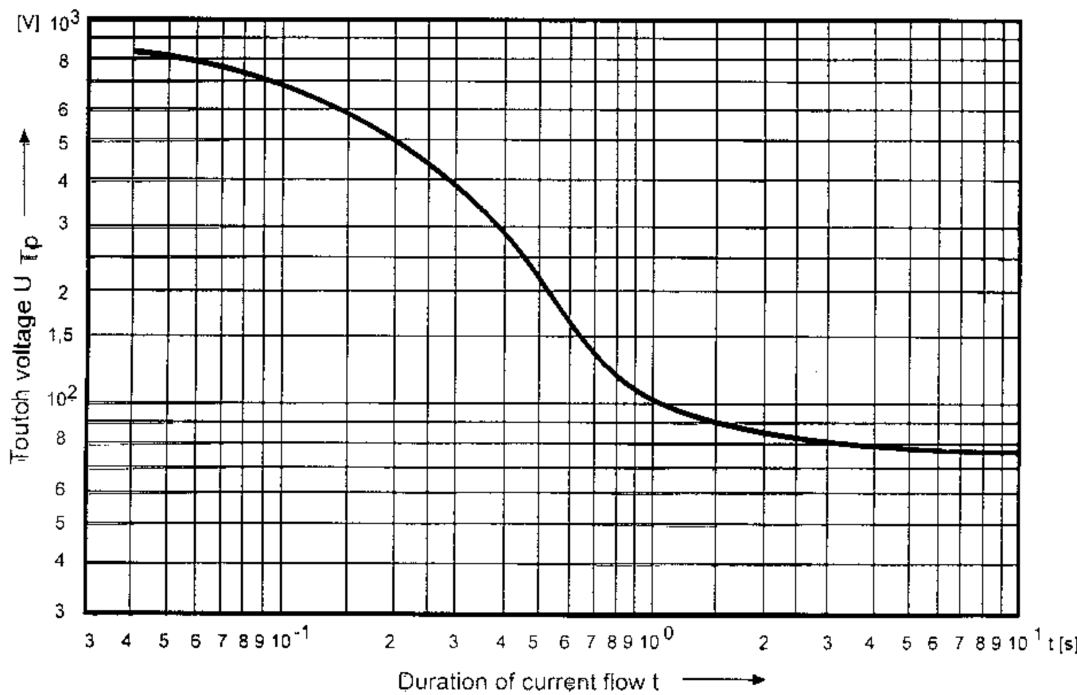
In base case, the earth fault voltage can be 2 x permissive touch voltage

⇔ The max touch voltage may be 0.5 * Ue = 200 V

From the figure:

200 V ⇔ 0.5 s

If we take 100 ms for CB operation, relay should trip in 0.4 seconds



Question 4

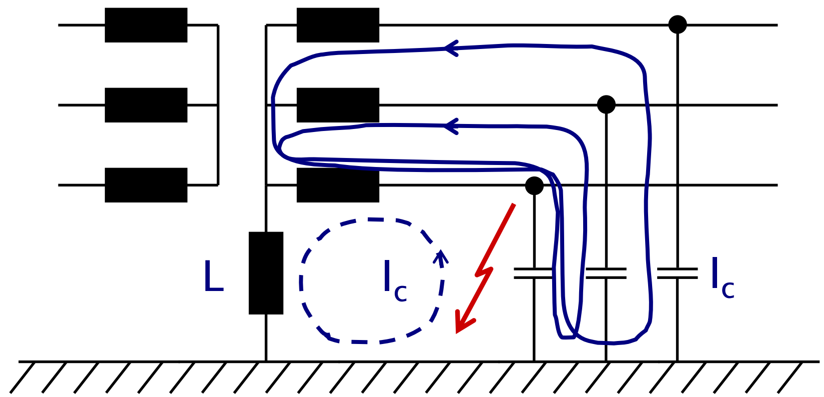
c)If ground fault current is reduced be a compensation coil, how big a coil is needed

Ground fault current can effectively be reduced by compensation coil located in neutral. It is hence in parallel to the earth capacitances and will be tuned in 50 Hz resonance.

$$X_L = X_C$$

$$\omega L = \frac{1}{3\omega C}$$

$$L = \frac{1}{3\omega^2 C}$$



C= 1.8μF & ω = 2πf & f=50 Hz ⇔ **L=1.9 H**