Aalto-yliopisto
Teknillinen korkeakoulu

Exercise Session 1

Power systems

Question 1

Show that the ratio of phase-to-phase-voltage and phase-to-earth-voltage is $\sqrt{3}$

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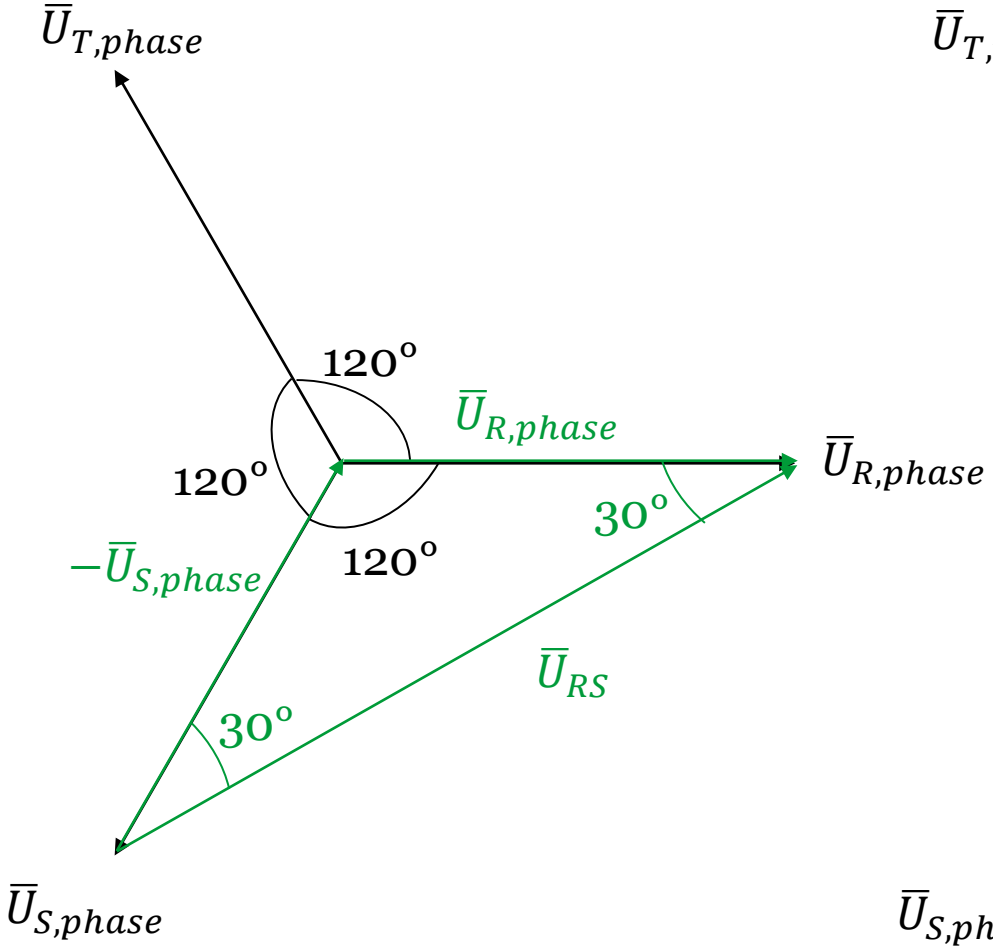
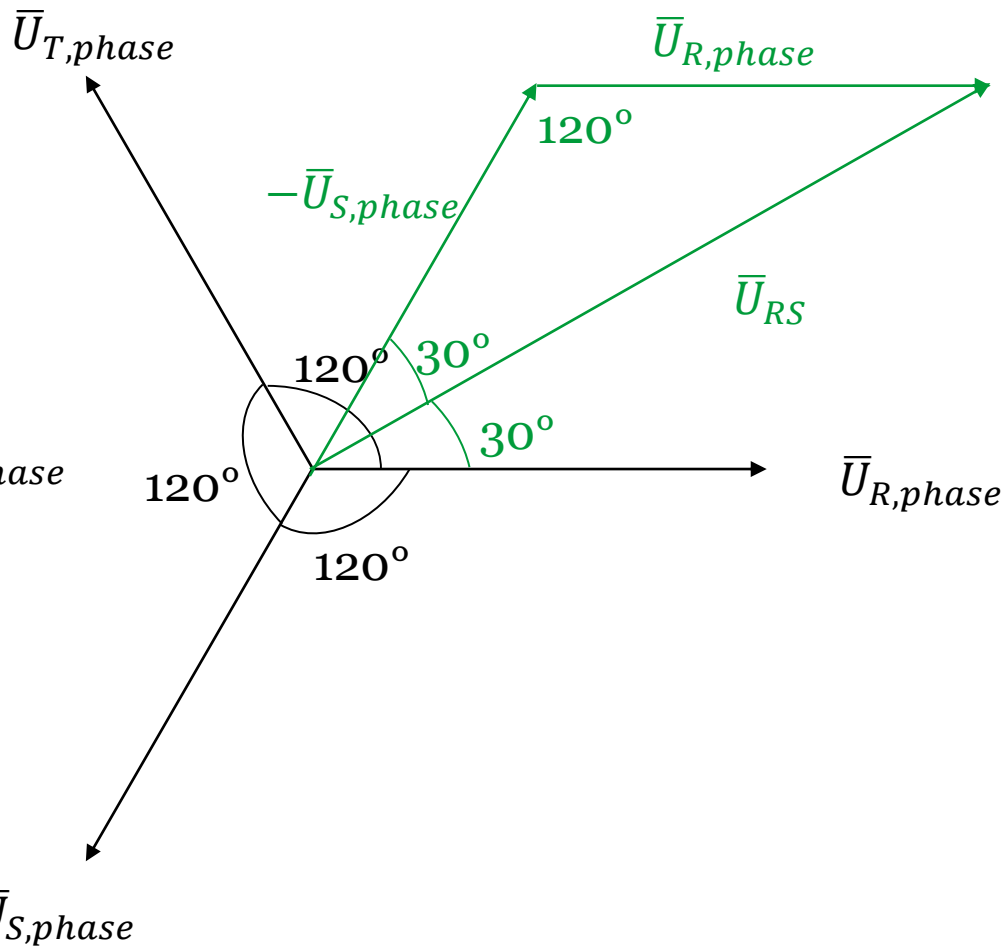
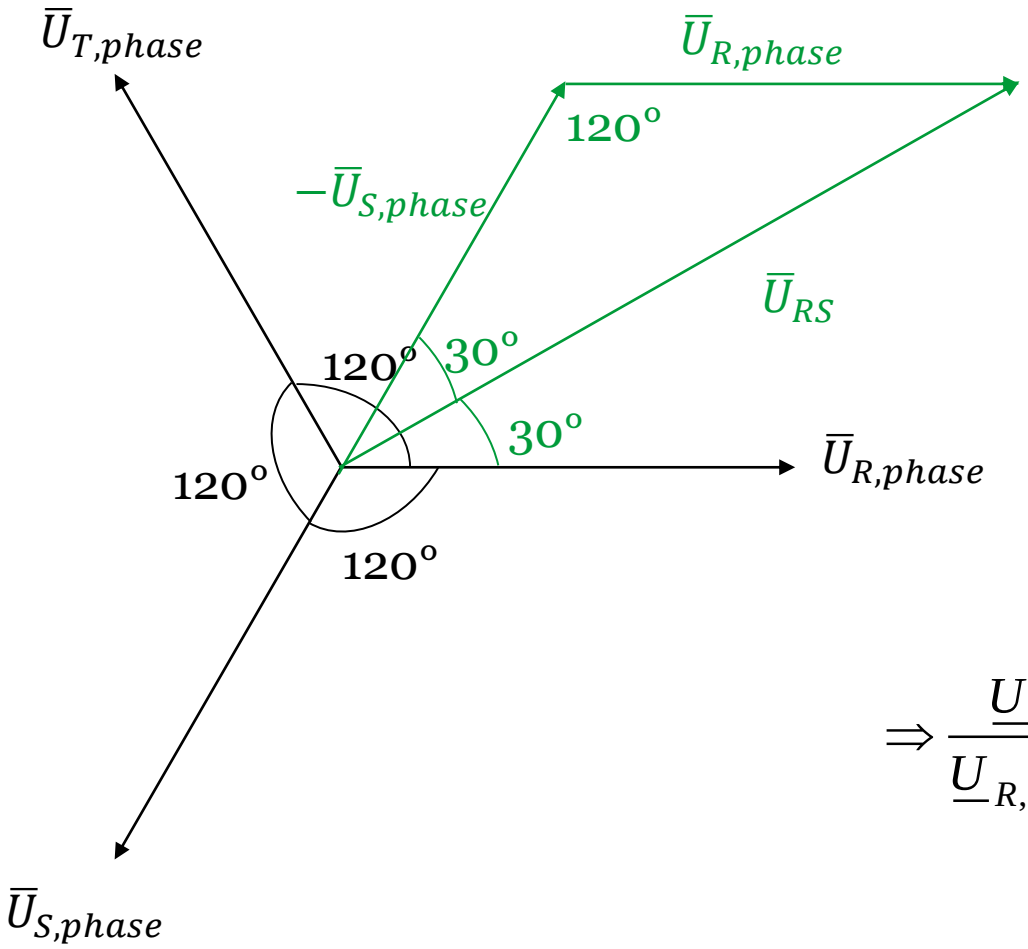




Question 1 Show that the ratio of phase-to-phase-voltage and phase-to-earth-voltage is $\sqrt{3}$



$$\underline{U}_{RS,phase} = \underline{U}_{R,phase} - \underline{U}_{S,phase}$$

by Sine Rule:

$$\frac{\underline{U}_{RS,phase}}{\sin 120°} = \frac{\underline{U}_{R,phase}}{\sin 30°}$$

$$\Rightarrow \frac{\underline{U}_{RS}}{\underline{U}_{R,phase}} = \frac{\sin 120°}{\sin 30°}$$

$$\Rightarrow \frac{\underline{U}}{\underline{U}_{phase}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \underline{\underline{\sqrt{3}}}$$

Question 2

Derive the equation for delta-star transformation.

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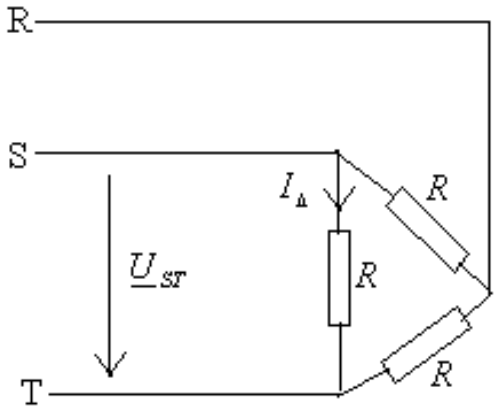
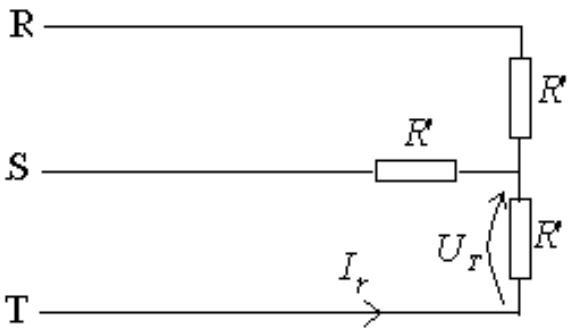




The power lost to resistance must remain unchanged

$$P_\Delta = 3UI_\Delta = 3U\frac{U}{R} = 3\frac{U^2}{R} \qquad P_\gamma = 3U_{phase}I_\gamma = 3U_{phase}\frac{U_{phase}}{R'} = 3\frac{U_{phase}^2}{R'}$$

$$P_\Delta = P_\gamma <=> 3\frac{U^2}{R} = 3\frac{U_{phase}^2}{R'} <=> \frac{U^2}{R} = \left(\frac{U}{\sqrt{3}}\right)^2\frac{1}{R'}$$

$$\Leftrightarrow R' = \frac{R}{3}$$

Question 3

A delta-connected three-phase load of (80+j60) Ω per phase is connected to a 440-V three-phase supply. Calculate:

a) the current in the load component

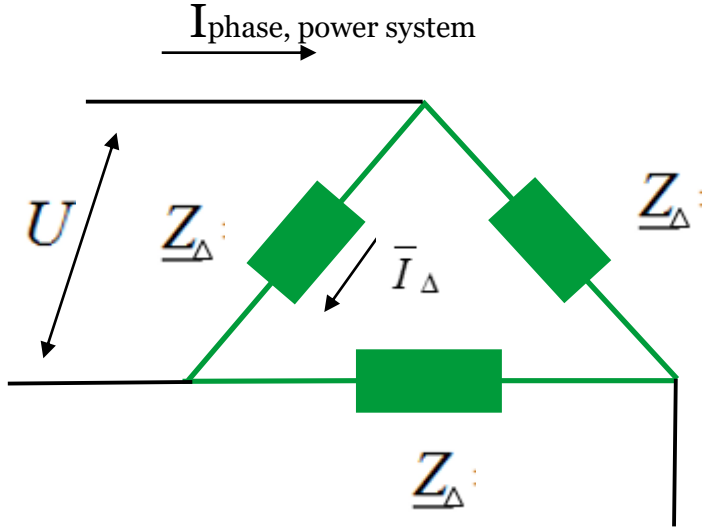
b) the current in the phase of power line

c) the total real power consumed (active power).

Question 3
a) Current in the load component

In delta connected load, the voltage across the load component is line voltage, i.e. phase-to-phase voltage:

$$\underline{Z}_{\Delta} = 80 + j60 = 100\underline{/36{,}87°}$$
$$U = 440V$$

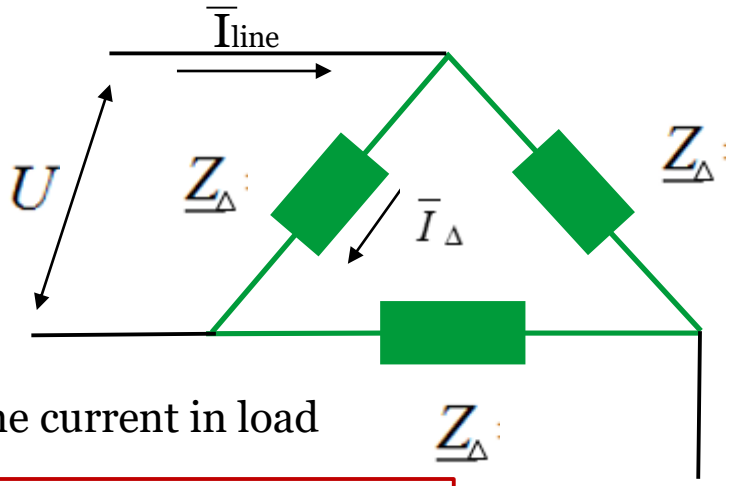


And the current

flowing through the load is

$$\bar{I}_{\text{load}} = \bar{I}_{\Delta} = \frac{\bar{U}}{\underline{Z}_{\Delta}} = \frac{440\angle 0°\text{V}}{100\angle 36.87°\Omega} = 4.4\angle -36.87°\text{A}$$

Question 3
b) Current

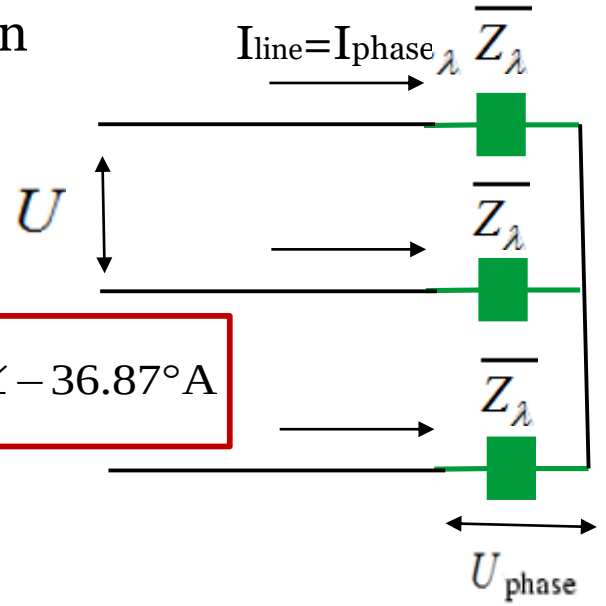


For Δ-connected load, current in power line is $\sqrt{3}$ times the current in load

$$\bar{I}_{\text{power line}} = \sqrt{3} * \bar{I}_\Delta = \sqrt{3} * 4.4\angle -36.87°\text{A} = 7.62\angle -36.87°\text{A}$$

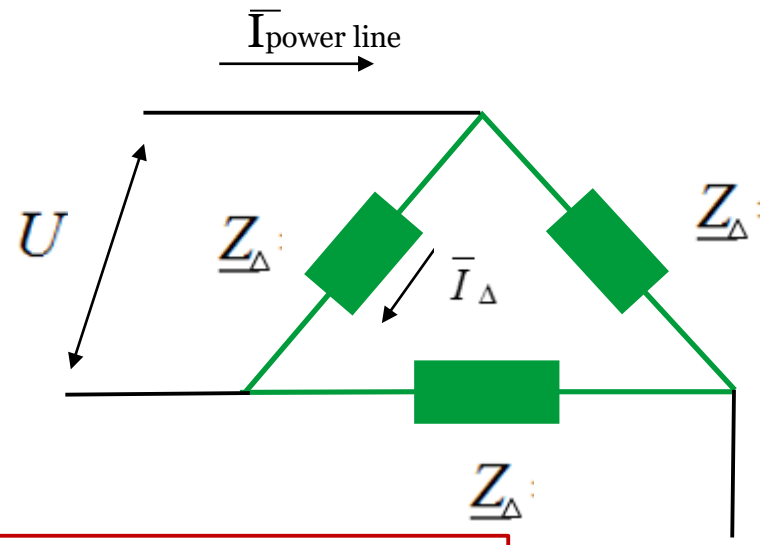
Option 2) Solution using delta-star-transformation

$$\bar{Z}_\lambda = \frac{\bar{Z}_\Delta}{3} = \left(\frac{80}{3} + j\frac{60}{3}\right)\Omega = 33.33\angle 36.87°\text{A}$$



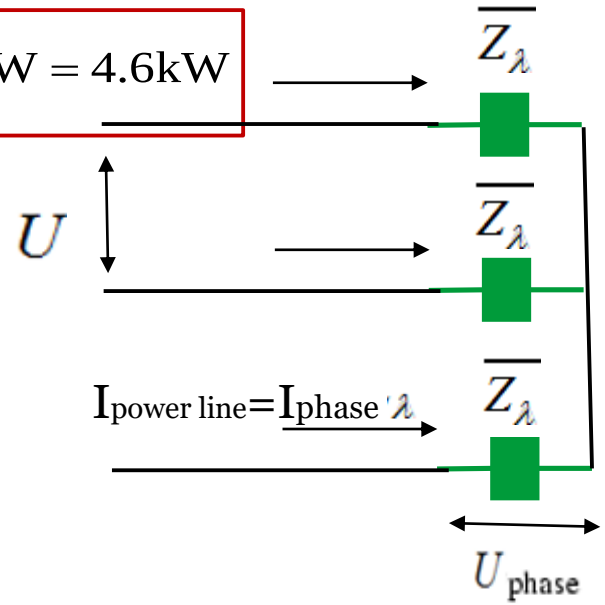
$$\bar{I}_{\text{power line}} = \frac{U_{\text{phase}}}{\bar{Z}_\lambda} = \frac{U_{\text{line}}}{\sqrt{3} * \bar{Z}_\lambda} = \frac{440\text{V}}{\sqrt{3} * 33.33\angle 36.87°\Omega} = 7.62\angle -36.87°\text{A}$$

Question 3
c) Active power



$$P_\Delta = 3 * U_{ph-ph} * I_\Delta \cos\varphi = 3 * 440\text{V} * 4.4 * \cos(36.87°) = 4646\text{W} = 4.6\text{kW}$$

$$P_\lambda = 3 * U_{\text{phase}} I_{\text{power line}} \cos\varphi = 3 * \frac{440\text{V}}{\sqrt{3}} * 7.62 \cos(36.87°) = 4645\text{W} = 4.6\text{kW}$$



Triple check...

$$\begin{cases} \underline{S}_\lambda = 3\underline{U}_{\text{phase}} \cdot \underline{I}^*_{\text{line}} = 3 \cdot \dfrac{440V}{\sqrt{3}} \cdot 7{,}62\underline{/-36{,}87°}^* = 5807{,}2\underline{/36{,}87°} \\ S_\Delta = 3U\,I_\Delta = 3 \cdot 440V \cdot 4{,}4\underline{/-36{,}87°}^* = 5807{,}2\underline{/36{,}87°} \end{cases}$$

$$= 4646 + j3484 = P + jQ \rightarrow P = 4.6\text{kW}$$

Question 4

A star-connected load consisting of a resistor of 80 Ω and and inductor of 0.191 H in each phase is connected to a 415-V, three-phase, 50-Hz supply. Calculate:

(a) the phase current I;
(b) the real power P consumed by the load; and
(c) the reactive power Q consumed by the load.
(d) From P and Q calculate the load phase angle ϕ, and show that:

$$P = \sqrt{3}VI\cos\phi$$

$$Q = \sqrt{3}VI\sin\phi$$

Question 4

a) Inductor:

$$X_L = 2\pi f L = 2 \times \pi \times 50 \times 0.191\Omega = 60\Omega$$

$$U_{Ph} = \frac{415V}{\sqrt{3}} = 240V$$

Lagging current (inductive load)

$$\Rightarrow \underline{I} = \frac{240}{(80 + j60)} A = \underline{\underline{2.4\angle -37^o A}}$$

b) $$\overline{S} = \sqrt{3}\overline{VI}^* = \sqrt{3} \times 415 \times \boxed{\frac{240}{(80 - j60)}} = (1380 + j1035)\text{VA}$$

$\boxed{= \underline{I}^*}$

$$= 1725\angle 37^o \text{ VA}$$

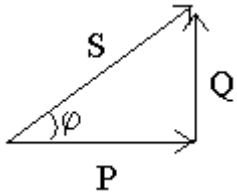
$$\Rightarrow P = \underline{\underline{1380\text{W}}}$$

c) $$\Rightarrow Q = \underline{\underline{1035\,\text{var}}}$$

d) $$P = \sqrt{3} \times U \times I \times \cos\varphi \qquad Q = \sqrt{3} \times U \times I \times \sin\varphi$$

$$\Rightarrow \tan\varphi = \frac{Q}{P}$$

$$\Rightarrow \varphi = \arctan\left(\frac{Q}{P}\right) = \arctan(0.75)$$



$$\approx \underline{\underline{37^o}}$$

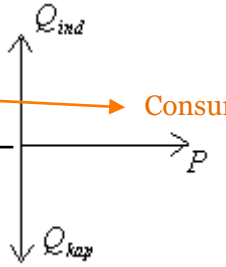
$$\Rightarrow P = \sqrt{3} \times UI \times \cos\varphi \approx 1380\text{W}$$

$$Q = \sqrt{3} \times UI \times \sin\varphi \approx 1035\text{var}$$

Use of reactance (ind)

Consumption

Generation



"Generation" of reactance (cap)