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# Exercise Session 2

## Power systems

# Question 1

- A 220-kV overhead line has the following properties:
  - s=200 km
  - r=0.07 Ω/km
  - x=0.32 Ω/km
  - b=3.6 μS/km

Define the parameters for medium-length line

a) Π-model

b) T-model

and calculate the open circuit voltage at the end of the line if the voltage at the beginning is 220 kV.

# Question 1

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  - s=200 km
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$$\underline{Z} = (r + j\omega l)\, s$$

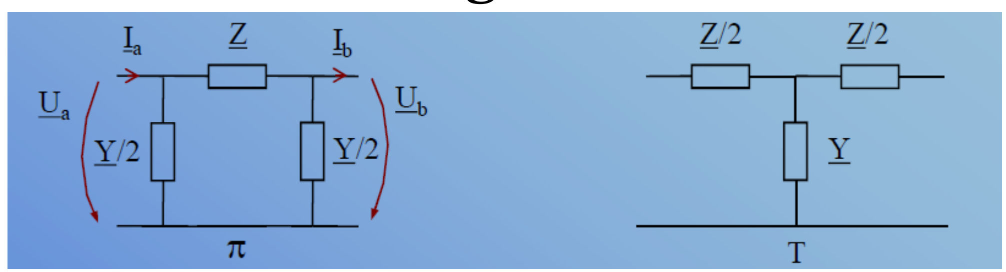
$$\underline{Y} = (g + j\omega c)\, s$$

- s = length
- r = resistance / s
- l = inductance / s
- g = conductance / s
- c = capacitance / s

Define the parameters for a medium-length line with the

a) Π-model

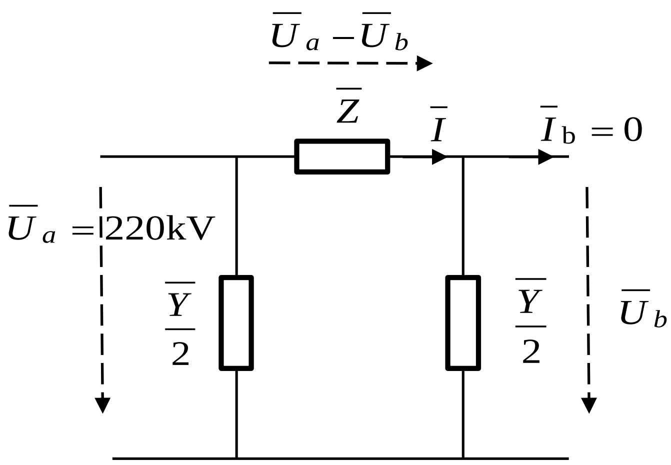
b) T-model




and calculate the open circuit voltage at the end of the line if the voltage at the beginning is 220 kV.

# Question 1

# a) Π-model




$$\overline{Z} = (r + j\omega l)s = (r + jx)s = \left(0.07\frac{\Omega}{\text{km}} + j0.32\frac{\Omega}{\text{km}}\right)200\text{km} = 65.51\angle 77.66°\Omega$$

$$\frac{\overline{Y}}{2} = \frac{(g + j\omega c)s}{2} = \frac{(g + jb)s}{2} = \frac{\left(0\frac{\mu\text{S}}{\text{km}} + j3.6\frac{\mu\text{S}}{\text{km}}\right)200\text{km}}{2} = 360\angle 90°\mu\text{S}$$

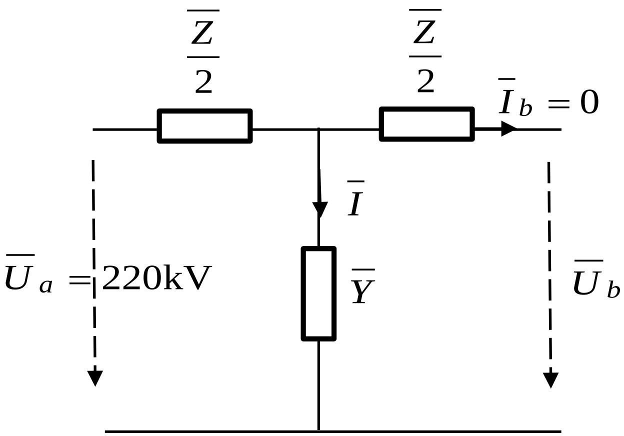
$$\overline{U}_b = \overline{I}\frac{2}{\overline{Y}} = \frac{\overline{U}_a - \overline{U}_b}{\overline{Z}}\frac{2}{\overline{Y}}$$

$$\Leftrightarrow \overline{U}_b\overline{Z}\overline{Y} = 2\overline{U}_a - 2\overline{U}_b$$

$$\Leftrightarrow \overline{U}_b\left(\overline{Z}\overline{Y} + 2\right) = 2\overline{U}_a$$

$$\overline{U}_b = \frac{2\overline{U}_a}{\left(\overline{Z}\overline{Y} + 2\right)} = \frac{2 \times 220\text{kV}}{(65.51\angle 77.66°\Omega \times 2 \times 360\angle 90°\mu\text{S} + 2)} = 225\angle -0.3°\text{kV}$$

# Question 1
# b) T-model




$$\frac{\overline{Z}}{2} = \frac{65.51\angle 77.66°\Omega}{2} = 32.76\angle 77.66°\Omega$$

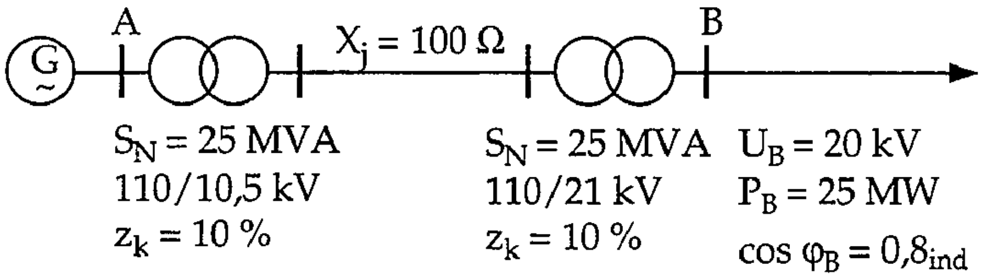
$$\overline{Y} = 2\times\frac{\overline{Y}}{2} = 2\times 360\angle 90°\mu S = 720\angle 90°\mu S$$

$$\overline{U}_b = \overline{I}\frac{1}{\overline{\overline{Y}}} = \frac{\overline{U}_a}{\frac{\overline{\overline{Z}}}{2}+\frac{1}{\overline{\overline{Y}}}}\frac{1}{\overline{\overline{Y}}} = \frac{2\overline{U}_a}{\overline{\overline{Z}}\,\overline{\overline{Y}}+2}$$

$$= \frac{2\times 220\text{kV}}{\left(65.51\angle 77.66°\Omega\times 720\angle 90°\mu S+2\right)} = 225\angle -0.3°\text{kV}$$

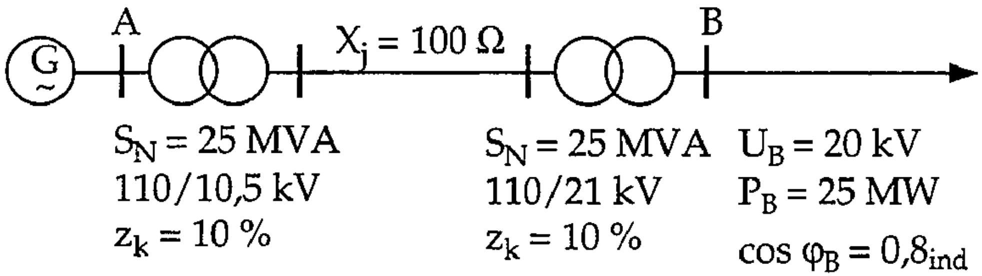
Both models give the same result!

# Question 2




- Calculate per-unit values for all the parameters in the picture above. Use base values: $S_b$=50 MVA and $U_b$=110 kV

# Question 2




- Calculate per-unit values for all the parameters in the picture above. Use base values: $S_b$=50 MVA and $U_b$=110 kV

$$x = \frac{X}{\frac{U_b^2}{S_b}} = \frac{Z_k \frac{U_n^2}{S_n}}{\frac{U_b^2}{S_b}}$$

Other base voltages by transforming ratios:

$$U_{2b} = (U_{2n}/U_{1n}) \cdot U_{1b}$$

Current base value by base power and base voltage:

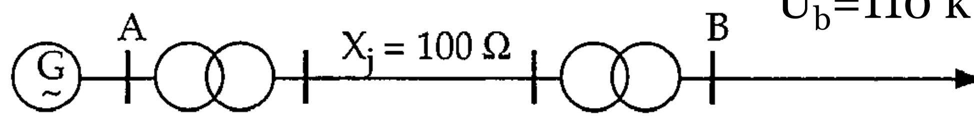
$$I_b = S_b / \sqrt{3}\, U_b$$

Impedance base value by base power and base voltage:

$$Z_b = U_b^{\,2} / S_b$$

# Question 2

$S_b$=50 MVA
$U_b$=110 kV




$S_N$ = 25 MVA
110/10,5 kV
$z_k$ = 10 %

$S_N$ = 25 MVA
110/21 kV
$z_k$ = 10 %

$U_B$ = 20 kV
$P_B$ = 25 MW
$\cos\varphi_B = 0{,}8_{ind}$

$$x_{T1} = \frac{Z_k \dfrac{U_N^2}{S_N}}{\dfrac{U_b^2}{S_b}} = \frac{0.10 \cdot \dfrac{(110\ \text{kV})^2}{25\ \text{MVA}}}{\dfrac{(110\ \text{kV})^2}{50\ \text{MVA}}} = \underline{\underline{0.200}}$$

$$x_j = \frac{X_j}{\dfrac{U_b^2}{S_b}} = \frac{100\ \Omega}{\dfrac{(110\ \text{kV})^2}{50\ \text{MVA}}} = \underline{\underline{0.413}}$$

$$x_{T2} = \frac{Z_k \dfrac{U_N^2}{S_N}}{\dfrac{U_b^2}{S_b}} = 0.10 \cdot \frac{\dfrac{(110\ \text{kV})^2}{25\ \text{MVA}}}{\dfrac{(110\ \text{kV})^2}{50\ \text{MVA}}} = \underline{\underline{0.200}}$$

$$u_B = \frac{20\ \text{kV}}{\left[U_b \cdot \dfrac{U_{N2}}{U_{N1}}\right]} = \frac{20\ \text{kV}}{\left[110\ \text{kV} \cdot \dfrac{21\ \text{kV}}{110\ \text{kV}}\right]} = \underline{\underline{0.952}}$$

$$p_B = \frac{P_B}{S_b} = \frac{25\ \text{MW}}{50\ \text{MVA}} = \underline{\underline{0.500}}$$

# Question 3

- Based on the results obtained from Question 2, calculate the voltage in busbar A (per-unit value and in volts).

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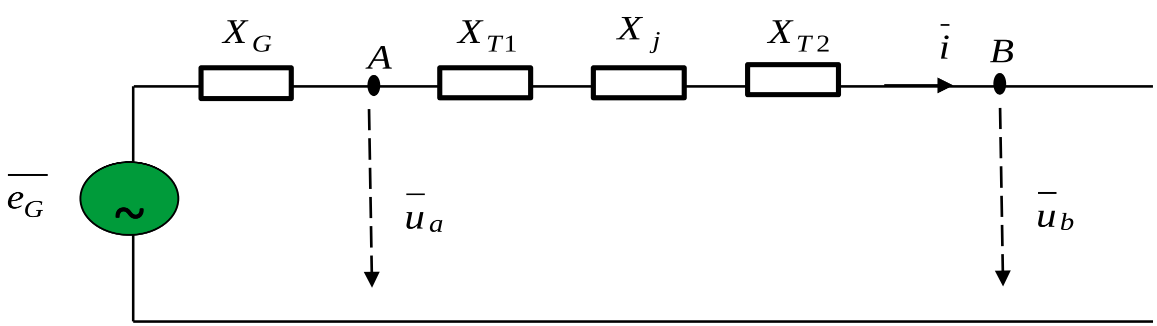
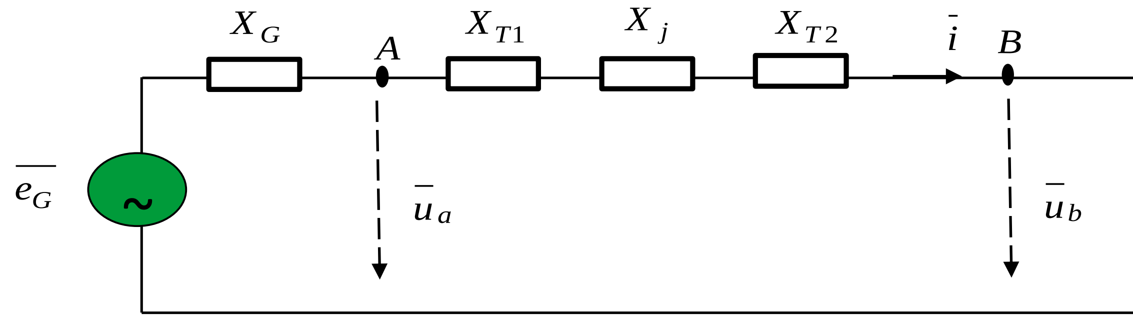



$x_{T2} = 0.200$ $\qquad x_{T1} = 0.200$ $\qquad u_B = 0.952$

$x_j = 0.413$ $\qquad p_B = 0.500$ $\qquad \cos\varphi_B = 0.8_{\text{ind}}$

# Question 3




$\underline{s}_B = \underline{u}_B \underline{i}^*$

$\cos\varphi_B = 0.8_{\text{ind}}$

$$\underline{i} = \frac{\underline{s}_B^*}{\underline{u}_B^*} = \left(\frac{p_B}{\cos\varphi}\angle -\varphi\right)\cdot\frac{1}{u_B} = \left(\frac{0.500}{0.8}\angle -\arccos 0.8\right)\cdot\frac{1}{0.952} \approx 0.656\underline{/-36.9^\circ}$$

$$\underline{u}_A = \underline{u}_B + j\left(x_{T1} + x_j + x_{T2}\right)\cdot\underline{i}$$

$$\underline{u}_A = 0.952\underline{/0^\circ} + j\left(0.200 + 0.413 + 0.200\right)\cdot 0.656\underline{/-36.9^\circ} \approx 1.34\underline{/18.5^\circ}$$

$$\underline{U}_A = \underline{u}_A \cdot U_b\left(\frac{U_{N2}}{U_{N1}}\right) = 1.34\underline{/18.5^\circ}\cdot 110\text{ kV}\cdot\left(\frac{10.5\text{ kV}}{110\text{ kV}}\right) \approx 14.1\text{ kV}\underline{/18.5^\circ}$$

Original base value was in 110-kV level (the line)

The "new" base value in 10.5-kV level (at point A). Note the reference over the transformer!

# Question 4

A 50-Hz, 50-MVA transformer with a 132-kV primary and a 33-kV secondary has a reactance of 0.1pu per phase. What is the reactance in ohms per phase:

(a) referred to the primary;

(b) referred to the secondary.

# Question 4

A 50-Hz, 50-MVA transformer with a 132-kV primary and a 33-kV secondary has a reactance of 0.1pu per phase. What is the reactance in ohms per phase:

(a) referred to the primary;

(b) referred to the secondary.

- Other base voltages by transforming ratios:

$$U_{2b} = (U_{2n}/U_{1n}) \cdot U_{1b}$$

- Current base value by base power and base voltage:

$$I_b = S_b / \sqrt{3}\, U_b$$

- Impedance base value by base power and base voltage:

$$Z_b = U_b^2 / S_b$$

# Question 4

## a) referred to the primary

Primary:

$$Z_B = \frac{U_B^2}{S_B} = \frac{(132\times10^3)^2}{50\times10^6}\Omega = 348.5\Omega = X_B$$

$$X = x \times X_B = 0.1\times348.5\Omega = \underline{\underline{34.85\Omega}}$$

## b) referred to the secondary

Secondary:

$$Z_B = \frac{(33\times10^3)^2}{50\times10^6}\Omega = 21.78\Omega = X_B$$

$$X = x \times X_B = 0.1\times21.78\Omega = \underline{\underline{2.178\Omega}}$$