Aalto-yliopisto
Teknillinen korkeakoulu

Exercise Session 3

Power systems

Question 1

Transform load $\underline{S}$=(200 + j150) kVA into equivalent resistance and reactance which are connected:

- in series
- in parallel

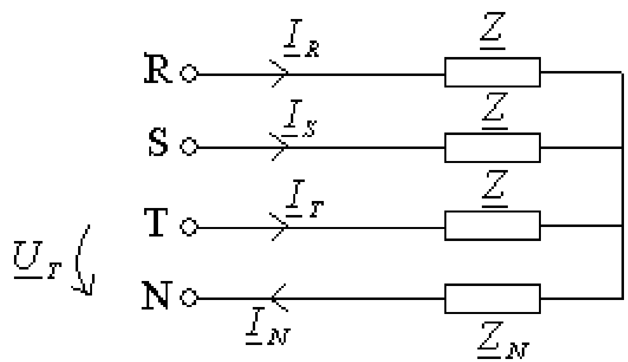
Voltage is 20 kV.

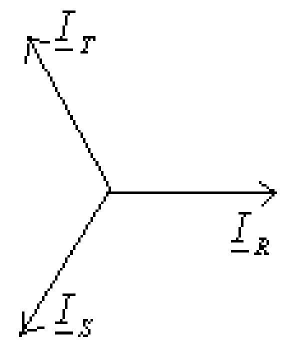
Question 1: Transform load $\underline{S}$=(200 + j150) kVA into equivalent resistance and reactance

- Three-phase equivalent circuit:

$$\underline{I}_{\mathrm{N}} = \underline{I}_{\mathrm{R}} + \underline{I}_{\mathrm{S}} + \underline{I}_{\mathrm{T}} = 0$$

$$\underline{S} = \underline{U}_{\mathrm{R}} \underline{I}_{\mathrm{R}}^{*} + \underline{U}_{\mathrm{S}} \underline{I}_{\mathrm{S}}^{*} + \underline{U}_{\mathrm{T}} \underline{I}_{\mathrm{T}}^{*}$$





$$\underline{S} = \underline{U}_{\mathrm{R}} \left(\frac{\underline{U}_{\mathrm{R}}}{\underline{Z}} \right)^{*} + \underline{U}_{\mathrm{S}} \left(\frac{\underline{U}_{\mathrm{S}}}{\underline{Z}} \right)^{*} + \underline{U}_{\mathrm{T}} \left(\frac{\underline{U}_{\mathrm{T}}}{\underline{Z}} \right)^{*}$$

$$\underline{U}_{\mathrm{R}} = \underline{Z}\,\underline{I}_{\mathrm{R}} = \frac{U}{\sqrt{3}}$$

$$\underline{U}_{\mathrm{S}} = \underline{Z}\,\underline{I}_{\mathrm{S}} = \frac{U}{\sqrt{3}} \underline{/-120^\circ}$$

$$\underline{U}_{\mathrm{T}} = \underline{Z}\,\underline{I}_{\mathrm{T}} = \frac{U}{\sqrt{3}} \underline{/120^\circ}$$

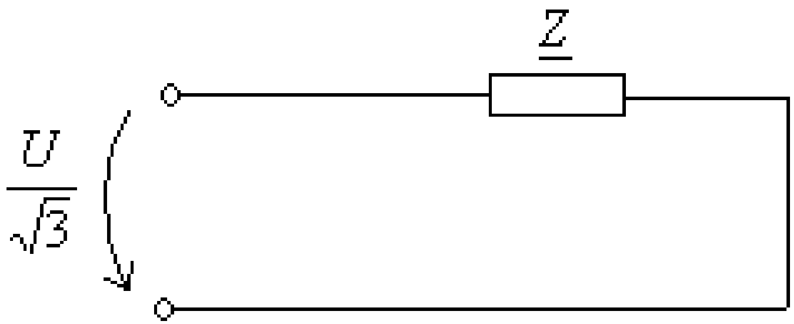
$$\underline{U}_{\mathrm{R}} \underline{U}_{\mathrm{R}}^{*} = U_{\mathrm{R}}^{2}$$

$$\underline{S} = \frac{U_{\mathrm{R}}^{2}}{\underline{Z}^{*}} + \frac{U_{\mathrm{S}}^{2}}{\underline{Z}^{*}} + \frac{U_{\mathrm{T}}^{2}}{\underline{Z}^{*}} = 3 \cdot \frac{\left(\frac{U}{\sqrt{3}} \right)^{2}}{\underline{Z}^{*}}$$

$$\boxed{\underline{S} = \frac{U^{2}}{\underline{Z}^{*}}}$$

Question 1: Transform load $\underline{S}$=(200 + j150) kVA into equivalent resistance and reactance

- One-phase equivalent circuit:



One-phase apparent power:

$$\frac{\underline{S}}{3} = \frac{\left(\frac{U}{\sqrt{3}}\right)^2}{\underline{Z}^*}$$

$\Leftrightarrow$

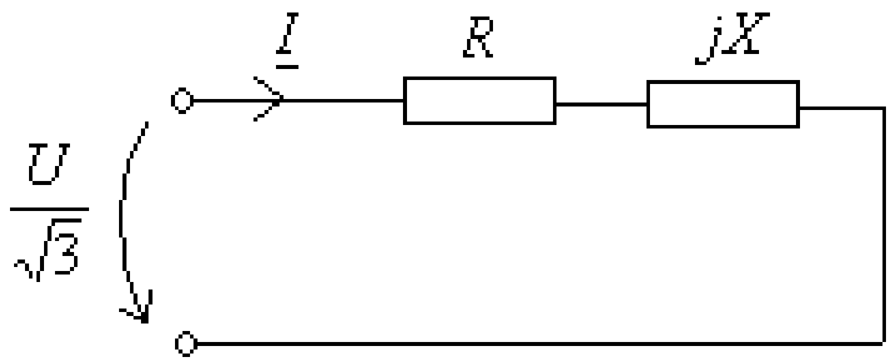
Three-phase apparent power:

$$\underline{S} = \frac{U^2}{\underline{Z}^*}$$

→ We can calculate the phase impedance $\underline{Z}$ with the line voltage and three-phase apparent power directly

Question 1: Transform load $\underline{S}$=(200 + j150) kVA into equivalent resistance and reactance

- a) In series:



$$\underline{Z} = R + jX$$

$$\underline{S} = \frac{U^2}{\underline{Z}^*} \rightarrow \quad \underline{Z}^* = R - jX = \frac{U^2}{\underline{S}} = \frac{U^2}{P + jQ} * \frac{P - jQ}{P - jQ} = \frac{U^2 P}{P^2 + Q^2} - j\frac{U^2 Q}{P^2 + Q^2}$$

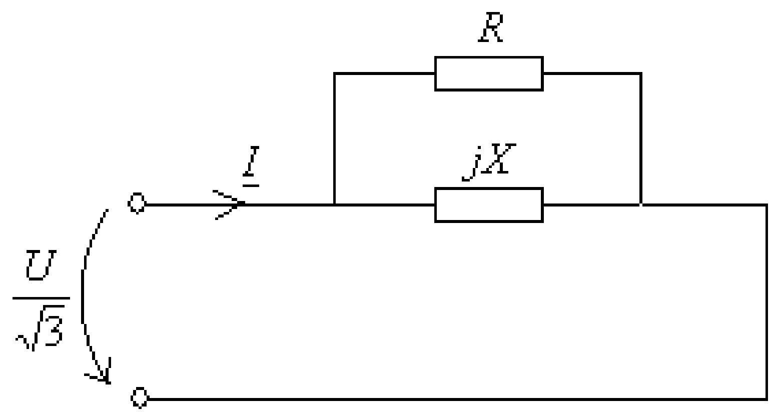
$$R = \frac{U^2 P}{P^2 + Q^2} = \frac{(20\ \text{kV})^2 \cdot 200\ \text{kW}}{\left[\left(200 \cdot 10^3\right)^2 + \left(150 \cdot 10^3\right)^2\right](\text{VA})^2} \approx \underline{\underline{1280\Omega}}$$

$$X = \frac{U^2 Q}{P^2 + Q^2} = \frac{(20\ \text{kV})^2 \cdot 150\ \text{kvar}}{\left[\left(200 \cdot 10^3\right)^2 + \left(150 \cdot 10^3\right)^2\right](\text{VA})^2} \approx \underline{\underline{960\ \Omega}}$$

Note: Reactance takes inductive reactive power ($Q > 0$)

Question 1: Transform load $\underline{S}$=(200 + j150) kVA into equivalent resistance and reactance

- b) In parallel:



Whole of the voltage is across resistance R and reactance X

$$\frac{P}{3} = \frac{\left(\frac{U}{\sqrt{3}}\right)^2}{R} = \frac{U^2}{3R} \qquad \frac{Q}{3} = \frac{\left(\frac{U}{\sqrt{3}}\right)^2}{X} = \frac{U^2}{3X}$$

$$R = \frac{U^2}{P} = \frac{(20\,\text{kV})^2}{200\,\text{kW}} \approx \underline{\underline{2.00\,\text{k}\Omega}}$$

$$X = \frac{U^2}{Q} = \frac{(20\,\text{kV})^2}{150\,\text{kvar}} \approx \underline{\underline{2.67\,\text{k}\Omega}}$$

Note: Reactance takes inductive reactive power $(Q > 0)$

Question 2

- A certain load curve can be presented with the following equation:

$$P(t) = \left(1 - 0.5\sin\left(\frac{t}{12\text{months}}\pi\right)\right)\text{MW}$$

Calculate:

a) Annual energy

b) mean power for one year P_k

c) load factor

d) load duration time t_k

Question 2

$$P(t) = \left(1 - 0.5\sin\left(\frac{t}{12\text{months}}\pi\right)\right)\text{MW}$$

- a) annual energy

$$W = \int_0^T P(t)\,dt$$

$$= \int_0^{12months} \left(1 - 0.5 \cdot \sin\left(\frac{t}{12\text{months}} \cdot \pi\right)\right) dt \text{ MW } = /_0^{12\text{months}} \left(t + \frac{0.5}{\frac{\pi}{12\text{months}}} \cdot \cos\left(\frac{t}{12\text{months}} \cdot \pi\right) \right) \text{MW}$$

$$= 8.1803 \text{ MW months} = 8.1803 \times \frac{8760}{12} \text{MWh} = 5971.6\text{MWh} \approx \underline{\underline{5.97 \text{ GWh}}}$$

Question 2

$$P(t) = \left(1 - 0.5\sin\left(\frac{t}{12\text{months}}\pi\right)\right)\text{MW}$$

- b) mean power for one year P_k

$$P_\text{k} = \frac{W}{T} = \frac{8.1803 \text{ MW months}}{12\text{months}} = 0.6817 \text{ MW} \approx \underline{\underline{682 \text{ kW}}}$$

Question 2

$$P(t) = \left(1 - 0.5\sin\left(\frac{t}{12\text{months}}\pi\right)\right)\text{MW}$$

- c) load factor

Load factor – the average load divided by the peak load in a specified time period

$$Load\ factor = \frac{P_{\text{average}}}{P_{\text{max}}}$$

$$\varepsilon = \frac{P_{\text{k}}}{P_{\text{max}}} = \frac{0.6817\ \text{MW}}{1\ \text{MW}} = 0.6817 \approx \underline{\underline{0.682}}$$

Question 2

$$P(t) = \left(1 - 0.5\sin\left(\frac{t}{12\text{months}}\pi\right)\right)\text{MW}$$

- d) load duration time

$$t_k = \frac{W}{P_{max}} = \frac{5971.6\ \text{MWh}}{1\ \text{MW}} = 5971.6\ \text{h} \approx \underline{\underline{5972\ \text{h}}}$$

Load factor	$\varepsilon = \frac{P_k}{P_{max}}$
Load duration time	$t_k = \frac{W_a}{P_{max}} = \frac{P_k T}{P_{max}}$ $= \varepsilon T$

W_a = annual energy
P_k = mean power
P_{max} = max power

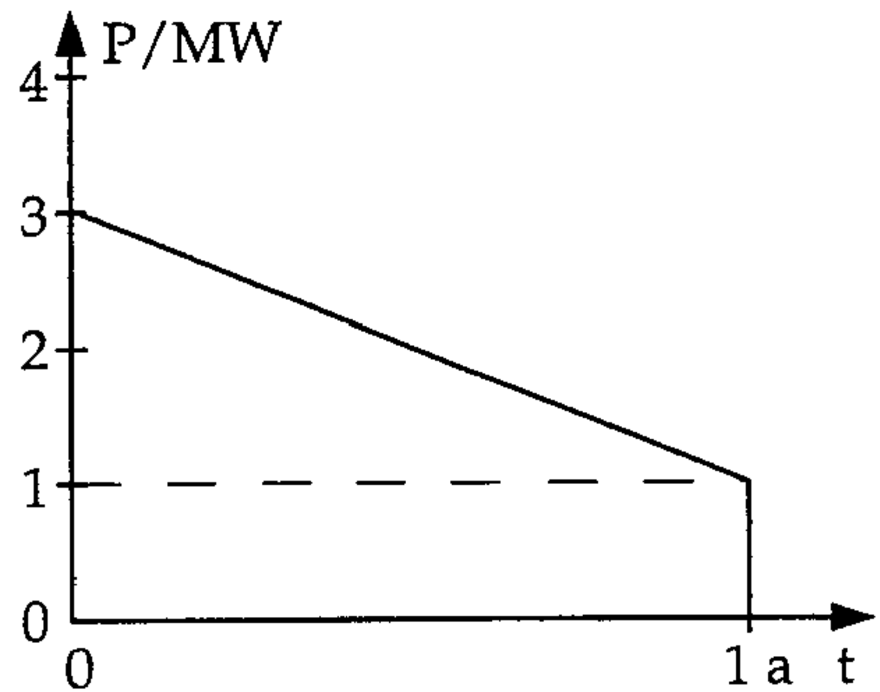
Question 3

At the end of a three-phase line the voltage is 20 kV. The line has following properties: resistance 0.8 Ω and reactance 0.6 Ω per phase. The load at the end of the line has a load duration curve as shown in the picture below. The power factor of the load is constant $\cos\varphi=0.8_{ind}$. Calculate:

a) active power losses P_{hmax} and P_{hmin}

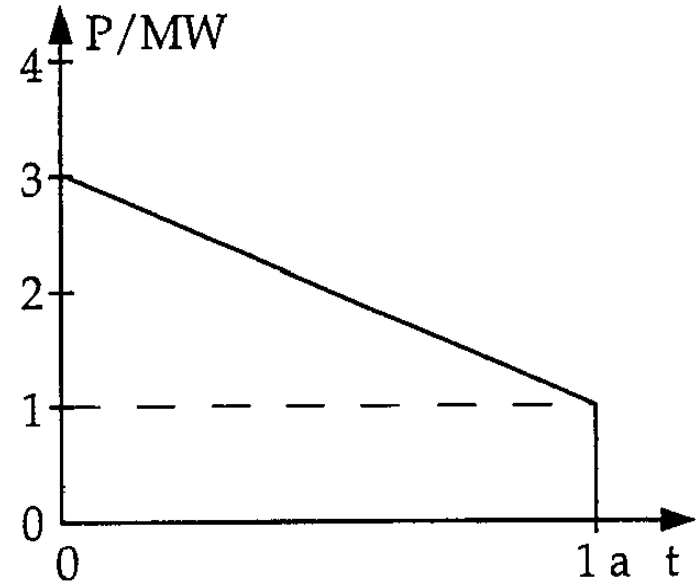
b) reactive power losses Q_{hmax} and Q_{hmin}

c) energy losses W_h



Question 3

$$P(t)=\left(3-2\cdot\frac{t}{a}\right)\text{MW}$$



$$P_{\text{hmax}} = P_h(0),\ P_{\text{hmin}} = P_{\text{h}}(1a),$$
$$P_{\text{h}}(t) = 3R[I(t)]^2$$

$$I(t)=\frac{P(t)}{\sqrt{3}\cdot U\cdot\cos\varphi}=\frac{\left(3-2\cdot\frac{t}{a}\right)\text{MW}}{\sqrt{3}\cdot 20\text{ kV}\cdot 0.8}=\frac{\left(3-2\cdot\frac{t}{a}\right)}{16\sqrt{3}}\text{kA}$$

$$P_{\text{h}}(t)=3R\left[I(t)\right]^2=3\cdot 0.8\,\Omega\cdot\frac{\left[\left(3-2\cdot\frac{t}{a}\right)\cdot 10^3\text{ A}\right]^2}{16^2\cdot 3}=\frac{\left(3-2\cdot\frac{t}{a}\right)^2}{320}\text{MW}$$

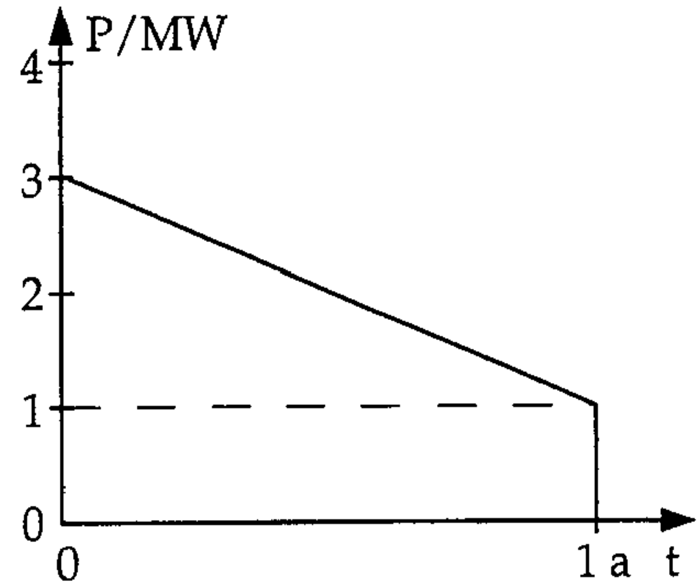
$$Q_{\text{h}}(t)=3X\left[I(t)\right]^2=\frac{X}{R}\cdot P_{\text{h}}(t)=\frac{0.6}{0.8}\cdot\frac{\left(3-2\cdot\frac{t}{a}\right)^2}{320}\text{Mvar}$$

Question 3

- a) active power losses P_{hmax} and P_{hmin}

$$P_{\text{hmax}} = P_{\text{h}}(0) = \frac{(3-2\cdot 0)^2}{320}\,\text{MW} = \underline{\underline{28.1\,\text{kW}}}$$

$$P_{\text{hmin}} = P_{\text{h}}(1a) = \frac{(3-2)^2}{320}\,\text{MW} = \underline{\underline{3.1\,\text{kW}}}$$



Question 3

- b) reactive power losses Q_{hmax} and Q_{hmin}

$$Q_{\text{hmax}} = Q_{\text{h}}(0) = \frac{0.6}{0.8} \cdot \frac{(3-2\cdot 0)^2}{320}\ \text{Mvar} = \underline{\underline{21.1\ \text{kvar}}}$$

$$Q_{\text{hmin}} = Q_{\text{h}}(1a) = \frac{0.6}{0.8} \cdot \frac{(3-2)^2}{320}\ \text{Mvar} = \underline{\underline{2.3\ \text{kvar}}}$$

Question 3

- c) energy losses W_h

$$W_\mathrm{h} = \int_0^{1a} P_\mathrm{h}(t)\,dt = \int_0^{1a} \frac{\left(3-2\cdot\frac{t}{a}\right)^2}{320}\,\mathrm{MW}\,dt$$

$$W_\mathrm{h} = \int_0^{1a} \frac{9-12\cdot\frac{t}{a}+4\cdot\frac{t^2}{a^2}}{320}\,\mathrm{MW}\,dt$$

$$W_\mathrm{h} = \left.\frac{9t-6\cdot\frac{t^2}{a}+\frac{4}{3}\cdot\frac{t^3}{a^2}}{320}\,\mathrm{MW}\right|_0^{1a}$$

$$W_\mathrm{h} = \frac{9-6+\frac{4}{3}}{320}\,\mathrm{MWa} = \frac{13}{3\cdot 320}\,\mathrm{MW}\cdot 8760\ \mathrm{h}$$

$$W_\mathrm{h} = 118.6\ \mathrm{MWh}$$

Question 4

The variation of load (P) with time (t) in a power supply system is given by the expression:

$$P(t) = 4000 + 8t - 0.00091t^2 \text{ (kW)}$$

where t is in hours over a total period of one year.

This load is supplied by three 10-MW generators and it is advantageous to fully load machine before connecting the others.

Determine:

a) the load factor on the system as a whole;
b) the total magnitude of installed load if the diversity factor is equal to 3;
c) the minimum number of hours each machine is in operation;
d) the approximate peak magnitude of installed load capacity to be cut off to enable only two generators to be used.

Question 4

a) Determine the load factor on the system as a whole:

Load factor – the average load divided by the peak load in a specified time period

$$Load\ factor = \frac{P_{\text{average}}}{P_{\max}}$$

Question 4

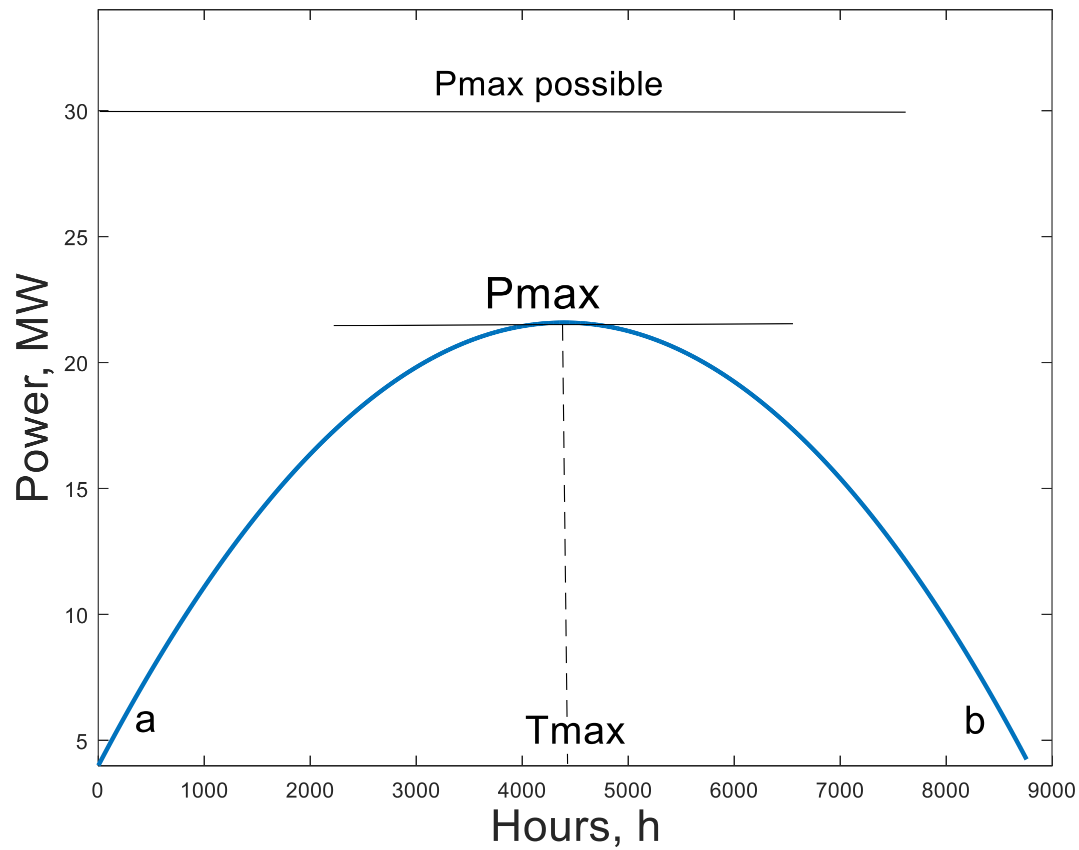
$$P(t) = 4000 + 8t - 0.00091t^2 \text{(kW)}$$

a) Determine the load factor on the system as a whole:

$$Load\ factor = \frac{P_{\text{average}}}{P_{\max}} = \frac{\frac{1}{b-a}\int_a^b P(t)dt}{P_{\max}},$$

where $P_{\max} = P(T_{\max})$

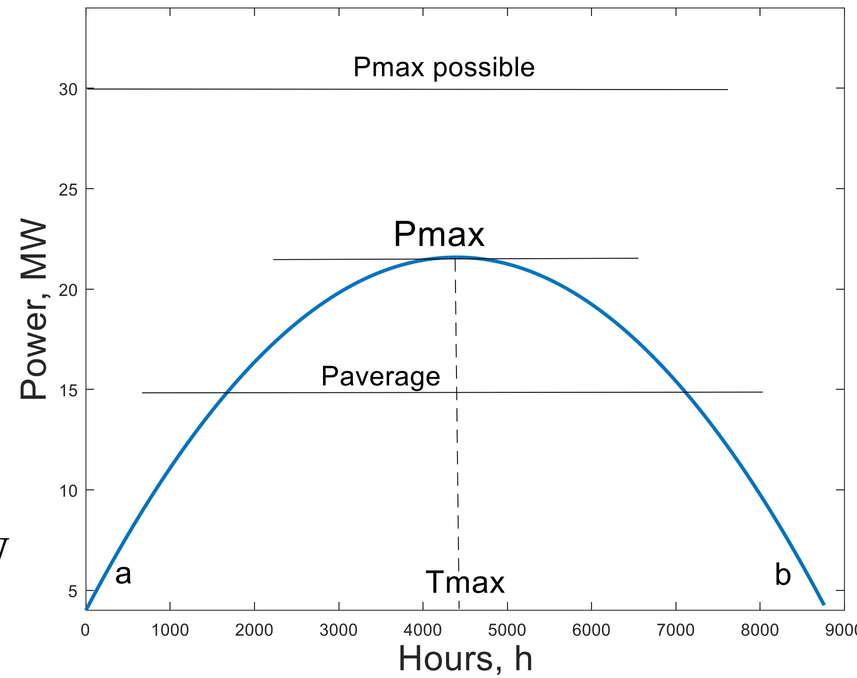
$$P_{\text{max possible}} = 3 * 10 \text{ MW} = 30 \text{ MW}$$



Question 4

$$P(t) = 4000 + 8t - 0.00091t^2 \, (\text{kW})$$

a) Determine the load factor on the system as a whole:



$$\frac{dP}{dt} = 8 - 0.00182t = 0$$

$$\Rightarrow$$

$$T_{\max} = \frac{8}{0.00182} = 4395.6\text{h}$$

$$P_{\max} = 4000 + 8 \times 4395.6 - 0.00091 \times 4395.6^2 = 21582\text{kW} = 21.6\text{MW}$$

$$P_{\text{average}} = \frac{1}{8760 - 0} \times \int_0^{8760} (4000 + 8t - 0.00091t^2)dt = \frac{1}{8760} \times (4000t + 4t^2 - \frac{0.00091}{3}t^3)\Big|_0^{8760} =$$

$$= \frac{1}{8760} \times (4000 \times 8760 + 4 \times 8760^2 - \frac{0.00091}{3} \times 8760^3) = \frac{138083249}{8760} = 15762.93 \text{ kW}$$

$$Load\ \ factor = \frac{P_{\text{average}}}{P_{\max}} = \frac{15762.93}{21582} = 0.730374$$

Question 4

b) The total magnitude of installed load if the diversity factor is equal to 3

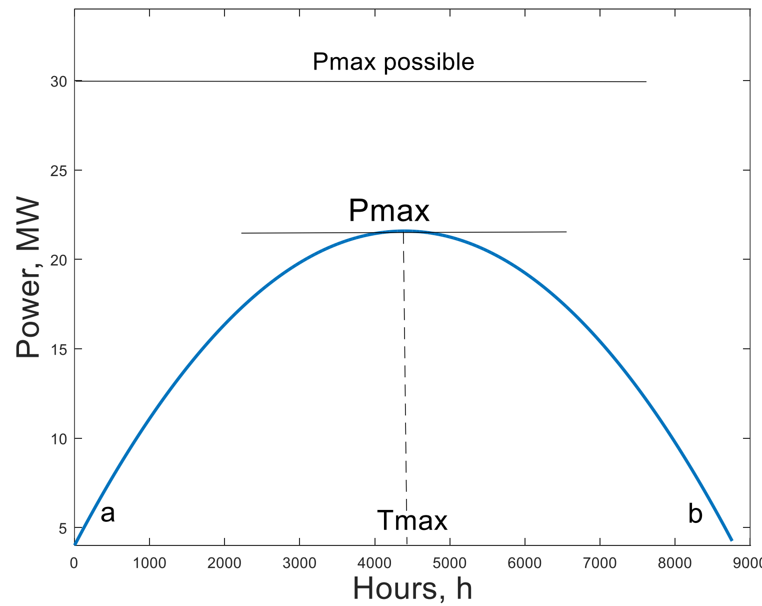
Diversity factor - defined as the sum of individual maximum demands of the consumers, divided by the maximum load on the system. This factor measures the diversification of the load and is concerned with the installation of sufficient generating and transmission plant. If all the demands occurred simultaneously, that is, unity diversity factor, many more generators would have to be installed. Fortunately, the factor is much higher than unity, especially for domestic loads.

$$k = \frac{P_l}{P_{\max}} = \frac{\sum_{i=1}^{n} P_i}{P_{\max}}, \quad Diversity\ factor = \frac{\text{installed load}}{\text{max running load}}$$

where P_l is sum of installed loads connected

$$k = 3 \Rightarrow 3 = \frac{P_l}{P_{\max}} \Rightarrow$$

$$\Rightarrow P_l = 3 \times P_{\max} = 3 \times 21582\text{kW} = 64\ 746\ \text{kW}$$



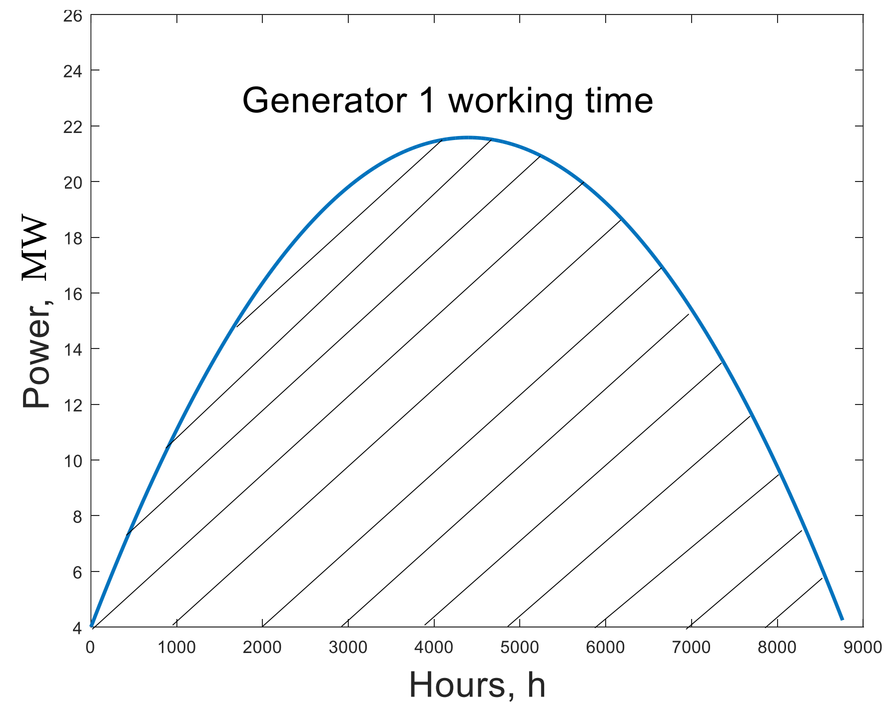
Question 4

$$P(t) = 4000 + 8t - 0.00091t^2 \text{ (kW)}$$

c) The minimum number of hours each machine is in operation:

This load is supplied by three 10-MW generators and it is advantageous to fully load machine before connecting the others.

Generator 1 works all year long (in this problem)



Question 4

$$P(t) = 4000 + 8t - 0.00091t^2 \text{(kW)}$$

c) The minimum number of hours each machine is in operation:

This load is supplied by three 10-MW generators and it is advantageous to fully load machine before connecting the others.

Generator 2 starts to work only when the load reaches over 10 MW

$$4000 + 8t - 0.00091t^2 = 10000$$

$$\Leftrightarrow -0.00091t^2 + 8t - 6000 = 0$$

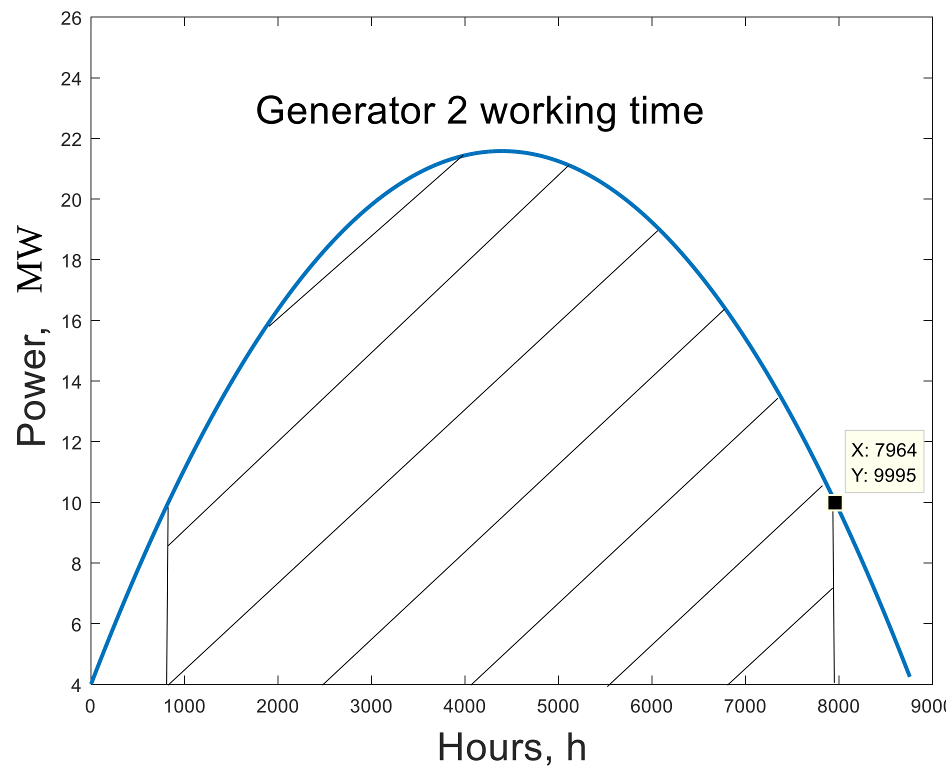
$$\Leftrightarrow$$

$$t_{1,2} = \frac{-8 \pm \sqrt{8^2 - 4\times(-0.00091)\times(-6000)}}{2\times(-0.00091)}$$

$$\Rightarrow$$

$$t_1 = 827.98\text{h} \rightarrow t_2 = 7963.23\text{h}$$

$$T_2 = t_2 - t_1 = 7963.23 - 827.98 = 7135.25\text{h}$$



Question 4

$$P(t) = 4000 + 8t - 0.00091t^2 \,(\text{kW})$$

c) The minimum number of hours each machine is in operation:

This load is supplied by three 10-MW generators and it is advantageous to fully load machine before connecting the others.

Generator 3 starts to work only when the load reaches over 20 MW

$$4000 + 8t - 0.00091t^2 = 20000$$

$$\Leftrightarrow -0.00091t^2 + 8t - 16000 = 0$$

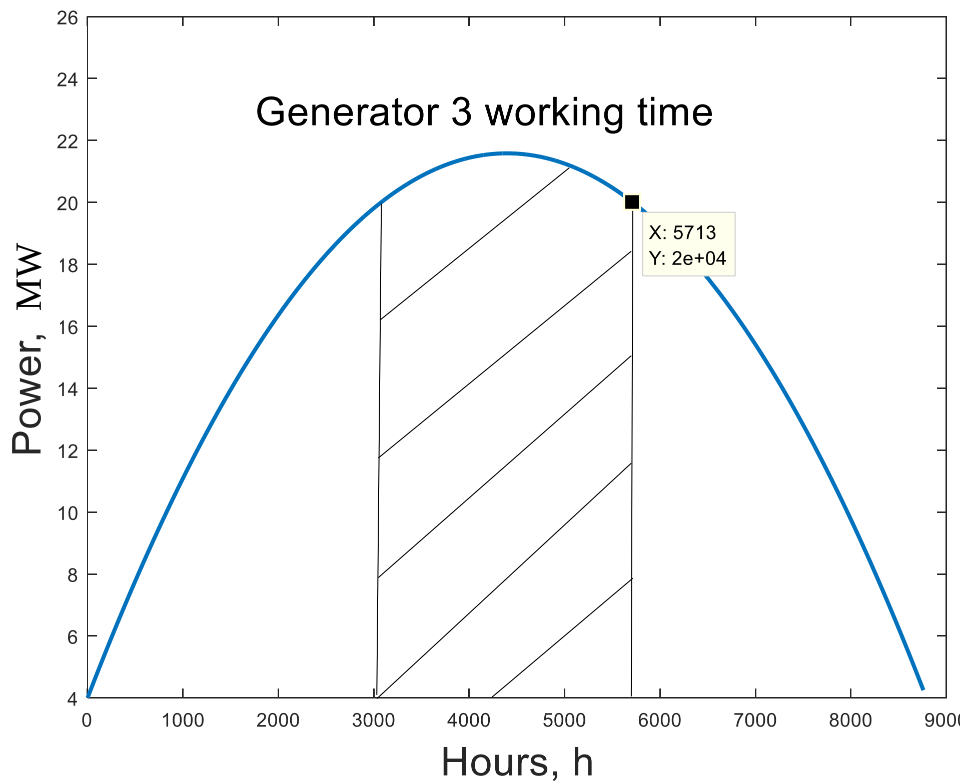
$$\Leftrightarrow$$

$$t_{1,2} = \frac{-8 \pm \sqrt{8^2 - 4 \times (-0.00091) \times (-16000)}}{2 \times (-0.00091)}$$

$$\Rightarrow$$

$$t_1 = 3076.92\text{h} \rightarrow t_2 = 5714.3\text{h}$$

$$T_3 = t_2 - t_1 = 5714.3 - 3076.92 = 2637.38\text{h}$$



Question 4

$$P(t) = 4000 + 8t - 0.00091t^2 \, (\text{kW})$$

c) The minimum number of hours each machine is in operation:

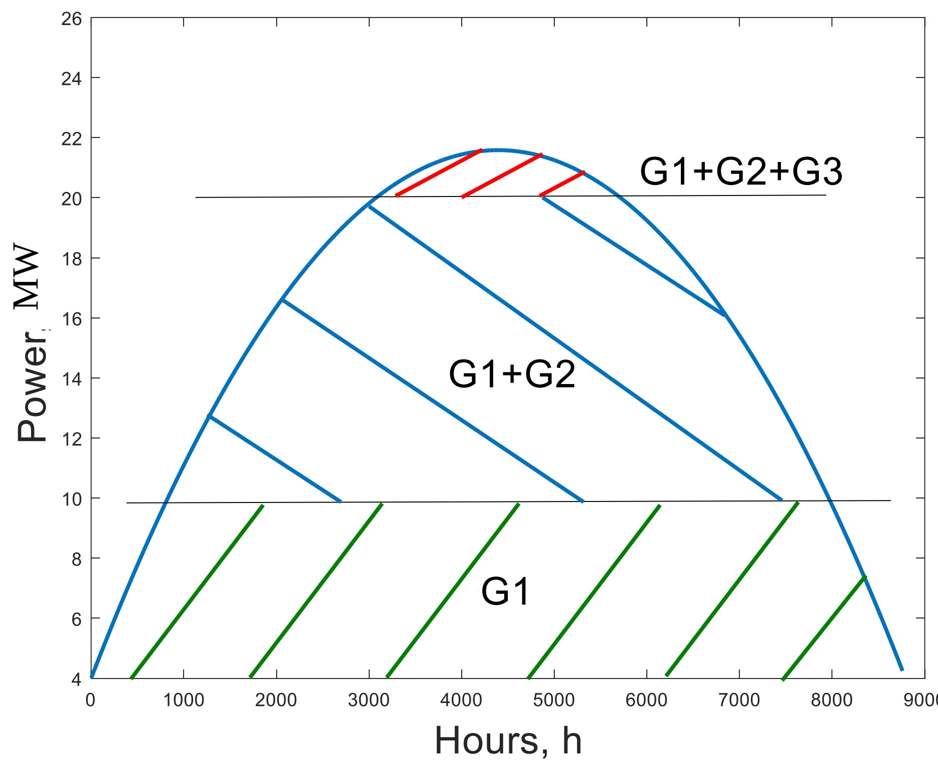
This load is supplied by three 10-MW generators and it is advantageous to fully load machine before connecting the others.

Generator 3 starts to work only when the load reaches over 20 MW

$$T_1 = 8760 \text{ h}$$

$$T_2 = 7135.25 \text{ h}$$

$$T_3 = 2637.36 \text{ h}$$



Question 4

d) The approximate peak magnitude of installed load capacity to be cut off to enable two generators to be used:

Total magnitude of installed load $P_l = \sum_{i=1}^{n} P_i = 64\ 746$ kW

2 generators can deliver 20 000 kW

If the diversity factor is 3, then ($k = \frac{P_l}{P_{\max}} = \frac{\sum_{i=1}^{n} P_i}{P_{\max}}$)

$P_{l2} = 3 * 20\ 000 = 60\ 000$ kW

We would need to cut $\Delta P = P_l - P_{l2}$= 64 746 - 60 000 = 4746 kW