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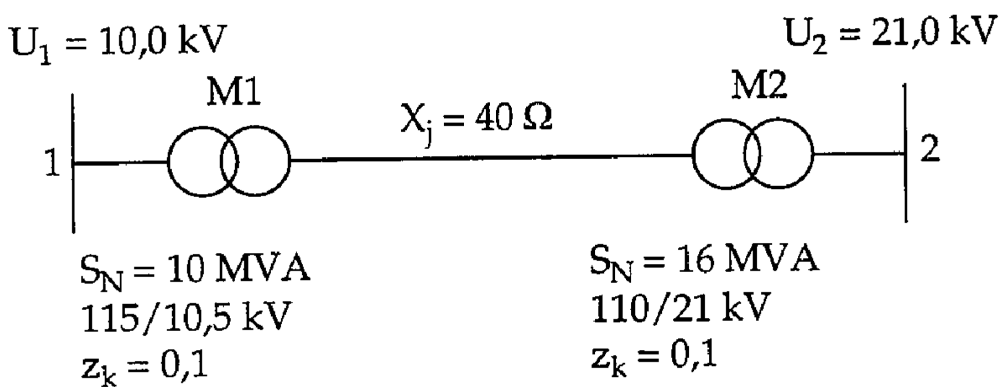
# Exercise Session 4

## Power systems

# Question 1

- Calculate the maximum active power $P_{max}$ that can be transferred from busbar 1 to busbar 2 using the voltages shown in the picture.

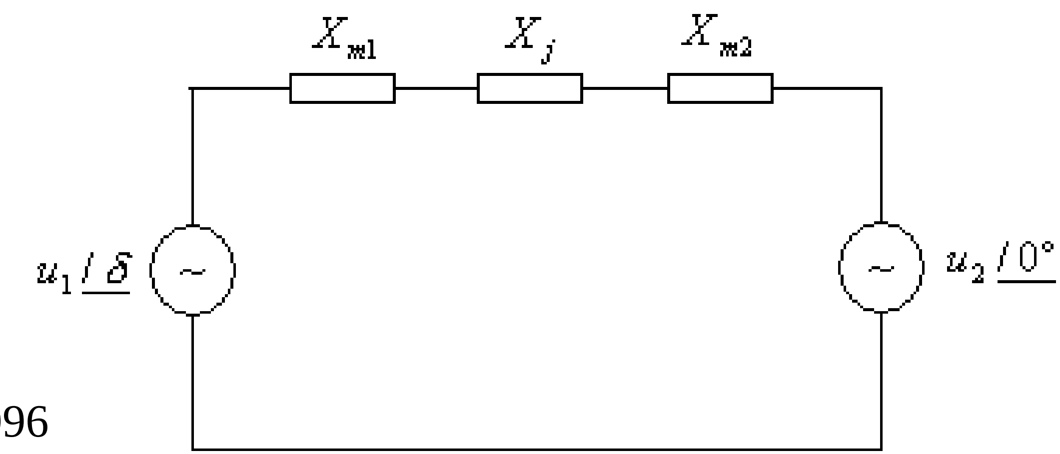


# Question 1

The equivalent scheme of the line, using reactances for P.U. computation.

Selecting:

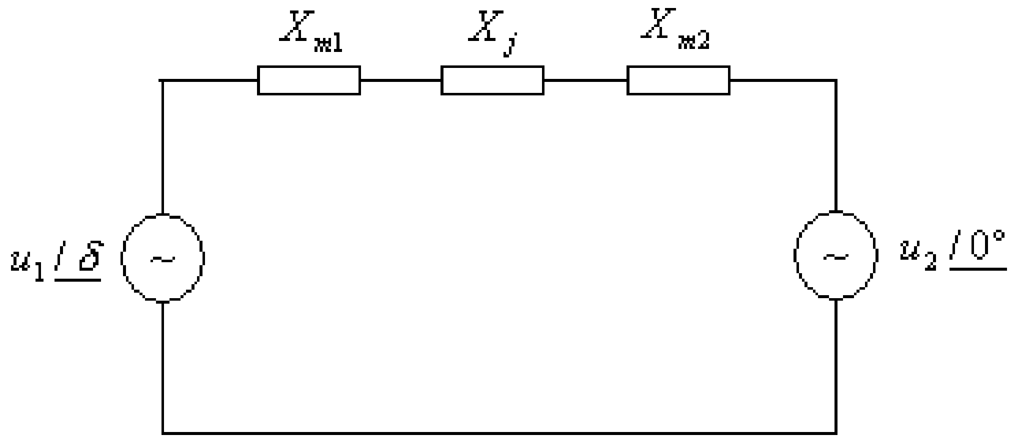
$S_b = 16$ MVA
$U_b = 110$ kV




$$u_1 = \frac{U_1}{U_b \cdot \left(\frac{U_{N2}}{U_{N1}}\right)} = \frac{10.0 \text{ kV}}{110 \text{ kV} \cdot \left(\frac{10.5}{115}\right)} \approx 0.996$$

$$u_2 = \frac{U_2}{U_b \left(\frac{U_{N2}}{U_{N1}}\right)} = \frac{21.0 \text{ kV}}{110 \text{ kV} \cdot \left(\frac{21}{110}\right)} \approx 1.000$$

# Question 1




Selecting:
$S_b = 16$ MVA
$U_b = 110$ kV

$$x_{m1} = Z_k \cdot \frac{U_N^2}{S_N} \bigg/ \frac{U_b^2}{S_b} = 0.1 \cdot \frac{(115\ \text{kV})^2}{10\ \text{MVA}} \bigg/ \frac{(110\ \text{kV})^2}{16\ \text{MVA}} = 0.175$$

$$x_{m2} = Z_k \cdot \frac{U_N^2}{S_N} \bigg/ \frac{U_b^2}{S_b} = 0.1 \cdot \frac{(110\ \text{kV})^2}{16\ \text{MVA}} \bigg/ \frac{(110\ \text{kV})^2}{16\ \text{MVA}} = 0.100$$

$$x_j = \frac{X_j}{\frac{U_b^2}{S_b}} = \frac{40\ \Omega}{\frac{(110\ \text{kV})^2}{16\ \text{MVA}}} \approx 0.053$$

# Question 1

Power-angle equation:

$$\boxed{P_1 = P_2 = \frac{U_1 U_2}{X} \cdot \sin\delta}$$

The highest power that can be transmitted:

$$p = \frac{u_1 u_2}{x_{m1} + x_j + x_{m2}} \cdot \sin\delta$$

Maximum value when:

$$\delta = 90^\circ \quad (\sin\delta = 1)$$

$$p_{\max} = \frac{u_1 u_2}{x_{m1} + x_j + x_{m2}} = \frac{0.996 \cdot 1.000}{0.175 + 0.100 + 0.053} \approx 3.038$$

$$\underline{\underline{P_{\max}}} = p_{\max} \cdot S_b = 3.038 \cdot 16\ \text{MVA} \approx \underline{\underline{48.60\ \text{MW}}}$$

# Question 2

- The power that can be transferred by a line is, usually, limited by the reactive power resources. A line has a series reactance X=100 Ω and the voltage at the beginning of the line is $U_1$=115 kV. The reactive power at the end of the line is $Q_2$=0. Derive the expressions for power transferred and reactive power $Q_1$ as the function of voltage angle δ. How much smaller is the maximum transferred power compared to a case where the necessary reactive power could be fed to the end of the line?

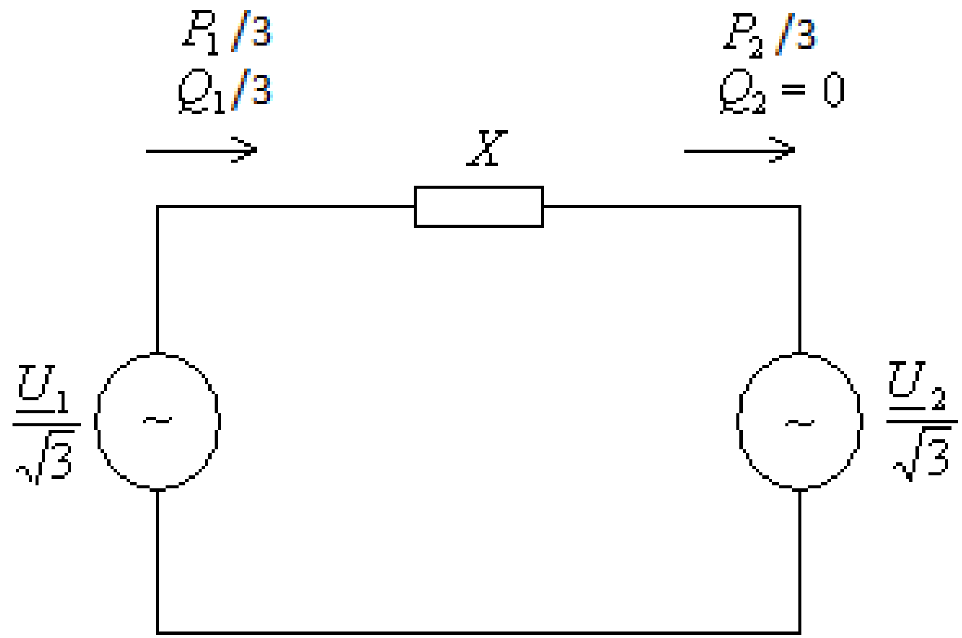
# Question 2

Power-angle equation:

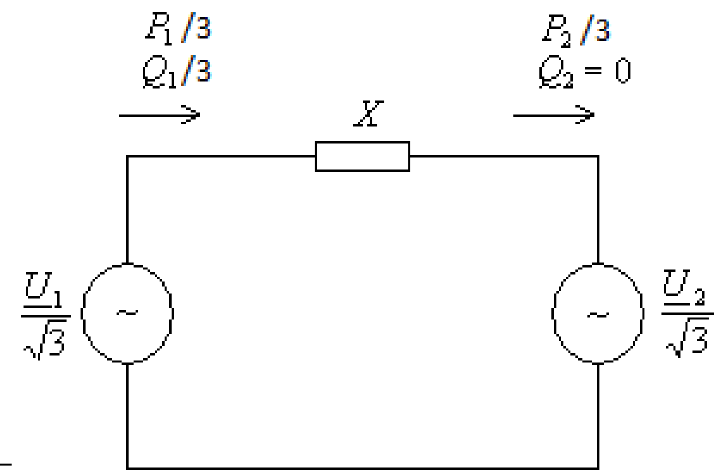
$$\begin{cases} P_1 = P_2 = \dfrac{U_1 U_2}{X} \cdot \sin\delta \\[2ex] Q_1 = \dfrac{U_1^2}{X} - \dfrac{U_1 U_2}{X} \cdot \cos\delta \end{cases}$$

$$Q_2 = \frac{U_1 U_2}{X} \cdot \cos\delta - \frac{U_2^2}{X}$$

$$Q_2 = 0 \quad \Leftrightarrow \quad U_2 = U_1 \cos\delta$$



# Question 2




$$Q_2 = 0 \quad \Leftrightarrow \quad U_2 = U_1 \cos\delta$$

$$\boxed{\sin(2\delta) = 2\sin(\delta)\cos(\delta)}$$

$$P_1 = P_2 = \frac{U_1 U_2}{X}\cdot\sin\delta = \frac{U_1^2}{X}\cdot\sin\delta\cdot\cos\delta = \frac{U_1^2}{2X}\cdot\sin 2\delta = \frac{(115\ \mathrm{kV})^2}{2\cdot 100\ \Omega}\cdot\sin 2\delta$$

$$\underline{\underline{P_1 = P_2 \approx 66.1\ \mathrm{MW}\cdot\sin 2\delta}}$$

$$\boxed{\sin^2\delta + \cos^2\delta = 1}$$

$$Q_1 = \frac{U_1^2}{X} - \frac{U_1 U_2}{X}\cdot\cos\delta = \frac{U_1^2}{X}\cdot\left(1-\cos^2\delta\right) = \frac{U_1^2}{X}\cdot\sin^2\delta = \frac{(115\ \mathrm{kV})^2}{100\ \Omega}\cdot\sin^2\delta$$

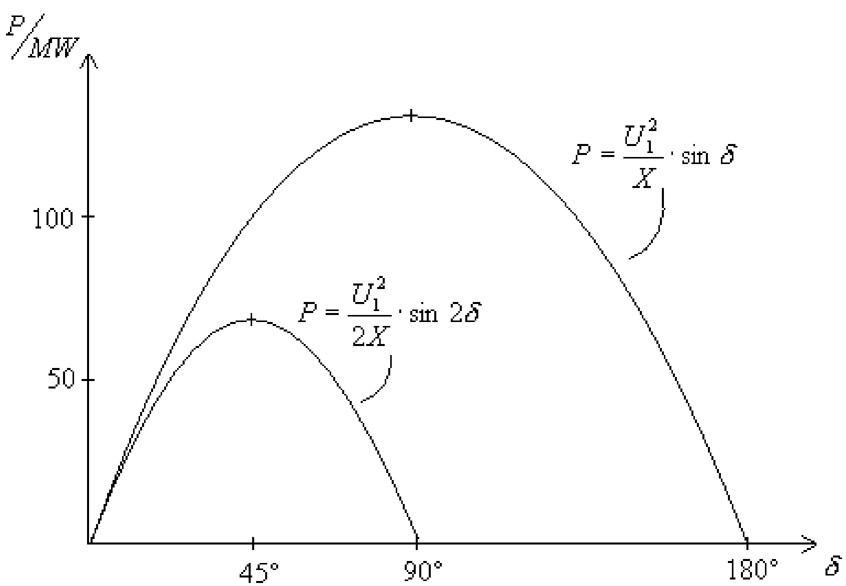
$$\underline{\underline{Q_1 \approx 132.3\ \mathrm{Mvar}\cdot\sin^2\delta}}$$

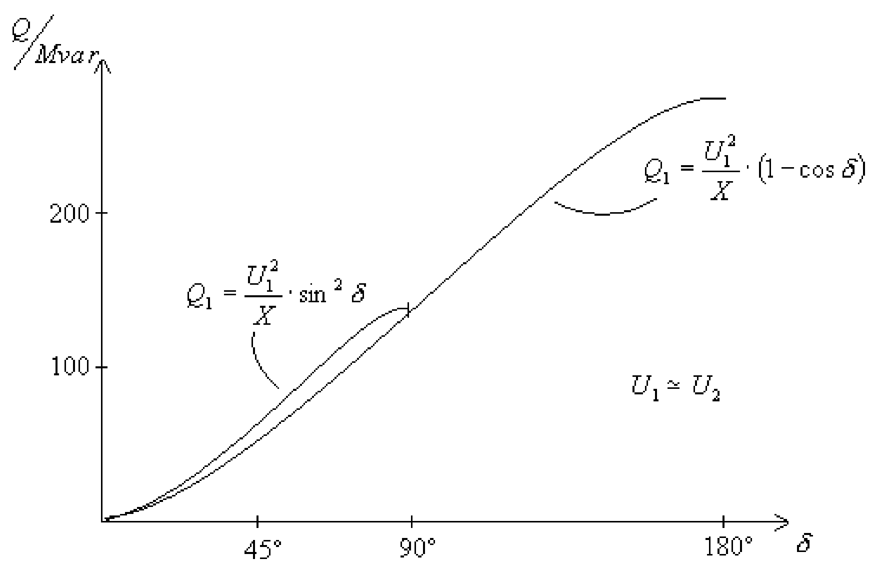
# Question 2

$$P_1 = P_2 = \frac{U_1^2}{2X}\cdot\sin 2\delta \qquad Q_1 = \frac{U_1^2}{X}\cdot\sin^2\delta \qquad Q_2 = 0 \iff P_{max} = \frac{U_1^2}{2X}$$

$$P = \frac{U_1 U_2}{X}\cdot\sin\delta = \frac{U_1^2}{X}\cdot\sin\delta \qquad Q_2 \neq 0\,,\ \underbrace{U_2 = U_1}_{ASSUMPTION} \iff P_{max} = \frac{U_1^2}{X}$$

$$\Rightarrow \underline{\underline{P_{max}\ 50\%\ smaller}}$$



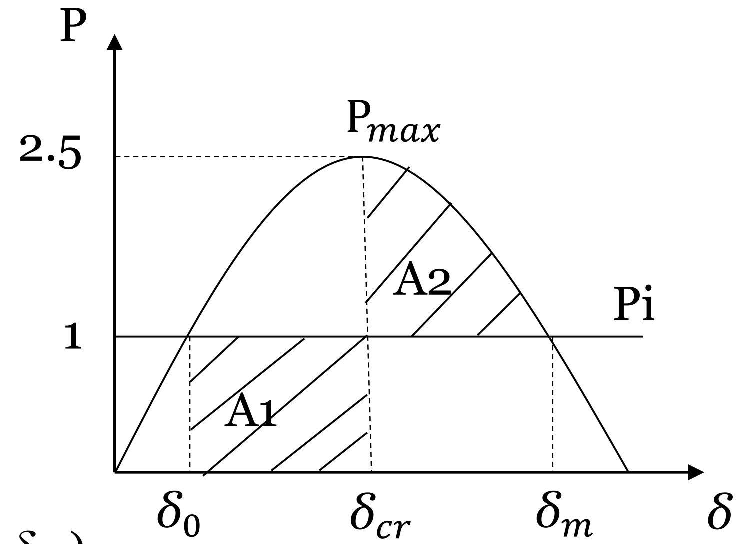



## Question 3: (for help, see Power System Analysis by Grainger, ch. 16 or other book)

- A generator having $H$ = 6.0 MJ/MVA is delivering power of 1.0 per unit to an infinite bus through a purely reactive network when the occurrence of a fault reduces the generator output to zero. The maximum power that could be delivered is 2.5 per unit. When the fault is cleared, the original network conditions again exist. Determine the critical angle and critical clearing time.

*Hint: For clearing time, $H = Wk/P$ & $Wk = \frac{1}{2}J\omega^2$ if using equation from lecture slides*

# Question 3: critical clearing angle




$$P_i = P_{\max} \sin\delta = 2.5 \sin\delta = 1.0$$

$$\Rightarrow \delta_0 = 23.58^o = 0.4115\text{rad}$$

$$\delta_m = \pi - \delta_0 = 154.42^o$$

A1 = A2, Stability Criterion

$$A_1 = P_i \times (\delta_{cr} - \delta_0) = A_2 = \int_{\delta_{cr}}^{\delta_m} P_{\max} \sin\delta d\delta - P_i \times (\delta_m - \delta_{cr})$$

$$\Leftrightarrow 1 \times (\delta_{cr} - \delta_0) = \int_{\delta_{cr}}^{\delta_m} P_{\max} \sin\delta d\delta - 1 \times (\delta_m - \delta_{cr})$$

$$\Leftrightarrow -\delta_0 = -P_{\max} \cos\delta_m + P_{\max} \cos\delta_{cr} - \delta_m$$

$$\Leftrightarrow -\delta_0 = -P_{\max} \cos\delta_m + P_{\max} \cos\delta_{cr} - (\pi - \delta_0)$$

$$\Leftrightarrow \pi - 2\delta_0 + P_{\max} \cos\delta_m = P_{\max} \cos\delta_{cr}$$

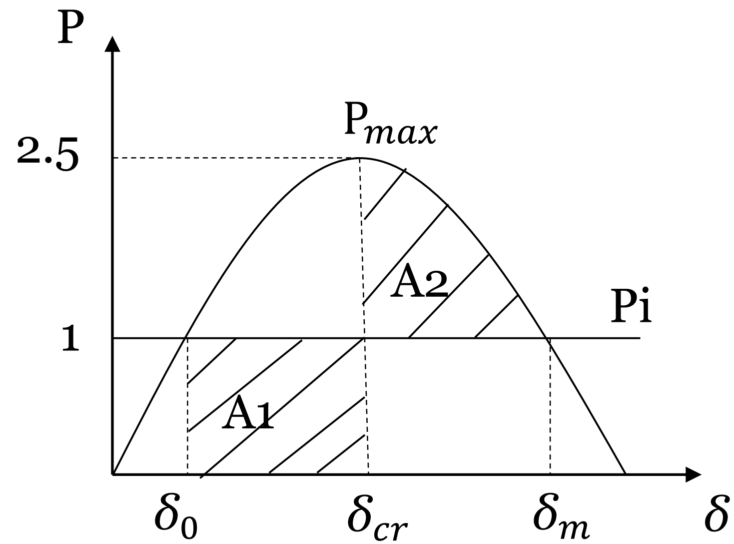
$$\cos\delta_{cr} = \frac{P_{\max} \cos\delta_m + \pi - 2\delta_0}{P_{\max}}$$

$$\delta_{cr} = \arccos\left(\frac{2.5\cos 154.42^\circ + \pi - 2 \times 0.4115}{2.5}\right) = \arccos(0.00274) = 89.84^\circ = 1.56\text{rad}$$

# Question 3: critical clearing time

$\delta_0 = 23.58^o = 0.4115\text{rad}$

$\delta_{cr} = 89.84° = 1.56\text{rad}$




$$t = \sqrt{2\frac{\mathrm{J}\omega}{\mathrm{P_m}}(\delta' - \delta_0')}$$

From lecture slides

$$H = \frac{W_k}{P_m} = \frac{\frac{1}{2}J\omega^2}{P_m}$$

Multiply by 4 and
Divide by ω

$$\frac{4H}{\omega} = 2\frac{\mathrm{J}\omega}{\mathrm{P_m}}$$

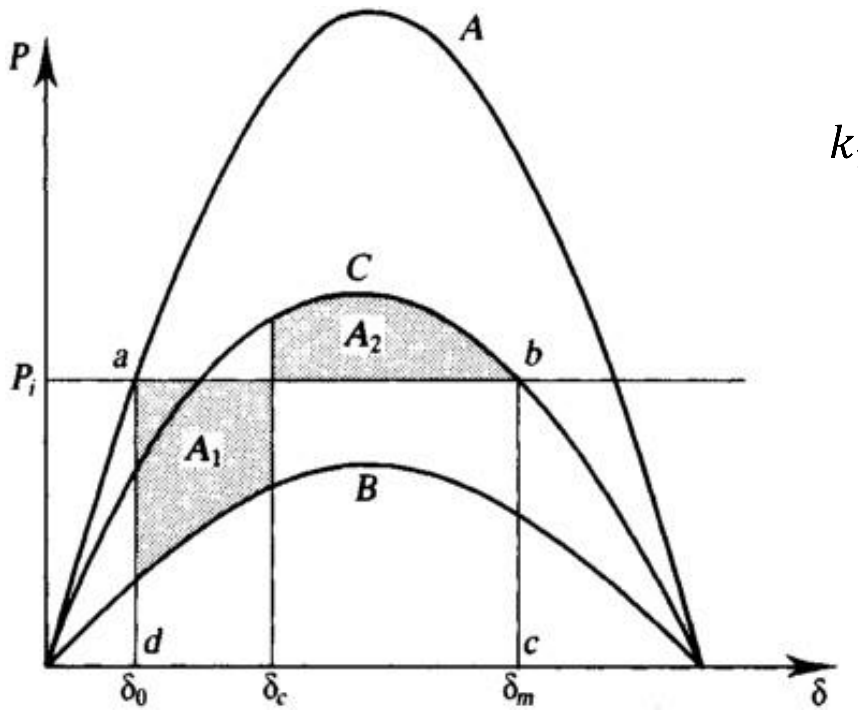
$$\boxed{t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_0)}{\omega_s}} = \sqrt{\frac{4\times 6(1.56 - 0.4115)}{2\times\pi\times 50Hz\times 1}} = \underline{\underline{0.296s}}}$$

# Question 4

- A 60-Hz generator is supplying 60% of $P_{max}$ to an infinite bus through a reactive network. A fault occurs which increases the reactance of the network between the generator internal voltage and the infinite bus by 400%. When the fault is cleared, the maximum power that can be delivered is 80% of the original maximum value. Determine the critical clearing angle for the condition described.

# Question 4: (for help, see Power System Analysis by Grainger, ch. 16 or other book)

Power-angle curve $A$ before a fault, $B$ during the fault, and $C$ after the fault such that $A = P_{max} \sin \delta$, $B = k_1 A$, and $C = k_2 A$, with $k_1 < k_2$. For stability, we must have area $A_1$ = area $A_2$.



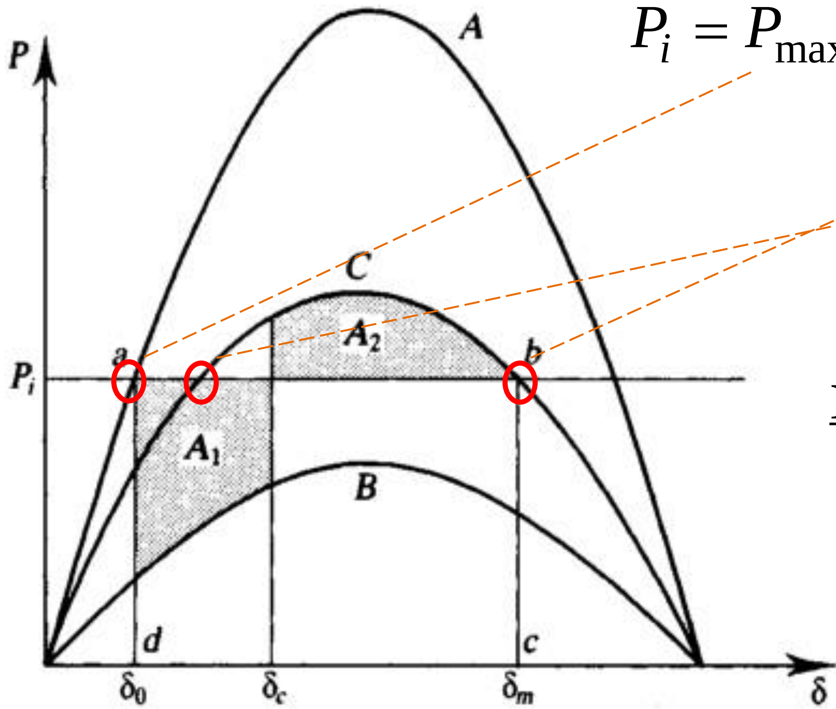

Note: picture illustrative - not from the problem

$$k_1 = \frac{X_{before\ fault}}{X_{during\ fault}} = \frac{X_{before\ fault}}{X_{before\ fault}(1.0 + 4.0)}$$

$$k_2 = \frac{X_{before\ fault}}{X_{after\ fault}}$$

$$\boxed{P_1 = P_2 = \frac{U_1 U_2}{X} \cdot \sin \delta}$$

# Question 4: Determine the critical clearing angle




$$P_i = P_{\max} \sin \delta_0 = k_2 P_{\max} \sin \delta_m = k_2 P_{\max} \sin(\pi - \delta_m)$$

$$\boxed{\Rightarrow \sin \delta_0 = k_2 \sin(\pi - \delta_m)}$$

<u>In our case:</u>

Before the fault:

$$P_i = P_{\max} \sin \delta_0 = 1.667 \sin \delta_0 = 1.0$$

$$\Rightarrow \quad \delta_0 = 36.86^o = 0.643\text{rad}$$

During the fault:

$$X'_f = 5X_f \Rightarrow P'_{\max} = \frac{1.667}{5} = 0.333$$

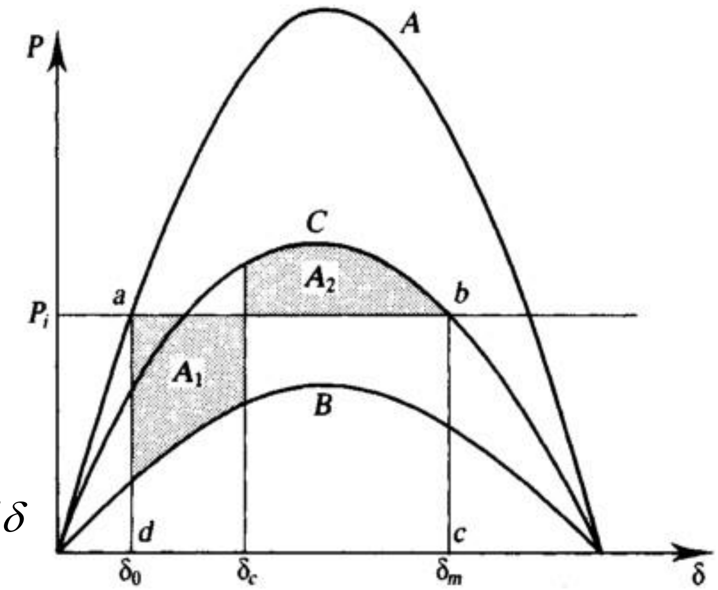
$$k_1 P_{\max} \sin \delta = 0.333 \sin \delta, \qquad k_1 = 0.2$$

After the fault:

$$P''_{\max} = 0.8 P_{\max}$$

$$k_2 P_{\max} \sin \delta = 1.33 \sin \delta, \qquad k_2 = 0.8$$

# Question 4: Determine the critical clearing angle




$A = P_{max} \sin\delta$, $B = k_1 A$, and $C = k_2 A$.
For stability, we must have area $A_1$ = area $A_2$.

$$A_1 = P_i \times (\delta_{cr} - \delta_0) - \int_{\delta_0}^{\delta_{cr}} B(\delta)d\delta = P_{max}\sin\delta_0 \times (\delta_{cr} - \delta_0) - \int_{\delta_0}^{\delta_{cr}} k_1 P_{max} \sin\delta d\delta$$

$$= P_{max}\left(\sin\delta_0 \times (\delta_{cr} - \delta_0) + k_1 \Big|_{\delta_0}^{\delta_{cr}} \cos\delta\right) = \underline{P_{max}(\sin\delta_0 \times (\delta_{cr} - \delta_0) + k_1 \cos\delta_{cr} - k_1 \cos\delta_0)}$$

$$A_2 = \int_{\delta_{cr}}^{\delta_m} C(\delta)d\delta - P_i \times (\delta_m - \delta_{cr}) = \int_{\delta_{cr}}^{\delta_m} k_2 P_{max} \sin\delta d\delta - P_{max}\sin\delta_0 \times (\delta_m - \delta_{cr})$$

$$= P_{max}\left(-k_2 \Big|_{\delta_{cr}}^{\delta_m} \cos\delta - \sin\delta_0 \times (\delta_m - \delta_{cr})\right) = \underline{P_{max}(k_2\cos\delta_{cr} - k_2\cos\delta_m - \sin\delta_0 \times (\delta_m - \delta_{cr}))}$$

$$A_1 = A_2$$

$$\Rightarrow \cos\delta_{cr}(k_2 - k_1) = \sin\delta_0(\delta_m - \delta_0) - k_1\cos\delta_0 + k_2\cos\delta_m$$

$$\Leftrightarrow \boxed{\cos\delta_{cr} = \frac{1}{(k_2 - k_1)}(\sin\delta_0(\delta_m - \delta_0) - k_1\cos\delta_0 + k_2\cos\delta_m)}$$

# Question 4: Determine the critical clearing angle

$$\boxed{\cos\delta_{cr} = \frac{1}{(k_2 - k_1)}\left(\sin\delta_0\left(\delta_m - \delta_0\right) - k_1\cos\delta_0 + k_2\cos\delta_m\right)}$$

$$\boxed{\sin\delta_0 = k_2\sin\left(\pi - \delta_m\right)}$$

Using the previously derived equations:

$$\sin\delta_0 = k_2\sin(\pi - \delta_m), \delta_m = \pi - \arcsin(\frac{P_i}{k_2 P_{\max}})$$

$$\delta_m = 180^o - \arcsin\left(\frac{1.0}{1.33}\right) = 131.2^o = 2.290\text{rad}$$

$$\cos\delta_{cr} = \frac{1}{(k_2 - k_1)}\left(\sin\delta_0\left(\delta_m - \delta_0\right) - k_1\cos\delta_0 + k_2\cos\delta_m\right)$$

$$= \left(\frac{1.0}{0.8 - 0.2}\right)\left(\sin 36.84^\circ \times \left(2.290 - 0.643\right) - 0.2\cos 36.86^o + 0.8\cos 131.2^o\right)$$

$$= 0.500$$

$$\boxed{\Rightarrow \delta_{cr} = \arccos(0.5) = \underline{\underline{60^o}}}$$