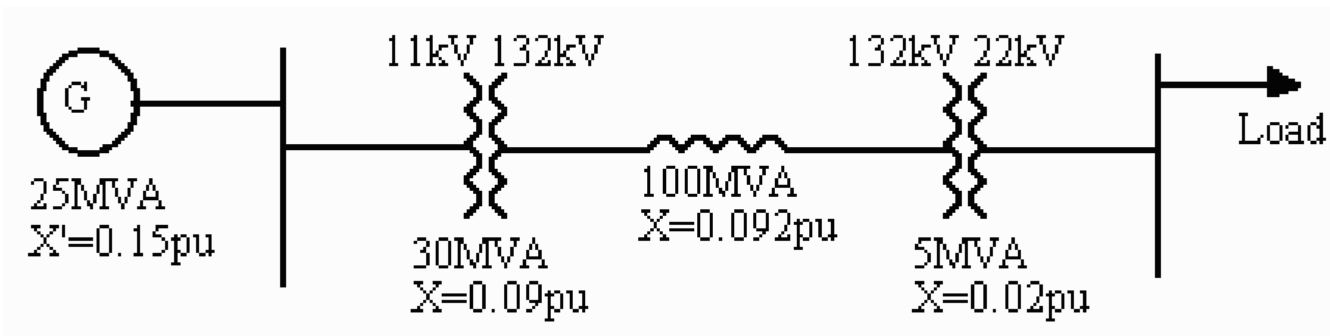
Aalto-yliopisto
Teknillinen korkeakoulu

Exercise 5

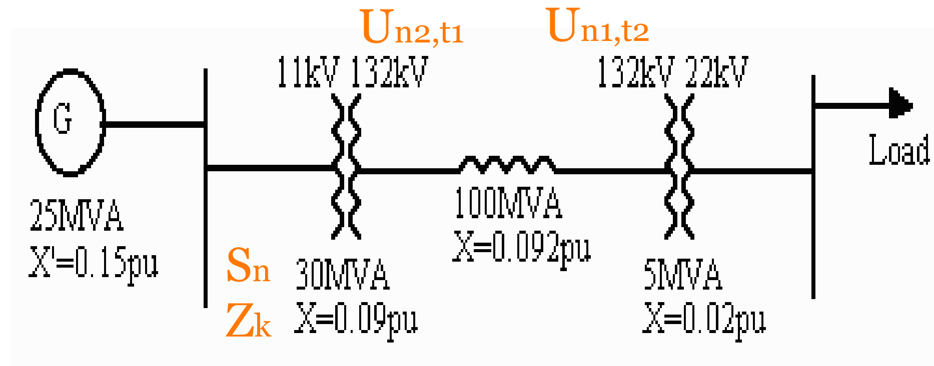
Power systems

Question 1

- A symmetrical three-phase short circuit occurs on the 22kV busbars of the circuit shown as a one-line diagram in the figure below. Calculate the fault current and the fault apparent power.



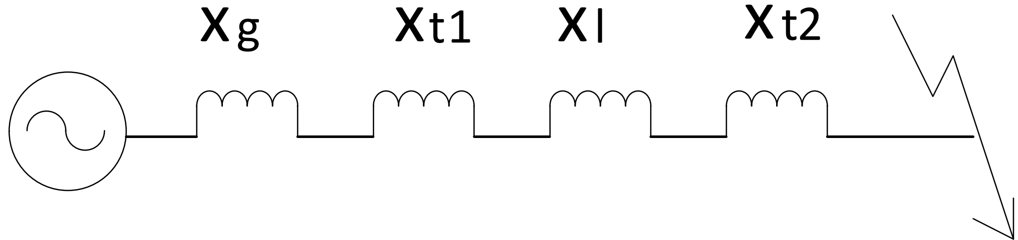
Question 1



1. Selecting base power $S_b = 100$ MVA
2. Drawing the equivalent circuit with reactances
3. Calculating Per Unit (p.u.) values for every component
4. Calculating the S_{sc} and I_{sc}

$$S_b = 100 \text{ MVA}$$

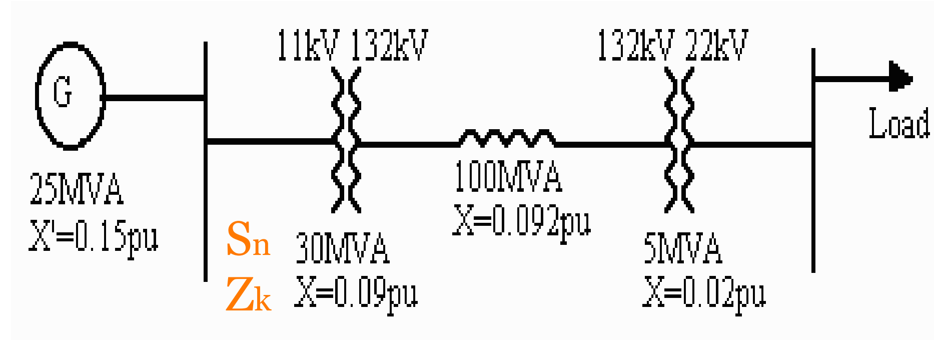
Equivalent circuit:



$$x_{p.u.} = \frac{X}{Z_b} = \frac{Z_k \frac{U_n^2}{S_n}}{\frac{U_b^2}{S_b}} = Z_k \frac{S_b}{S_N} \frac{U_n^2}{U_b^2} \xrightarrow{\text{if selected } U_b = U_n} x_{p.u.} = Z_k \frac{S_b}{S_n}$$

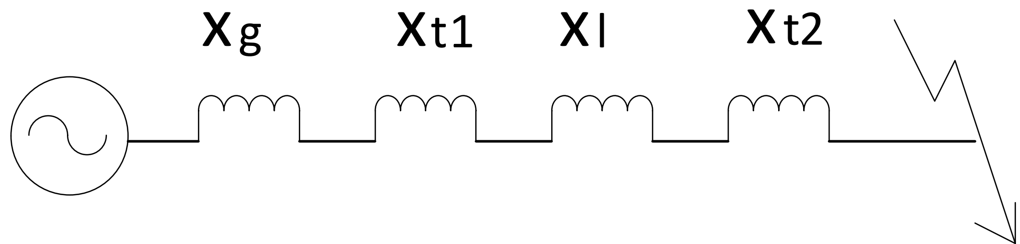
Note: this simplifies calculation if $U_{n2,t1} = U_{n1,t2}$

Question 1

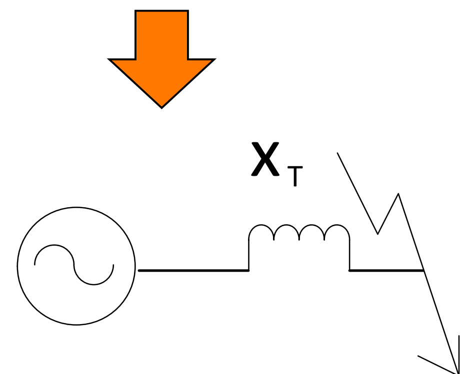


1. Selecting base power $S_b = 100$ MVA
2. Drawing the equivalent circuit with reactances
3. Calculating Per Unit (p.u.) values for every component
4. Calculating the S_{sc} and I_{sc}

$$x_{p.u.} = Z_k \frac{S_b}{S_n}$$

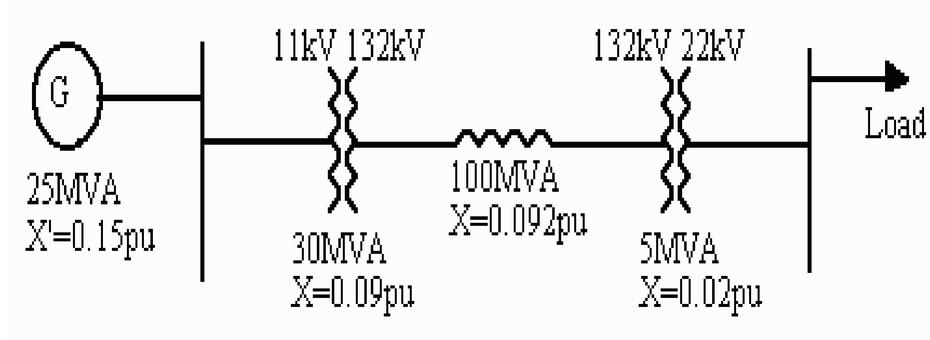


Name	Calculation	Reactance
Generator (x_g)	$j0.15 \times 100/25$	0.6 p.u.
Transformer 1 (11/132) (x_{t1})	$j0.09 \times 100/30$	0.3 p.u.
Transformer 2 (132/22) (x_{t2})	$j0.02 \times 100/5$	0.4 p.u.
Line (x_l)	$j0.092 \times 100/100$	0.092 p.u



$$x_T = x_g + x_{t1} + x_{t2} + x_l = 1.392$$

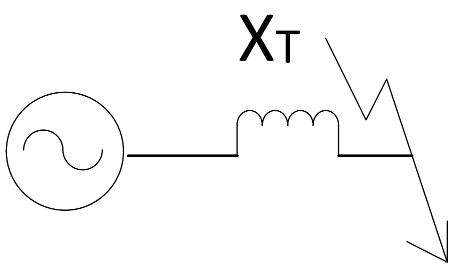
Question 1



1. Selecting base power $S_b = 100$ MVA
2. Drawing the equivalent circuit with reactance
3. Calculating Per Unit (p.u.) values for every component
4. Calculating the S_{sc} and I_{sc}

$$S_b = 100 \text{ MVA}$$

Equivalent circuit:



$$x_{T,p.u} = z_{sc,p.u.} = \frac{Z_{sc}}{Z_b} = \frac{\frac{U_{sc}^2}{S_{sc}}}{\frac{U_b^2}{S_b}}$$

we calculated $x_{T,p.u}$ presuming that $U_b = U_n = U_{sc}$ $\rightarrow x_{T,p.u} = \frac{S_b}{S_{sc}}$

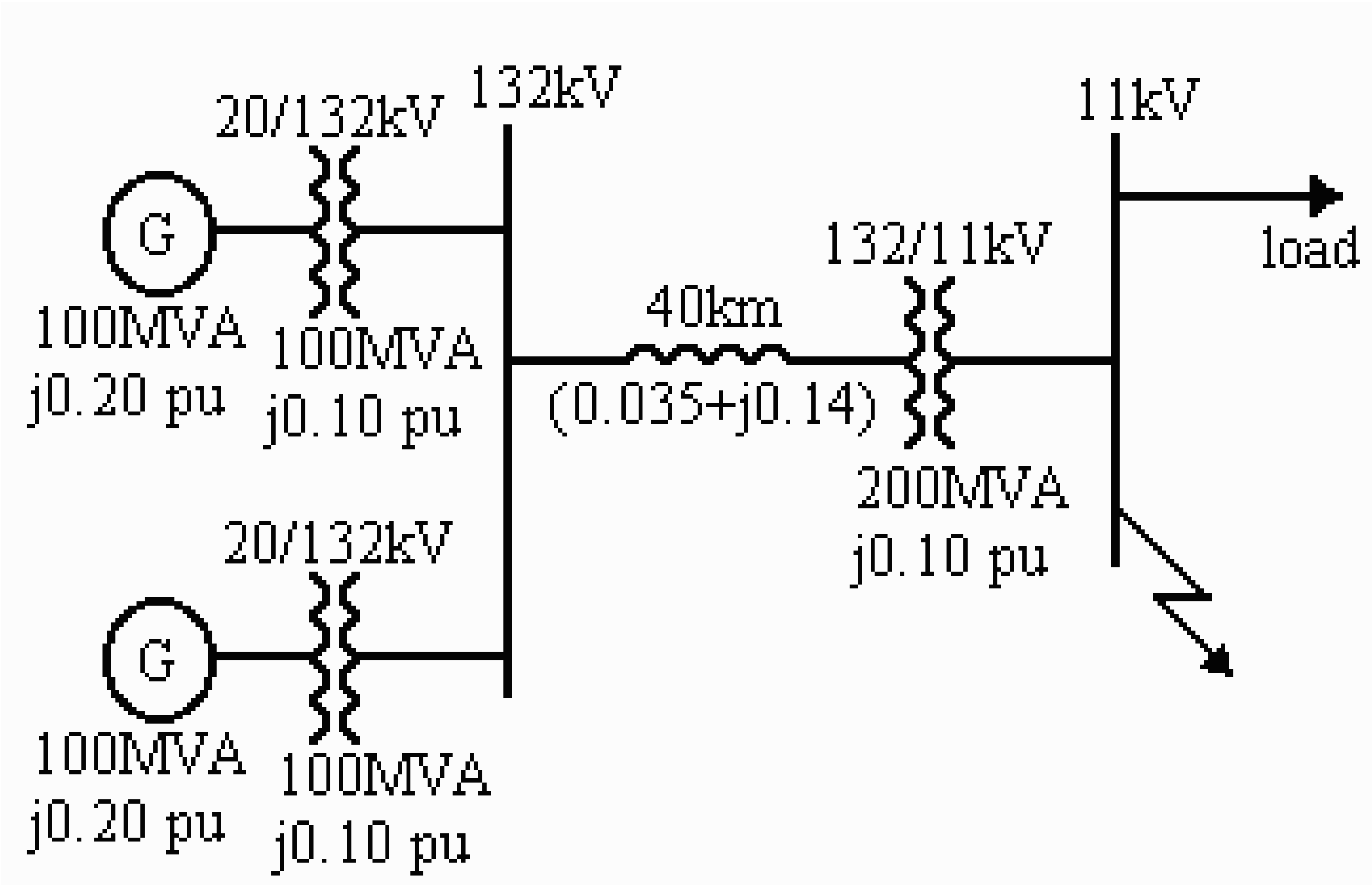
$$\rightarrow S_{sc} = \frac{S_b}{x_T} = \frac{100}{1.392} = 71.8 \text{ MVA}$$

$$I_{sc} = \frac{I_b}{x_T} = \frac{S_b}{\sqrt{3} \times U_b \times x_T} = \frac{100}{\sqrt{3} \times 22 \times 1.392} = 1.885 \text{ kA}$$

Question 2

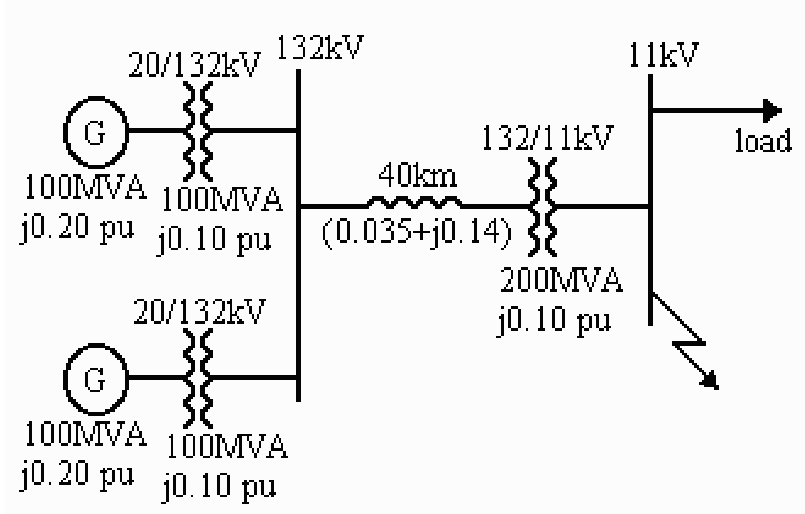
- Two 100-MVA, 20-kV turbo generators (each of transient reactance 0.2 pu) are connected, each through its own 100-MVA, 0.1 pu reactance transformer, to a common 132-kV busbar. From this busbar, a 132-kV feeder, 40 km in length, supplies an 11-kV load through a 132/11-kV transformer of 200 MVA rating and reactance 0.1 pu. If a balanced three-phase short circuit occurs on the low voltage terminals of the load transformer, determine, using a 100-MVA base, the fault current in the feeder and the rating of a suitable circuit breaker at the load end of the feeder. The feeder impedance per phase is $(0.035+j0.14)\Omega$/km.

Question 2



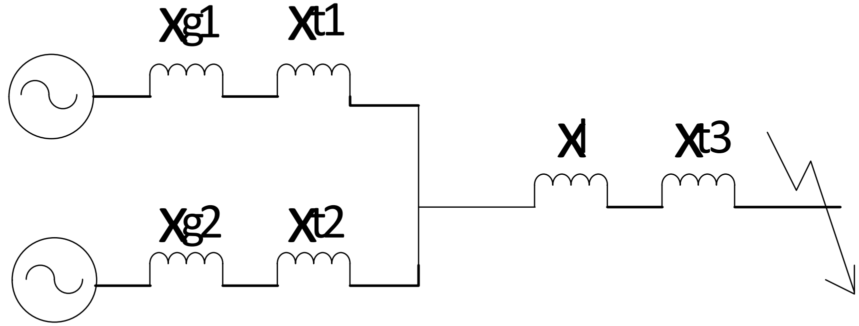
20/132kV
132kV
11kV
G
load
100MVA
j0.20 pu
100MVA
j0.10 pu
132/11kV
40km
(0.035+j0.14)
200MVA
j0.10 pu
20/132kV
G
100MVA
j0.20 pu
100MVA
j0.10 pu

Question 2



1. Selecting base power $S_b = 100$ MVA
2. Drawing the equivalent circuit with reactance
3. Calculating Per Unit (p.u.) values for every component
4. Simplifying the circuit
5. Calculating the I_{sc} and S_{sc}

Equivalent circuit:



Calculating reactance of the line (feeder): $x_l = \frac{Z_l}{Z_b}$

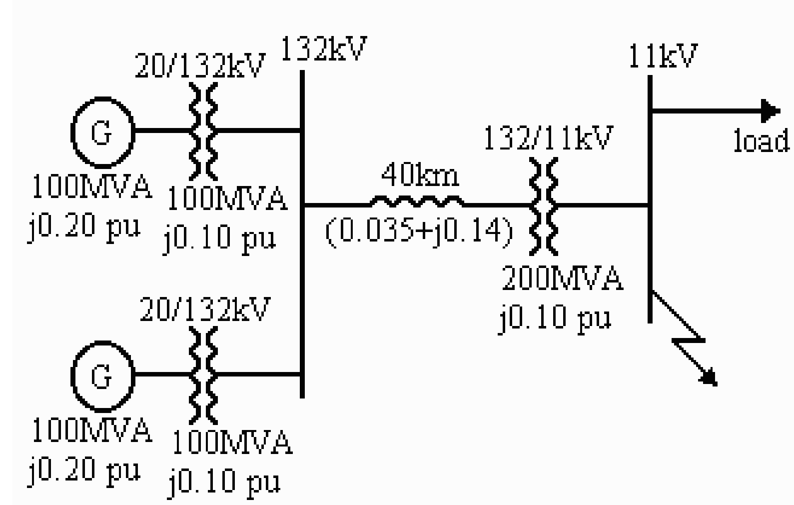
Line impedance $Z_l = 40 * (0.035 + j0.14) = 1.4 + j5.6 = 5.77 < 76° \ \Omega$

Base impedance $Z_b = \frac{U_b^2}{S_b} = \frac{132^2}{100} = 174.24\Omega$

Per Unit value $x_l = \frac{Z_l}{Z_b} = \frac{5.77<76°}{174.24} = 0.033 < 76°\ p.u.$

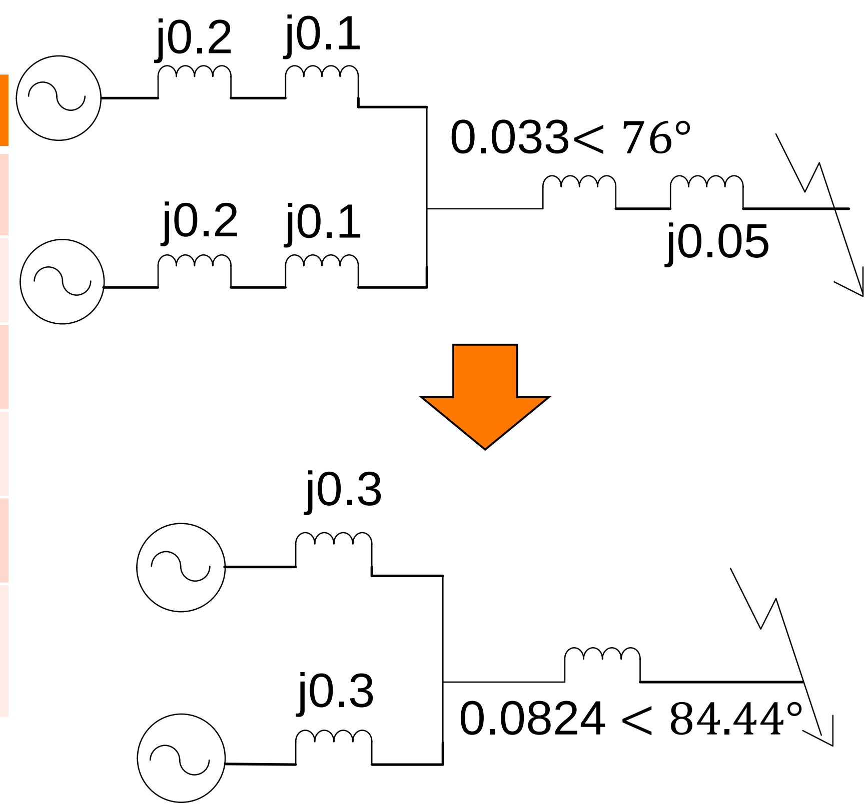
Question 2

3. Calculating Per Unit (p.u.) values for every component
4. Simplifying the circuit
5. Calculating the I_{sc} and S_{sc}

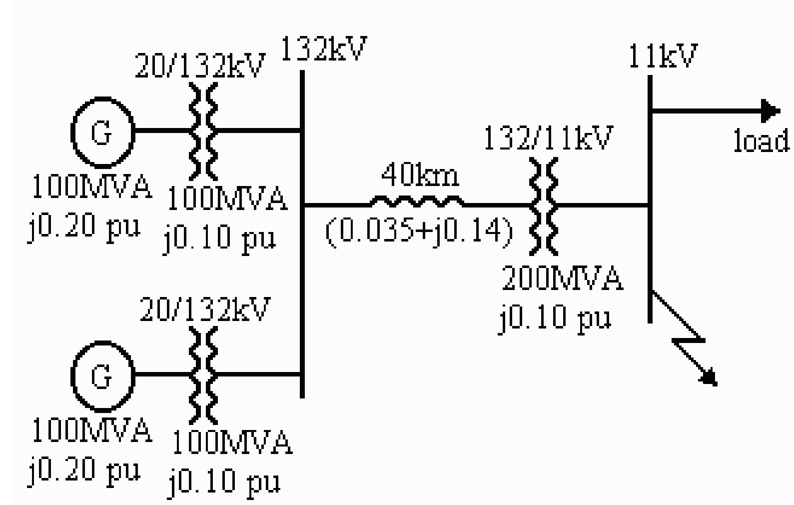


$$x_{p.u.} = Z_k \frac{S_b}{S_n}$$

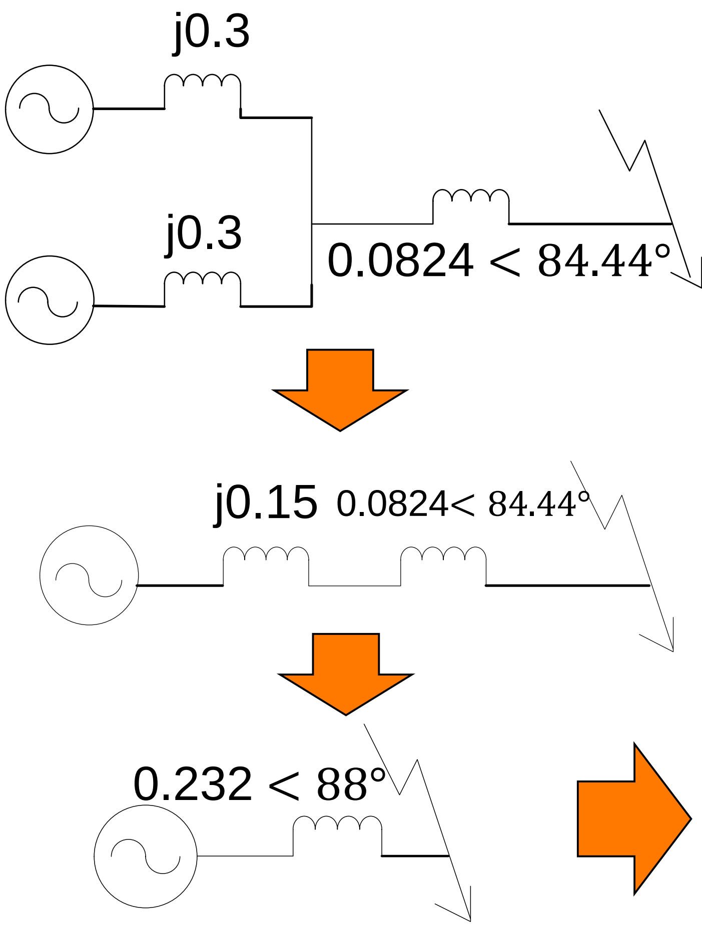
Name	Calculation	Impedance
Generator 1(x_{g1})	$j0.20 \times 100/100$	j0.2 p.u.
Generator2(x_{g2})	$j0.20 \times 100/100$	j0.2 p.u.
Transformer1(x_{t1})	$j0.10 \times 100/100$	j0.1 p.u.
Transformer2(x_{t2})	$j0.10 \times 100/100$	j0.1 p.u.
Transformer3(x_{t3})	$j0.10 \times 100/200$	j0.05 p.u.
Line(feeder)(x_l)	Z_l/Z_b	0.033< 76° p.u.



Question 2



4. Simplifying the circuit
5. Calculating the I_{sc} and S_{sc}



Parallel: $x_{eq} = \frac{j0.3 \times j0.3}{j0.3 + j0.3} = j0.15 p.u.$

$$\bar{z}_T = 0.232 < 88° p.u. \rightarrow z_T = 0.232 p.u.$$

$$I_{sc} = \frac{I_b}{z_T} = \frac{S_b}{\sqrt{3} \times U_b \times z_T} = \frac{100}{\sqrt{3} \times 11 \times 0.232} = 22.62 \text{ kA}$$

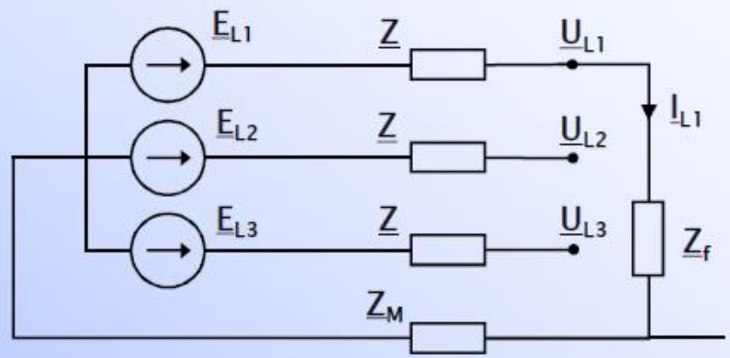
$$S_{sc} = \frac{S_b}{z_T} = \frac{100}{0.232} = 431 \text{ MVA}$$

Question 3

- A single line-to-earth fault occurs in a radial transmission system. The following sequences exist between the source of supply (an infinite busbar) of voltage 1 pu to the point of the fault: Z_1 = (0.3+j0.6)pu, Z_2 = (0.3+j0.55)pu, Z_0 = (1+j0.78)pu. The fault path to earth has a resistance of 0.66 pu. Determine the fault current and the voltage at the point of the fault.

Some background first

One phase earth fault



Solution using symmetric components

During the earth fault: $\underline{U}_{L1} = \underline{Z}_f \underline{I}_{L1}$

$$\underline{I}_{L2} = 0$$
$$\underline{I}_{L3} = 0$$

It follows:

$$\underline{U}_0 + \underline{U}_1 + \underline{U}_2 = \underline{Z}_f \underline{I}_{L1} \quad *$$

$$\left.\begin{aligned} \underline{I}_0 + a^2 \underline{I}_1 + a\,\underline{I}_2 &= 0 \\ \underline{I}_0 + a\,\underline{I}_1 + a^2 \underline{I}_2 &= 0 \end{aligned}\right\} \quad \underline{I}_0 = \underline{I}_1 = \underline{I}_2$$

Inverse matrix also provides the same solution

$$\begin{vmatrix} I_0 \\ I_1 \\ I_2 \end{vmatrix} = \frac{1}{3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{vmatrix} \begin{vmatrix} I_R \\ I_S \\ I_T \end{vmatrix}$$

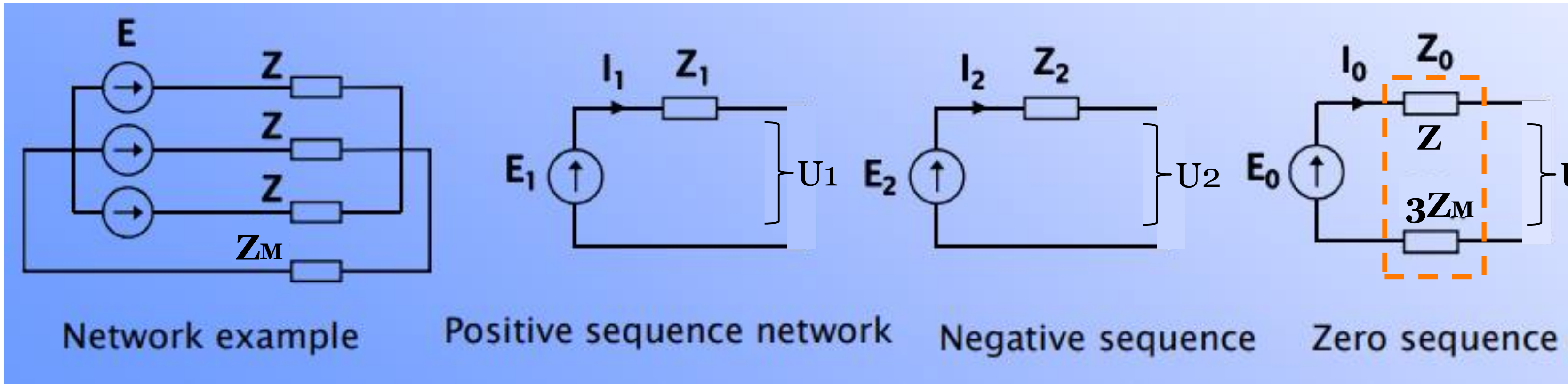
→ $I_0 = 1/3 \times (I_R + 0 + 0)$
→ $I_R = 3\, I_0\ (= I_{L1})$

$I_S = 0\ (= I_{L2})$

$I_T = 0\ (= I_{L3})$

Question 3

Generally:



Voltage source is symmetric:

$\Rightarrow \quad E_1 = E_R \;;\quad E_2 = 0 \;;\quad E_0 = 0$

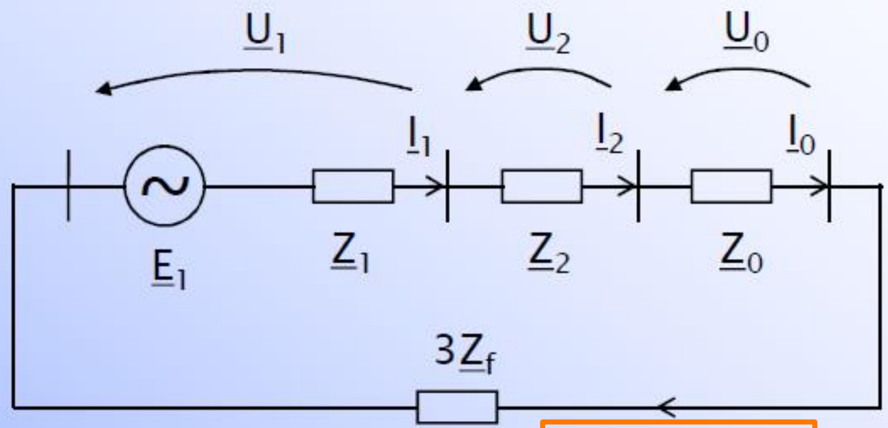
$$\Rightarrow \quad \underline{U}_0 = -\underline{Z}_0\,\underline{I}_0$$
$$\underline{U}_1 = \underline{E}_1 - \underline{Z}_1\,\underline{I}_1$$
$$\underline{U}_2 = -\underline{Z}_2\,\underline{I}_2$$

From previous slide:

$\underline{U}_0 + \underline{U}_1 + \underline{U}_2 = \underline{Z}_f\,\underline{I}_{L1} \quad *$ and $I_{L1} = 3\,I_0$

→ $* \quad -\underline{Z}_0\,\underline{I}_0 + \underline{E}_1 - \underline{Z}_1\,\underline{I}_1 - \underline{Z}_2\,\underline{I}_2 = 3\,\underline{Z}_f\,\underline{I}_0 \quad \rightarrow$

One phase earth fault



$\underline{I}_0 = \underline{I}_1 = \underline{I}_2$

Component networks are in series connection in one-phase earth fault

Which gives for the zero sequence current:

$$\underline{I}_0 = \frac{\underline{E}_1}{\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2 + 3\,\underline{Z}_f} = \frac{\underline{E}_1}{3\,\underline{Z} + 3\,\underline{Z}_M + 3\,\underline{Z}_f}$$

The total fault current is three times the zero sequence current:

$$\underline{I}_f = 3\,\underline{I}_0 = \frac{3\,\underline{E}_1}{\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2 + 3\,\underline{Z}_f} = \frac{\underline{E}_1}{\underline{Z} + \underline{Z}_M + \underline{Z}_f} = I_{L1}$$

Question 3

So we can utilize the following equations:

$$\bar{I}_f = \bar{I}_1 = \frac{3U}{\bar{Z}_1 + \bar{Z}_2 + \bar{Z}_0 + 3\bar{Z}_f} = \frac{3\times 1}{(3.58 + j1.93)} = 0.649 - j.035 = 0.738\angle -28.3°$$

$$\Rightarrow \underline{\underline{I_f = 0.738\,pu}}$$

$$\bar{U}_f = \bar{I}_f \bar{Z}_f = (0.649 - j0.35)\times 0.66 = (0.43 - j0.23)pu = 0.487\angle -28.3°$$

$$\Rightarrow \underline{\underline{U_f = 0.487\,pu}}$$