Aalto-yliopisto
Teknillinen korkeakoulu

# Exercise 6

## Power systems

# Question 1

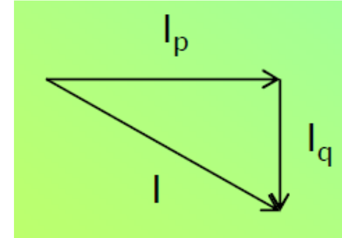
a) Derive the following equation
for the voltage drop over a transmission line

$$U_{loss} \approx I\,R \cdot \cos\varphi + I\,X \cdot \sin\varphi$$

b) Can this equation be applied in all situations?

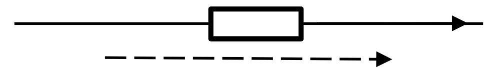
c) Draw the phasor diagrams related to the equation
in cases where the reactive power is inductive and capacitive.

# Question 1




Transmission line is mostly inductive reactance

$$\overline{Z} = R + jX \qquad \overline{I} = I_p - jI_q$$



$$\Delta \underline{U}_{phase} = \underline{U}_{loss}$$

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# Question 1

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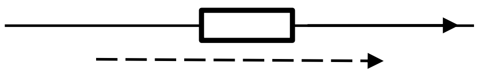
$\overline{Z} = R + jX \qquad \overline{I} = I_p - \underbrace{jI_q}_{inductive}$



$\Delta\underline{U}_{phase} = \underline{U}_{loss}$

$$\Delta\underline{U}_{phase} = \underline{U}_{loss} = \underline{Z}\,\underline{I} = (R + jX)\cdot(I_p - jI_q) \qquad ;\ I_p \text{ is active current, } I_q \text{ is reactive current}$$

$$\underline{U}_{loss} = RI_p - jRI_q + jIX_p - j^2XI_q = \underbrace{(RI_p + XI_q)}_{longitunal} + j\underbrace{(IX_p - RI_q)}_{transverse}$$

$$i^2 = i \times i = -1.$$

# Question 1

## a) Derive the following equation $\underline{\underline{U_{loss} \approx \; I\,R\cdot\cos\varphi + I\,X\cdot\sin\varphi}}$

The voltage drop is typically considered as the difference between the absolute values of the phase voltage ($U_{1,ph}$ – $U_{2,ph}$). A good **approximate** for this can be attained by projecting the voltage phasor with the larger magnitude onto the smaller voltage phasor.

→ We can approximate that the imaginary part of $\underline{U}_{loss}$ is zero (projection onto the smaller voltage phasor with angle of zero).

Also, knowing that

$$I_p = I\cdot\cos\varphi \;\;,\;\; I_q = I\cdot\sin\varphi$$

We get: $$\underline{U}_{loss} = \left(RI_p + XI_q\right) + \underbrace{j\left(IX_p - RI_q\right)}_{\approx 0}$$

□

$$\Rightarrow U_{loss} \approx IR\cdot\cos\varphi + IX\cdot\sin\varphi,$$

where $\varphi$ is the phase angle at the end of the line

# Question 1

## b) Can this equation be applied in all situations

The equation $\underline{\underline{U_{loss} \approx \quad I\,R\cdot\cos\varphi + I\,X\cdot\sin\varphi}}$

is <u>only approximate and can be applied in a case with inductive current</u>.

For capacity current, q-component is positive:

$$\bar{I} = I_p + jI_q$$

The equation in this case is

$$\underline{U}_{loss} = (R + jX)\cdot(I_p + jI_q) = RI_p + jRI_q + jIX_p + j^2XI_q = (RI_p - XI_q) + j(RI_q + IX_p)$$

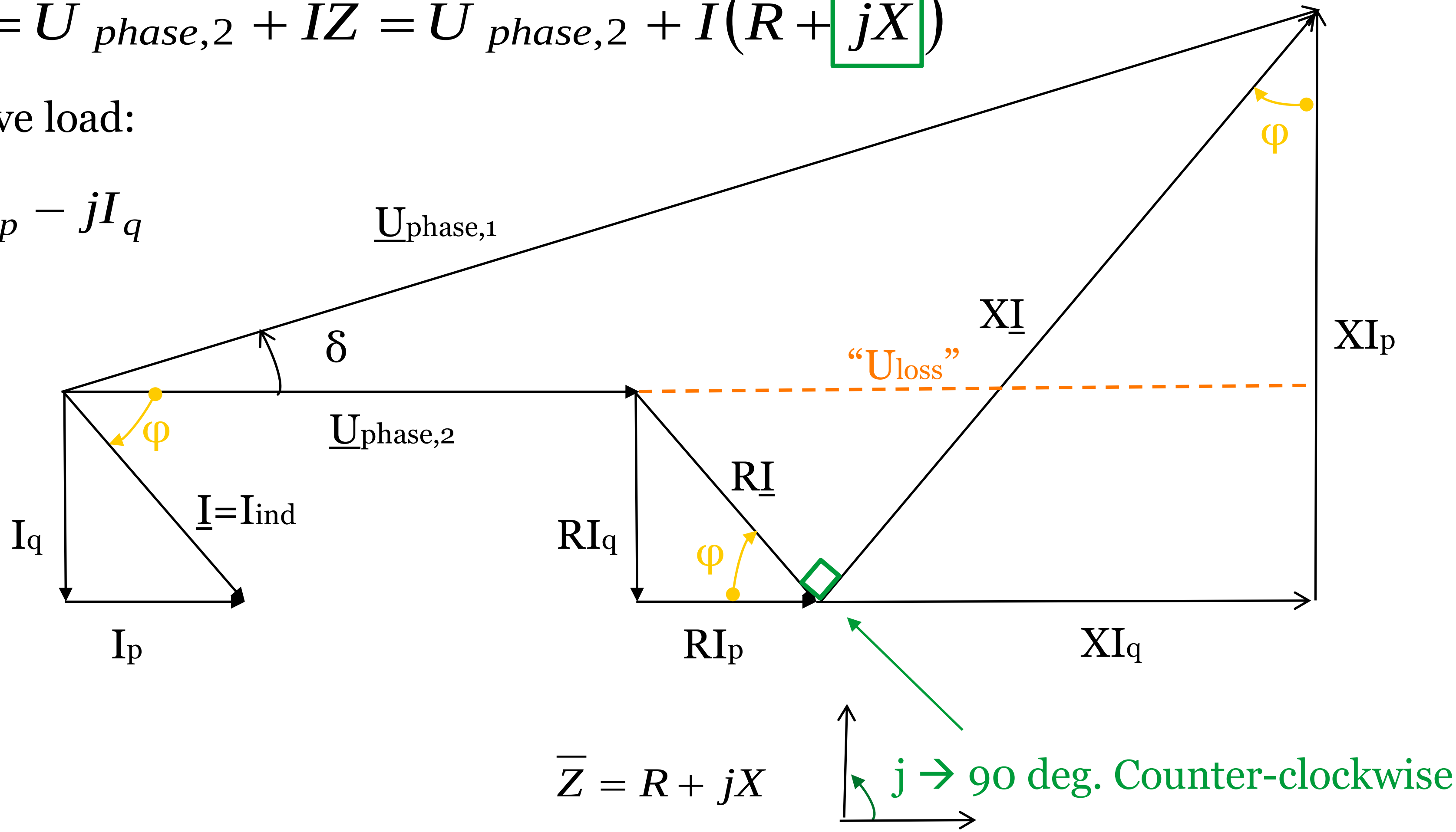
# Question 1

## c) Draw the phasor diagrams

$$\overline{U}_{phase,1} = \overline{U}_{phase,2} + \overline{I}\overline{Z} = \overline{U}_{phase,2} + \overline{I}(R + jX)$$

Inductive load:

$$\overline{I} = I_p - jI_q$$




$$\overline{Z} = R + jX$$

Note: the figure is out of perspective

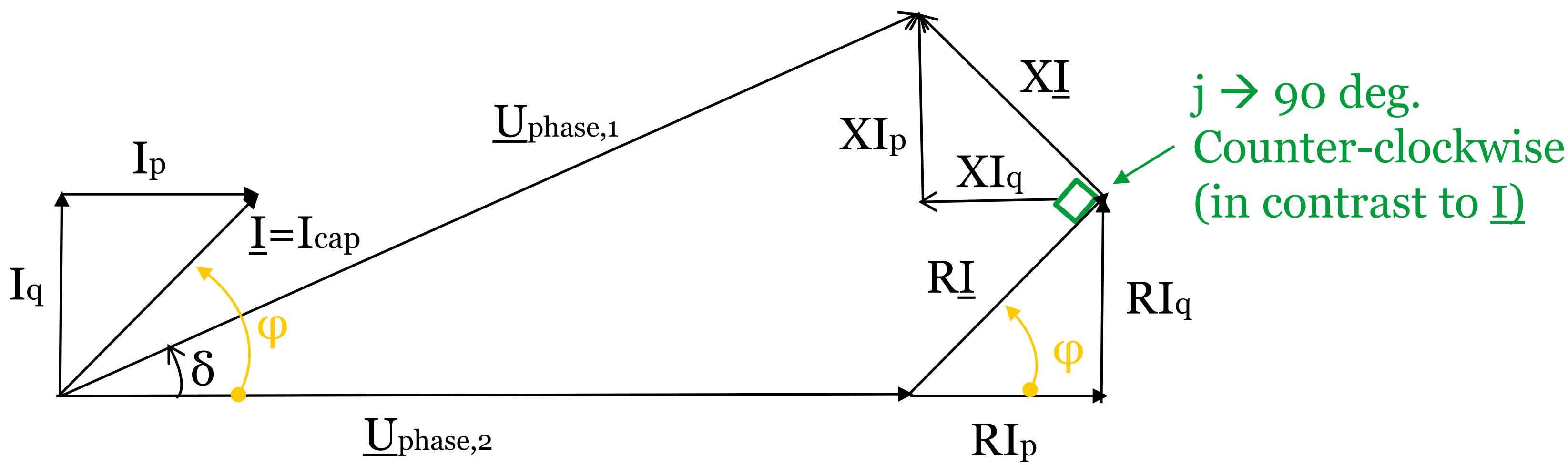
# Question 1

## c) Draw the phasor diagrams

$$\overline{U}_{phase,1} = \overline{U}_{phase,2} + \overline{IZ} = \overline{U}_{phase,2} + \overline{I}(R + jX)$$

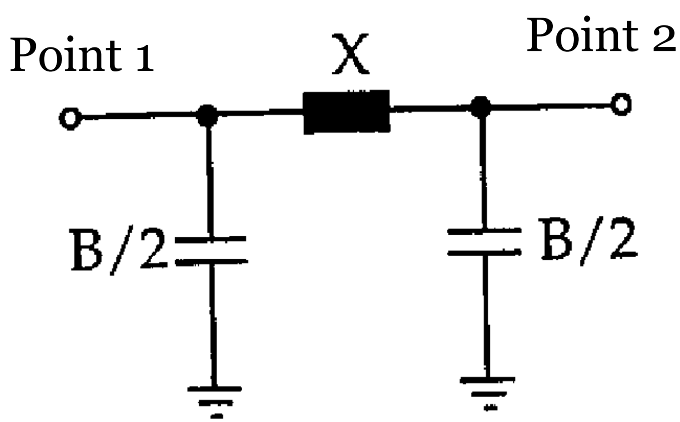
Capacitive load:

$$\overline{I} = I_p + jI_q$$




Note: the figure is out of perspective

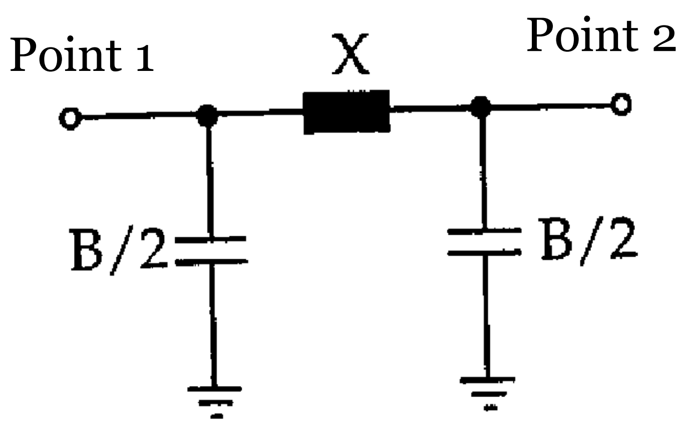
# Question 2




$X = 168\ \Omega$
$B = 0{,}87\ mS$

10 MW of active power is transferred using a three-phase line shown in the picture.
Line-to-line voltages in the both ends of the line are 110 kV.
**Calculate the power factor (cosφ) of the load.**

# Question 2




X = 168 Ω
B = 0,87 mS

$$P = \frac{U_1 U_2}{X} \sin \delta$$

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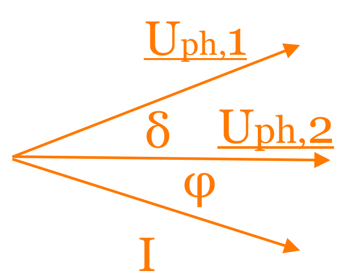
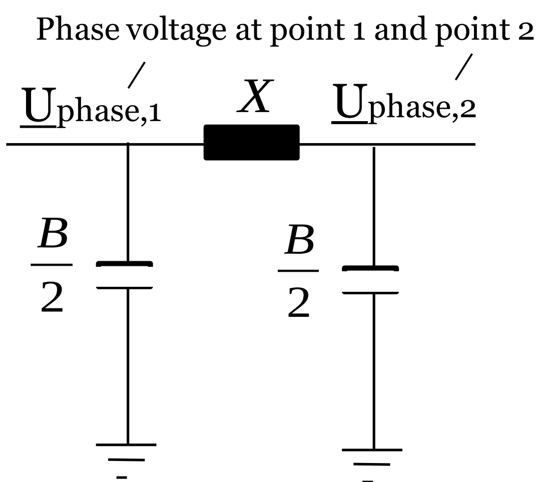
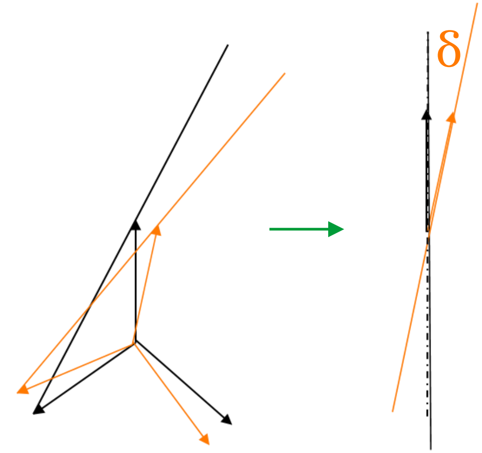



– In transmission system X >> R

# Question 2




$$U_{line} = 110\text{kV}, \text{ three phases}$$




Power angle equation for the three phase system

$$P = \frac{U_1 U_2}{X} \sin\delta$$

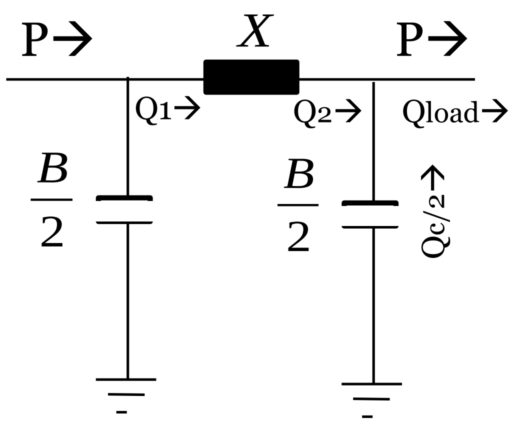
$$Note: P = 3 \times \frac{U_{phase,1} U_{phase,2}}{X} \sin\delta = 3 \times \frac{\frac{U_{line,1}}{\sqrt{3}} \cdot \frac{U_{line,2}}{\sqrt{3}}}{X} \sin\delta = \frac{U_{line,1} \cdot U_{line,2}}{X} \sin\delta$$

The voltage amplitudes are now kept constant with voltage control: $U_{phase,1} = U_{phase,2}$
The angle difference (power angle) between the voltages is:

$$\delta = \arcsin\left(\frac{PX}{U_1 U_2}\right) = \arcsin\left(\frac{10\text{ MW} \cdot 168\ \Omega}{110\text{ kV} \cdot 110\text{ kV}}\right) \approx 7.98°$$

# Question 2

## Calculate the power factor of the load.




To calculate the power factor of the load, we need to know $P_{load}$ and $Q_{load}$.
Now, the resistive losses are neglected:
$P_1=P_2=P_{load}=P$

In order to keep the voltage level constant, a certain amount of reactive power has to flow into the line from point 2 and the shunt capacitance

Reactive power angle equation for the three phase system (between voltage U1 and U2 with a reactance between) and reactive power produced by shunt capacitance (lecture 6):

$$Q_2 = \frac{U_1 U_2}{X}\cos\delta - \frac{U_2^2}{X}$$

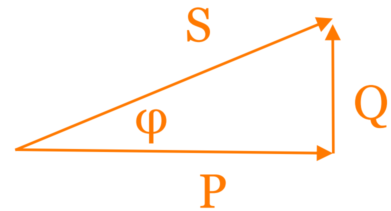
$$Q_c = YU^2 = BU^2$$

$$Q_{load} = Q_2 + \frac{Q_c}{2} = \frac{U_1 U_2}{X}\cos\delta - \frac{U_2^2}{X} + \frac{B}{2}U_2^2$$

$$\approx 4.57\text{Mvar}$$

# Question 2

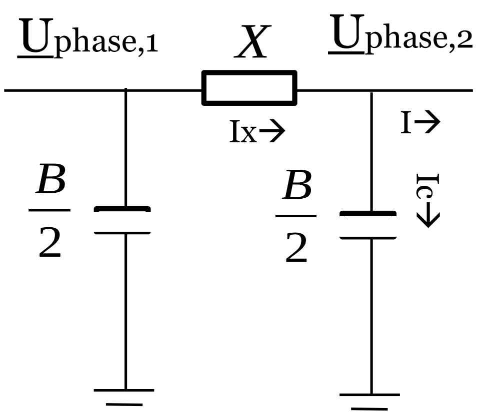
## Calculate the power factor of the load.




$$\cos\varphi = \frac{P_{load}}{S_{load}} = \frac{P}{\sqrt{P^2 + Q_{load}^2}} = \frac{10\text{MW}}{\sqrt{(10\text{MW})^2 + (4.57\text{MVAr})^2}}$$

$$\underline{\underline{\cos\varphi \approx 0.91_{\text{ind}}}}\text{, inductive since } \text{Q}_{\text{load}} > 0$$

Another way:




$$\underline{I} = \underbrace{\frac{U_1\angle\delta - U_2\angle 0^\circ}{\sqrt{3}\cdot jX}}_{\underline{I}x} - \underbrace{j\frac{B}{2}\cdot\frac{U_2}{\sqrt{3}}}_{\underline{I}c} = \frac{110\text{ kV}\angle 7.98^\circ - 110\text{ kV}}{\sqrt{3}\cdot j168\ \Omega} - j\frac{0.87\text{ mS}}{2}\cdot\frac{110\text{ kV}}{\sqrt{3}}$$

$$\underline{I} \approx 52.6\text{ A}\angle 4.0^\circ - 27.6\text{ A}\angle 90^\circ = 57.7\text{ A}\angle -24.5^\circ$$

$\rightarrow$ lagging 24.5 degrees to $\text{U}_2$

$\rightarrow \cos\varphi = \cos(24.5^\circ) \approx 0.91_{\text{ ind}}$

# Question 3

A turbine generator is delivering 20MW at 50Hz to a local load;
it is not connected to the grid.

The load suddenly drops to 15MW;
and the turbine governor starts to close the steam valve after a delay of 0.5s.

The stored energy in the rotating parts is 80MJ at 3000rev/min. What is the generated frequency at the end of the 0.5s delay?

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The kinetic energy in rotating masses :

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# Question 3

## What is the generated frequency at the end of the 0.5s delay?

Generated steady-state power = 20 MW.

Power usage drops to 15 MW.

Excess power of 20-15 = 5 MW accelerates the generator (frequency is increased)

Rotational energy is thus increased by:

$$\Delta Energy = P \times t = 5 \times 10^6 \mathrm{W} \times 0.5\mathrm{s} = 2.5\mathrm{MJ}$$

Rotational energy now:

$$E_2 = E_1 + \Delta E = 80 + 2.5 = 82.5\mathrm{MJ}$$

$$\frac{f_1^2}{f_2^2} = \frac{E_1}{E_2} \quad \Rightarrow \quad f_2 = \sqrt{\frac{f_1^2 \times E_2}{E_1}}$$

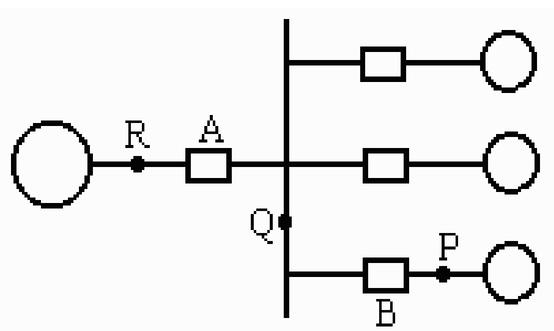
and

$$f_2 = \sqrt{\frac{50^2 \times 82.5}{80}} = \underline{\underline{50.775\mathrm{Hz}}}$$

# Question 4

A 625-kVA 2.4 kV generator with $X''_d = 0.20$ per unit is connected to a bus through a circuit breaker, as shown in the figure below. Connected through circuit breakers to the same bus are three synchronous motors rated 250 hp, 2.4 kV, 1.0 power factor, 90% efficiency, with $X''_d = 0.20$ per unit. The motors are operating at full load, unity power factor, and rated voltage, with the load equally divided among the machines.

For interrupting current, presume that transient reactance for the synchronous motors is 1.5 times the subtransient reactance. For the generator, apply the subtransient reactance.



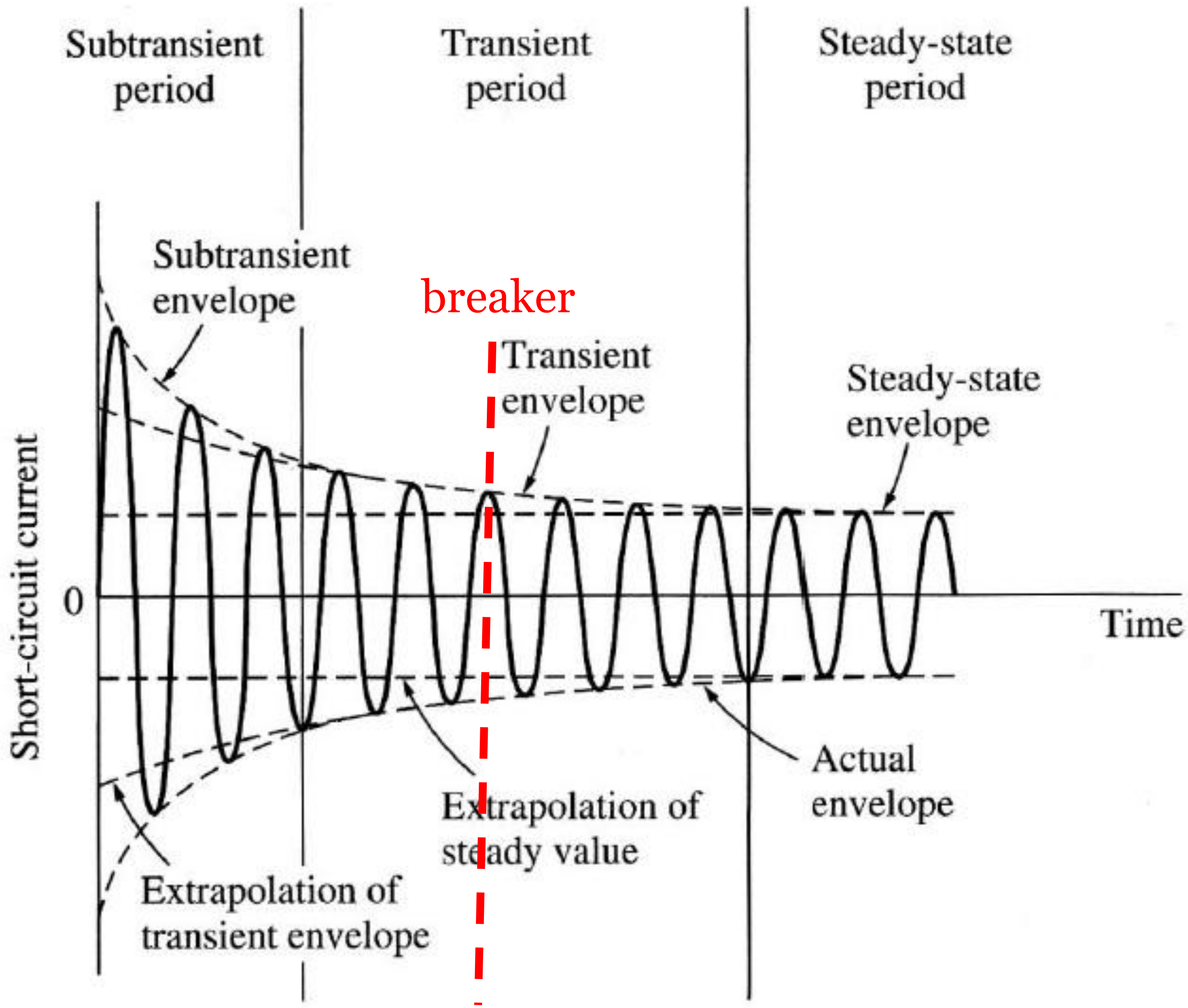

**(a)** Draw the impedance diagram with the impedances marked in per unit on a base of 625 kVA, 2.4 kV.

**(b)** Find the symmetrical short-circuit current in amperes, which must be interrupted by breakers A and B for a three-phase fault at point Q. Simplify the calculations by neglecting the pre-fault current.

**(c)** Repeat part (b) for a three-phase fault at point P.

# Question 4
# General thoughts

Circuit breakers typically become active around 100ms after the fault. This equals around 5 cycles (50Hz → 20ms per cycle). In this case, we are in a transient state rather than in the substransient state.

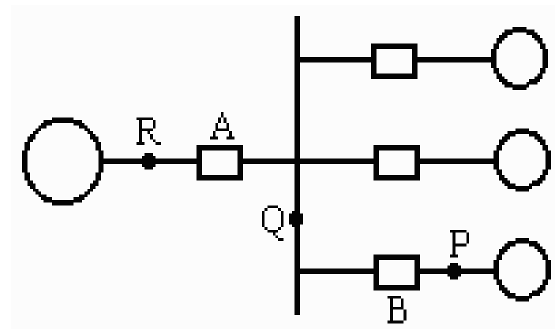



http://www.egr.unlv.edu/~eebag/Fault%20Analysis.pdf

# Question 4

(a) Draw the impedance diagram marked in per unit on a base of 625kVA, 2.4kV.

Base of 625kVA.
Motors: Output 250hp, 2.4kV, 1.0 power factor, 90% efficiency.



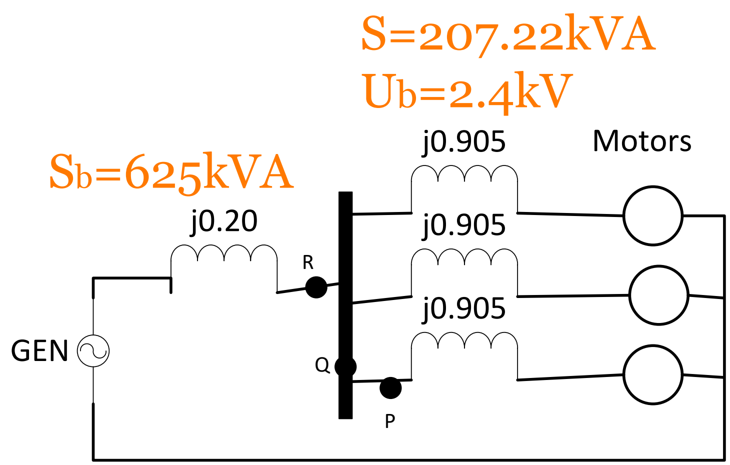

Motor input:

$$S = \frac{P}{\cos(\varphi)} = \frac{\frac{250\text{hp}}{0.9} \times 746 \frac{\text{W}}{\text{hp}}}{1} = 207.22\text{kVA}.$$

Motor subtransient reactance:

$$x''_m = X''_{d,m} \frac{S_b}{S} = j0.2 \times \frac{625\text{kVA}}{207.22\text{kVA}} = j0.603\,pu$$

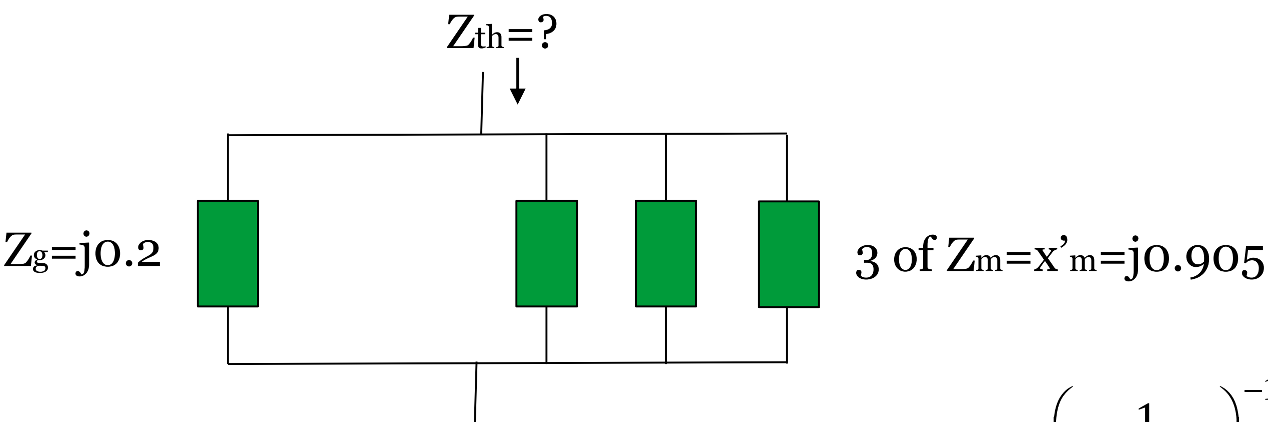
Motor reactance in transient state: $x'_m = 1.5 \times x''_m = 1.5 \times j0.603\,pu = j0.905\,pu$

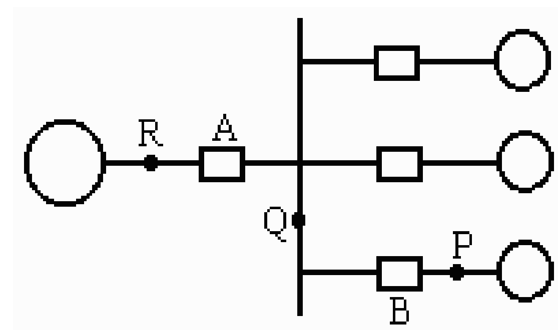


# Question 4 b)

## Let's calculate the transient current in Q using the Thevenin's theorem

Any possible fault occurs between the generator and the motors.
One phase diagram (one generator, three motors):



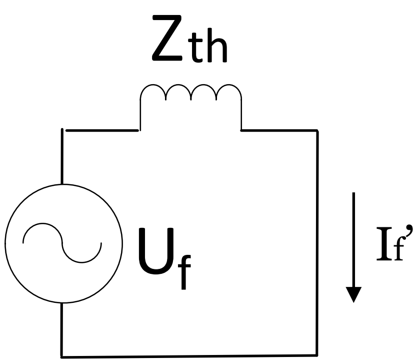

$$Z_{motors} = \left(\frac{1}{Z_{motors}}\right)^{-1} = \left(3\times\frac{1}{Z_m}\right)^{-1} = 0.302\,pu$$

$$Z_{th} = \frac{Z_{motors}Z_{gen}}{Z_{motors}+Z_{gen}} = \frac{j0.302\times j0.2}{j0.302+j0.2} = j0.1203\,pu$$

Thevenin voltage is voltage at the start of the fault :

$$U_f = 1.0\angle 0^\circ\,pu$$

$$\rightarrow I'_f = \frac{U_f}{Z_{th}} = \frac{1}{j0.1203} = -j8.315\,pu$$



# Question 4 b

Let's calculate the transient current in Q (symmetrical short-circuit current) using the Thevenin's theorem

We can calculate the value of base current and absolute value of transient current:

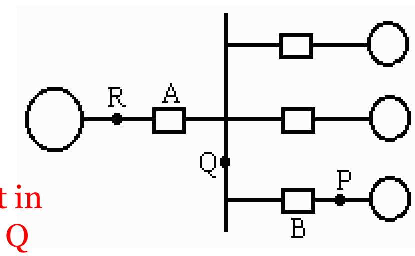
$S_b$=625kVA

$U_b$=2.4kV

$$\left|I_B\right| = \frac{S_B}{\sqrt{3}\times U_B} = \frac{625}{\sqrt{3}\times 2.4} = 150.35\text{A}$$

$$\left|I'_f\right| = 8.315\times 150.35 = 1250.2\text{A}$$

This is current in Fault location Q

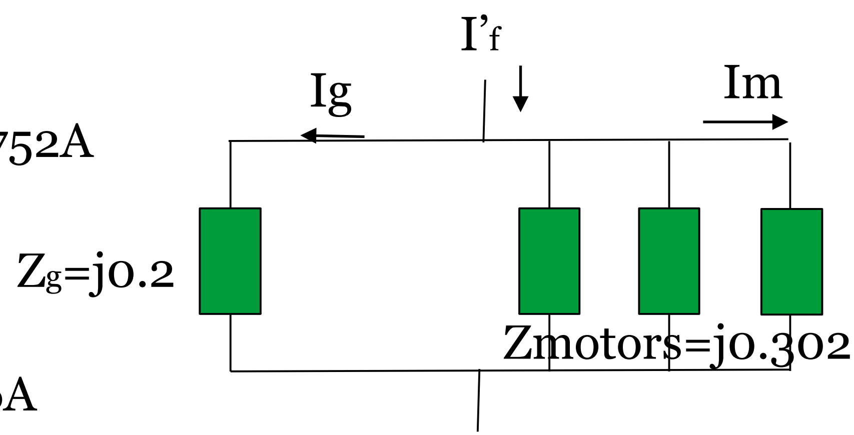



Current of breaker A:

$$I_g = I'_f \frac{Z_{motors}}{Z_g + Z_{motors}} = -j8.315\times\frac{0.302}{0.502} = -j5.0\,pu \Leftrightarrow 752\text{A}$$

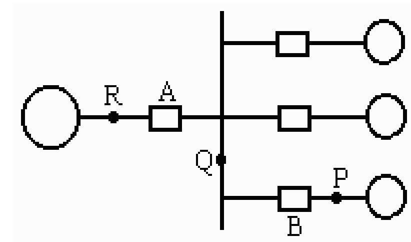
Current of breaker B:

$$I_m = -j\frac{8.315-5.0}{3} = -j1.105\,pu \Leftrightarrow 1.105^*150.35\text{A}=166\text{A}$$



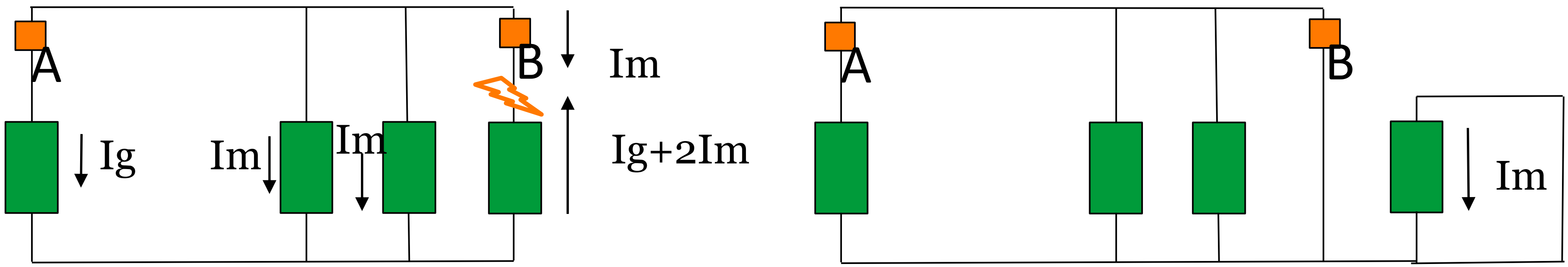
# Question 4 c)

b) fault at point P




Fault at P:

In this case breaker A carries the current fed by generator as in b)
Breaker B carries currents of generator and the other two motors




Currents to be interrupted:

$$\text{Breaker A}: \quad I = I_g = -j5.0\,pu = 752\text{A}$$

$$\text{Breaker B}: \quad I = I_g + 2 \times I_m = -j5.0 + 2(-j1.105) = -j7.21\,pu = 1084\text{A}$$