Aalto-yliopisto
Teknillinen korkeakoulu

Exercise 7

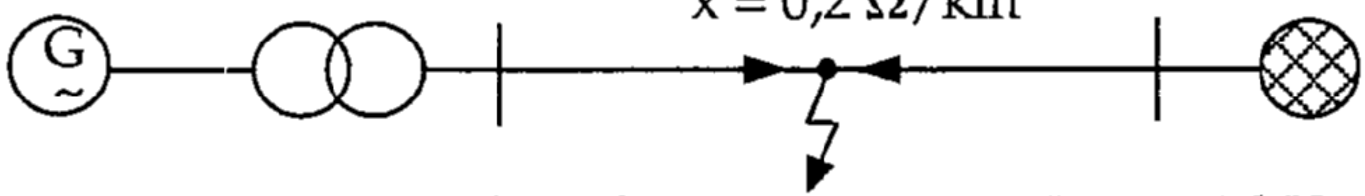
Power systems

Question 1

In the middle of the line there is a zero resistance three-phase short circuit. **What is the voltage at the fault location prior to the fault?** Calculate

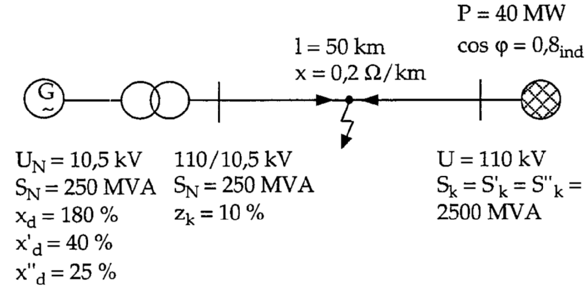
a) sub-transient
b) transient
c) steady state

short-circuit current.

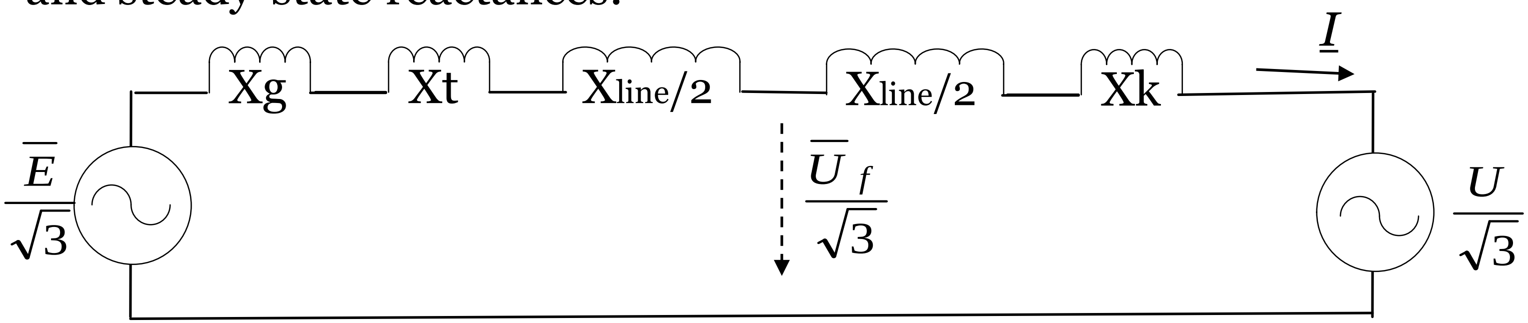


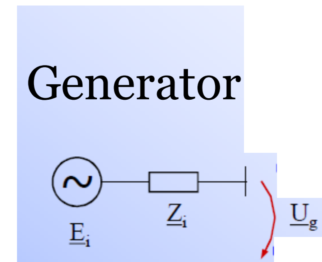
Question 1

What is the voltage at the fault location prior to the fault?



Let's calculate the voltage *U*f with the help of given information: *P* , *U*, power factor and steady-state reactances:



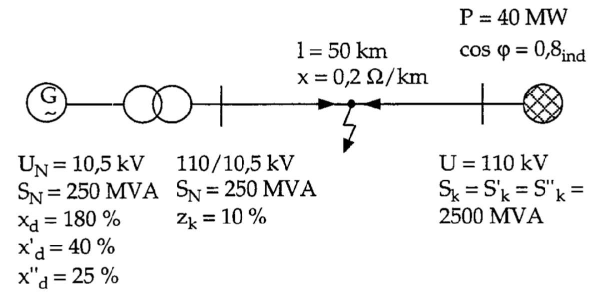


First, calculate the current flowing through each component:

$$\underline{S} = 3 \cdot U_{ph} \cdot \underline{I}^* = 3 \cdot \frac{U}{\sqrt{3}} \cdot \underline{I}^* = \sqrt{3}\sqrt{3} \cdot \frac{U}{\sqrt{3}} \cdot I = \sqrt{3}U \cdot \underline{I}^*$$

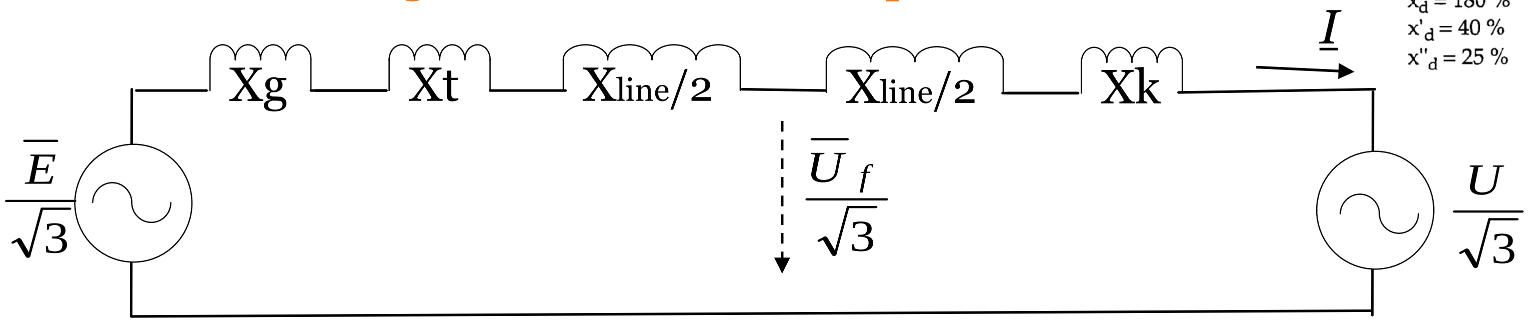
$$\underline{S} = \frac{P}{\cos\varphi} \angle \underbrace{\arccos(0.8)}_{positive(ind.)}$$

$$\Rightarrow \underline{I} = \left(\frac{1}{\sqrt{3}U} \frac{P}{\cos\varphi} \angle 36.9° \right)^* = \frac{1}{\sqrt{3} \cdot 110\text{kV}} \frac{40\text{MW}}{0.8} \angle -36.9° = 262.4\text{A} \angle -36.9°$$



Question 1

What is the voltage at the fault location prior to the fault?



Next, the reactance values as seen from the line (110-kV level):

$$X_g = x_d \cdot \frac{U_N^2}{S_N} \cdot \left(\frac{U_{N1}}{U_{N2}}\right)^2 = 1.80 \cdot \frac{(10.5\ \text{kV})^2}{250\ \text{MVA}} \cdot \left(\frac{110}{10.5}\right)^2 = 87.12\ \Omega$$

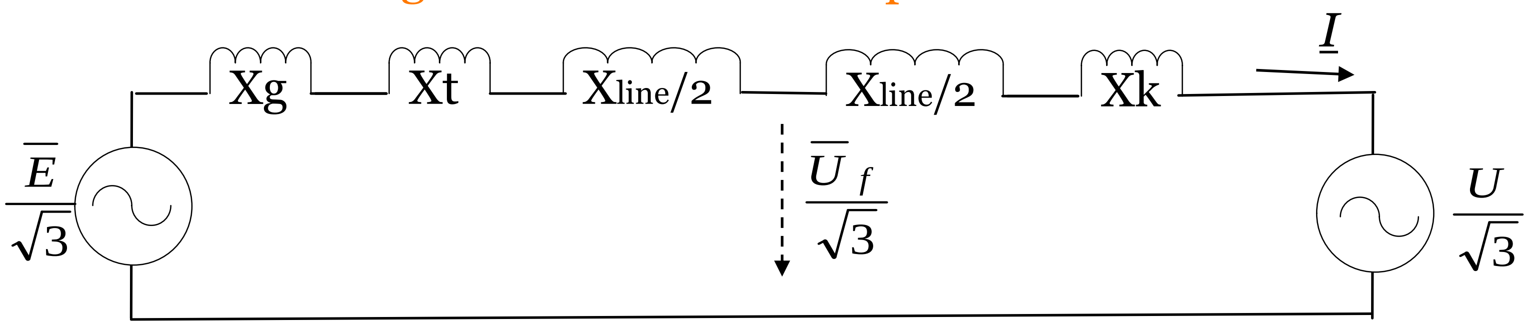
$$X_t = z_k \cdot \frac{U_N^2}{S_N} = 0.10 \cdot \frac{(110\ \text{kV})^2}{250\ \text{MVA}} = 4.84\ \Omega$$

$$\frac{X_{line}}{2} = x \cdot \frac{l}{2} = 0.2 \frac{\Omega}{\text{km}} \cdot 25\text{km} = 5\ \Omega$$

$$X_k = X_k^{'} = X_k^{''} = \frac{U^2}{S_k} = \frac{(110\ \text{kV})^2}{2500\ \text{MVA}} = 4.84\ \Omega \qquad \text{since } S_k = S_k^{'} = S_k^{''}$$

Question 1

What is the voltage at the fault location prior to the fault?



The fault voltage is the grid voltage plus voltage over Xk and half of the line (Xline/2)

$$\frac{\underline{U}_f}{\sqrt{3}} = \frac{U}{\sqrt{3}} + j\left(X_k + \frac{X_{line}}{2}\right)\cdot \underline{I}$$

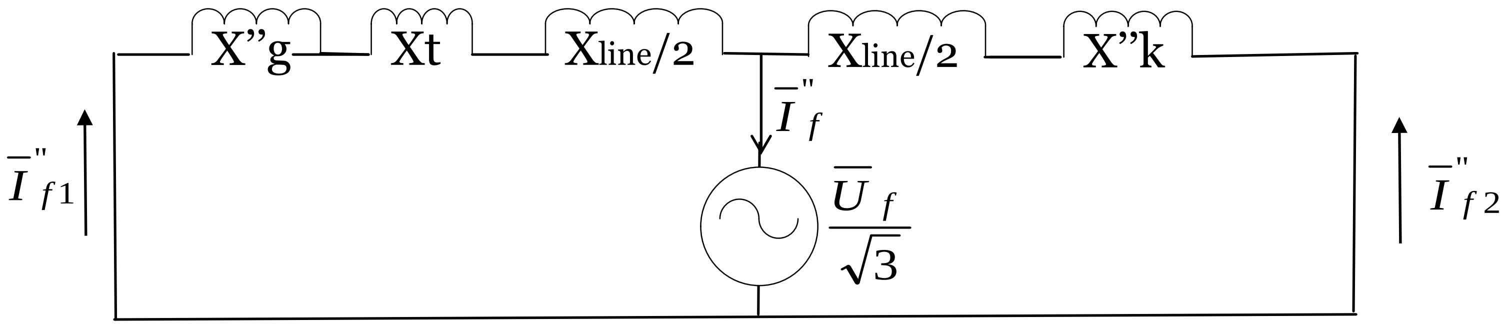
$$\underline{U}_f = U + j\sqrt{3}\cdot\left(X_k + \frac{X_{line}}{2}\right)\cdot \underline{I} = 110\text{ kV} + j\sqrt{3}\cdot(4.84+5)\,\Omega\cdot 262.4\text{ A}\underline{/-36.9^\circ}$$

$$\approx 112.74\,\text{kV}\angle 1.82^\circ$$

$$U_f = \underline{\underline{112.74\,\text{kV}}}$$

Question 1

a) Calculate sub-transient currents (from the grid and from the generator)



The subtransient reactances are:

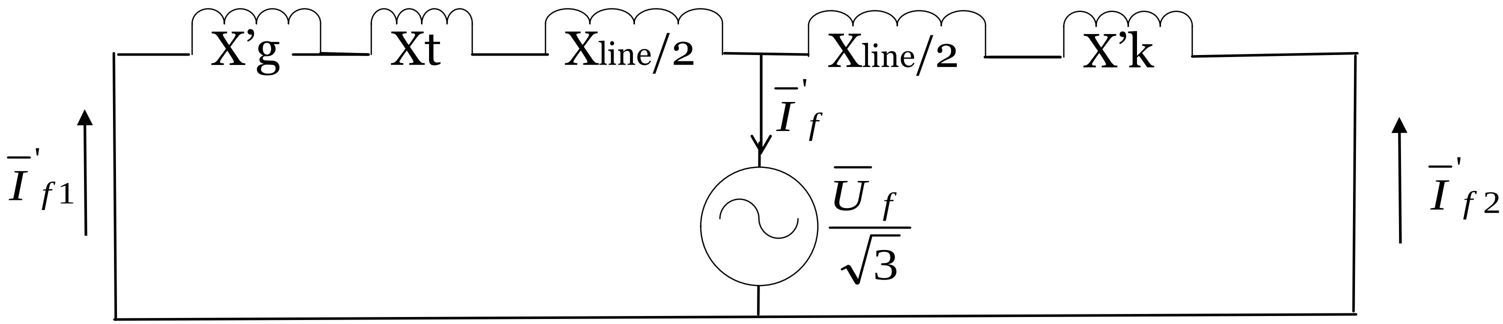
$$X''_g = \frac{x''_d}{x_d} \cdot X_g = \frac{0.25}{1.80} \cdot 87.12\,\Omega = 12.10\,\Omega \qquad X''_k = 4.84\,\Omega$$

$$\underline{I}''_{f1} = \frac{\frac{\underline{U}_f}{\sqrt{3}}}{j\left(X''_g + X_t + \frac{X_{line}}{2}\right)} = \frac{\frac{112.74\,\text{kV}}{\sqrt{3}}\underline{/1.82^\circ}}{j(12.10 + 4.84 + 5)\,\Omega} \approx 2.97\,\text{kA}\,\underline{/-88.2^\circ} \qquad I''_{f1} = \underline{\underline{2.97\,\text{kA}}}$$

$$\underline{\underline{I''_{f2}}} = \frac{\frac{\underline{U}_f}{\sqrt{3}}}{j\left(\frac{X_{line}}{2} + X''_k\right)} = \frac{\frac{112.74\,\text{kV}}{\sqrt{3}}\angle 1.82^\circ}{j(5 + 4.84)\,\Omega} \approx 6.61\,\text{kA}\,\underline{/-88.2^\circ} \qquad I''_{f2} = \underline{\underline{6.61\,\text{kA}}}$$

Question 1

b) Calculate transient currents (from the grid and from the generator)



The transient reactances are:

$$X'_g = \frac{x'_d}{x_d} \cdot X_g = \frac{0.4}{1.80} \cdot 87.12\ \Omega = 19.36\ \Omega \qquad X'_k = 4.84\ \Omega$$

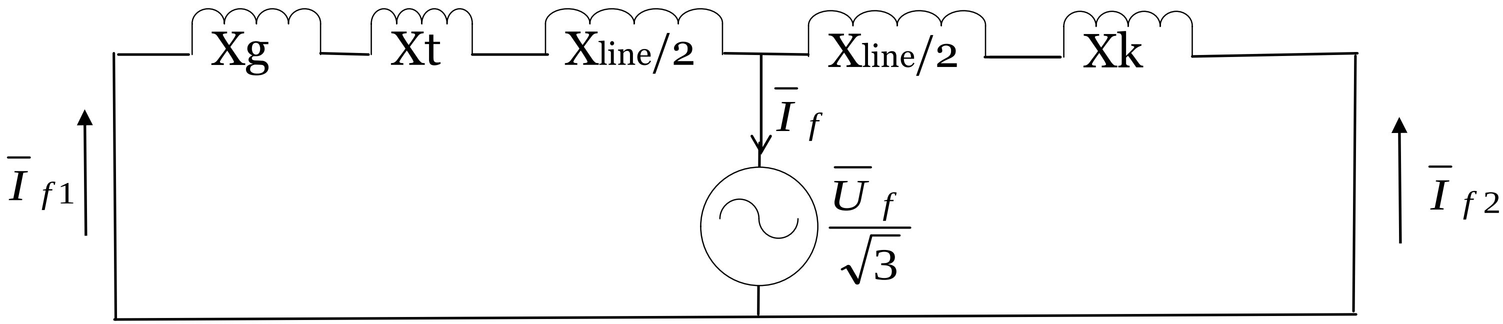
$$\underline{I}'_{f1} = \frac{\frac{\underline{U}_f}{\sqrt{3}}}{j\left(X'_g + X_t + \frac{X_{line}}{2}\right)} = \frac{\frac{112.74\,\text{kV}}{\sqrt{3}}\angle 1.82°}{j(19.36 + 4.84 + 5)\,\Omega} \approx \underline{\underline{2.23\,\text{kA}\angle -88.2°}} \qquad I'_{f1} = \underline{\underline{2.23\,\text{kA}}}$$

$$\underline{I}'_{f2} = \frac{\frac{\underline{U}_f}{\sqrt{3}}}{j\left(\frac{X_{line}}{2} + X'_k\right)} = \underline{I}''_{f2} \approx 6.61\,\text{kA}\angle -88.2° \qquad I'_{f2} = \underline{\underline{6.61\,\text{kA}}}$$

$$X'_k = X''_k$$

Question 1

c) Calculate steady-state currents (from the grid and from the generator)

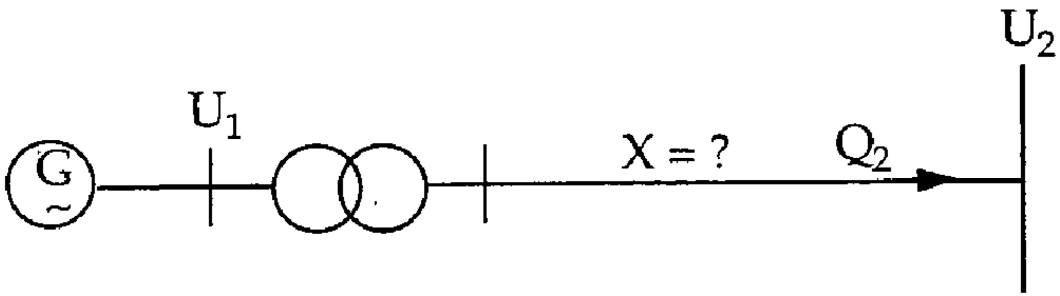


$$\underline{I}_{f1} = \frac{\frac{\underline{U}_f}{\sqrt{3}}}{j\left(X_g + X_t + \frac{X_{line}}{2}\right)} = \frac{\frac{112.74\,\text{kV}}{\sqrt{3}}\underline{/1.82^\circ}}{j(87.12+4.84+5)\,\Omega} \approx 0.67\text{ kA}\underline{/-88.2^\circ} \qquad I_{f1} = \underline{\underline{0.67\,\text{kA}}}$$

$$\underline{I}_{f2} = \frac{\frac{\underline{U}_f}{\sqrt{3}}}{j\left(\frac{X_{line}}{2} + X_k\right)} = \underline{I}''_{f2} \approx 6.61\,\text{kA}\underline{/-88.2^\circ} \qquad \underline{\underline{I_{f2} = 6.61\text{kA}}}$$

$$X_k = X''_k$$

Question 2



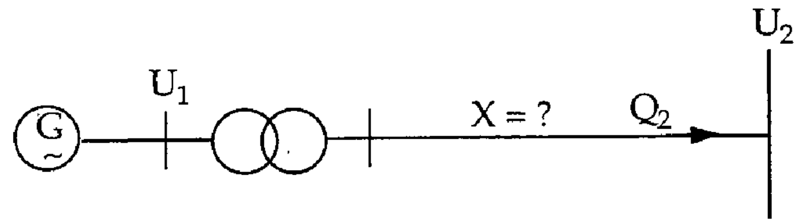
Generator
$P_N = 880$ MW
$S_N = 980$ MVA
$U_N = 20$ kV

Transformer
$S_N = 1000$ MVA
410/20 kV
$z_k = 0{,}15$

Starting from the power-angle equations, **define the maximum line reactance X so that the power plant can feed its full active power P_N without exceeding the maximum apparent power S_N**. Voltage of the transmission network is constant $U_2 = 410$ kV and it doesn't consume any reactive power, that is $Q_2 = 0$.

Question 2

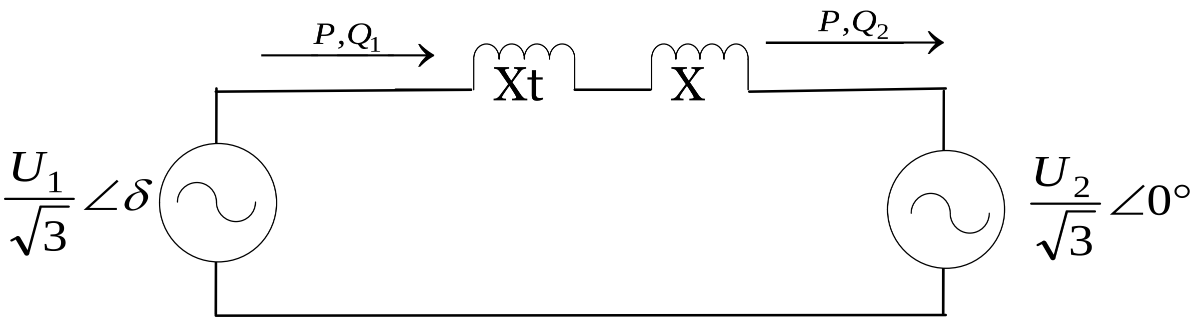
Define maximum line reactance



Generator
$P_N = 880$ MW
$S_N = 980$ MVA
$U_N = 20$ kV

Transformer
$S_N = 1000$ MVA
410/20 kV
$z_k = 0{,}15$

U2=410kV
Q2=0 var

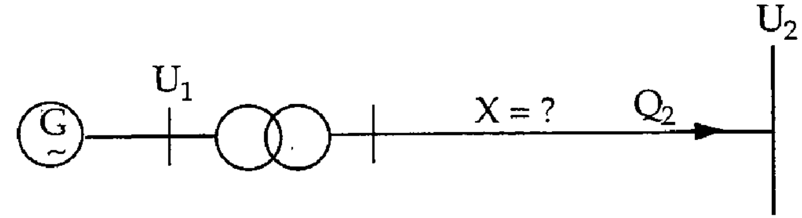


Resistances are neglected: $\Rightarrow \quad P_1 = P_2 = P = P_N$

$$X_t = z_k \cdot \frac{U_N^2}{S_N} = 0.15 \cdot \frac{(410\ \text{kV})^2}{1000\ \text{MVA}} \approx 25.2\ \Omega$$

Question 2

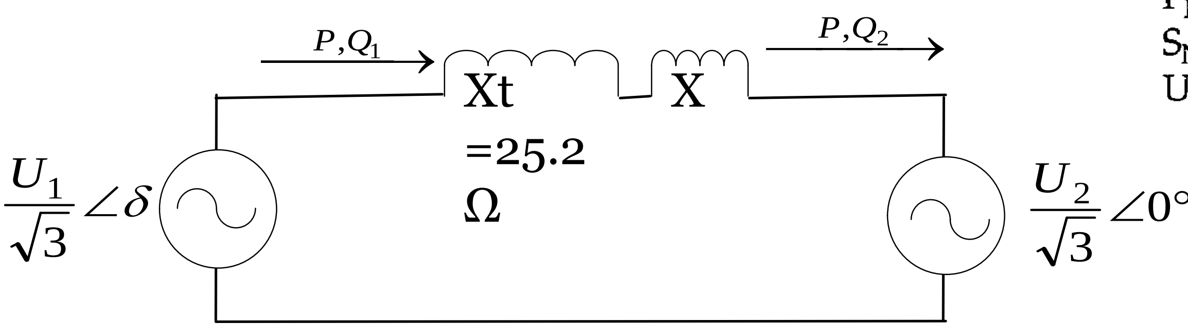
Define maximum line reactance



Generator	Transformer
$P_N = 880$ MW	$S_N = 1000$ MVA
$S_N = 980$ MVA	410/20 kV
$U_N = 20$ kV	$z_k = 0{,}15$

U2=410kV

Q2=0 var



$$S_1 = \sqrt{P^2 + Q_1^2} = S_N$$

$$\begin{cases} P = \dfrac{U_1 U_2}{X_t + X}\sin\delta = P_N \\ Q_1 = \dfrac{U_1^2 - U_1 U_2\cos\delta}{X_t + X} = \sqrt{S_N^2 - P_N^2} \\ Q_2 = \dfrac{U_1 U_2\cos\delta - U_2^2}{X_t + X} = 0 \quad \Leftrightarrow \quad U_1\cos\delta = U_2 \end{cases}$$

→

$$\begin{cases} \sin\delta = \dfrac{P_N\left(X_t + X\right)}{U_1 U_2} \\ \dfrac{U_1^2 - U_2^2}{X_t + X} = \sqrt{S_N^2 - P_N^2} \\ \cos\delta = \dfrac{U_2}{U_1} \end{cases}$$

Question 2

Define maximum line reactance

$$\begin{cases} \sin\delta = \dfrac{P_N\left(X_t + X\right)}{U_1 U_2} \\ \dfrac{U_1^2 - U_2^2}{X_t + X} = \sqrt{S_N^2 - P_N^2} \\ \cos\delta = \dfrac{U_2}{U_1} \end{cases}$$

1

$$\sin^2\delta + \cos^2\delta = 1$$

$$\Leftrightarrow \quad \frac{P_N^2 \cdot \left(X_t + X\right)^2}{U_1^2 U_2^2} + \frac{U_2^2}{U_1^2} = 1 \qquad \Big| \cdot U_1^2$$

2

$$\Leftrightarrow \quad U_1^2 = \frac{P_N^2 \cdot \left(X_t + X\right)^2}{U_2^2} + U_2^2$$

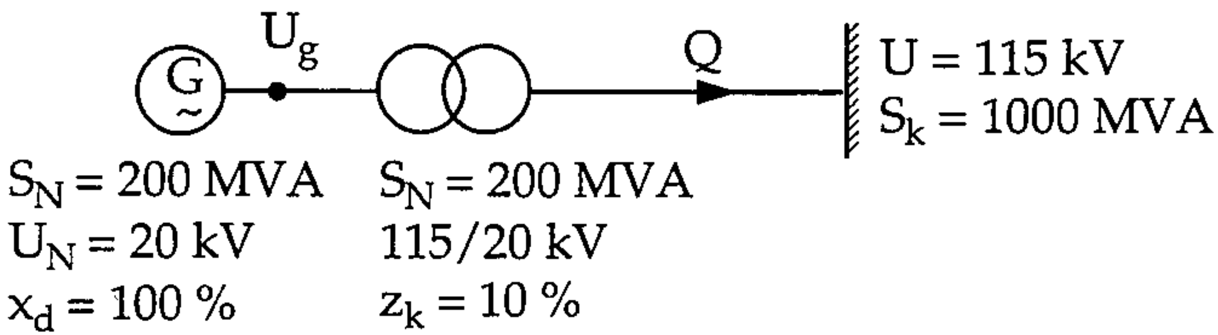
3

$$\frac{U_1^2 - U_2^2}{X_t + X} = \frac{\left(\dfrac{P_N^2 \cdot \left(X_t + X\right)^2}{U_2^2} + U_2^2\right) - U_2^2}{X_t + X} = \frac{P_N^2 \cdot \left(X_t + X\right)}{U_2^2} = \sqrt{S_N^2 - P_N^2}$$

$$\Leftrightarrow \quad X = \frac{U_2^2}{P_N^2} \cdot \sqrt{S_N^2 - P_N^2} - X_t = \frac{\left(410\ \text{kV}\right)^2}{\left(880\ \text{MW}\right)^2} \cdot \sqrt{\left(980\ \text{MVA}\right)^2 - \left(880\ \text{MW}\right)^2} - 25.2\ \Omega$$

Maximum reactance: $X \approx 68.4\ \Omega$

Question 3

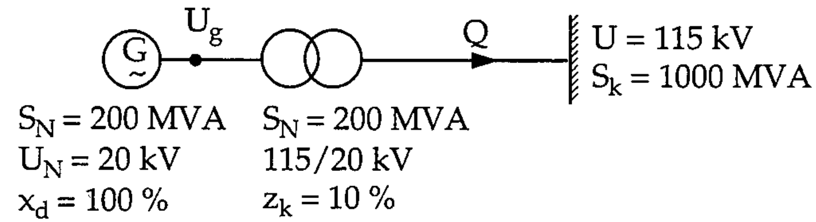


A synchronous generator is synchronized through a transformer to a bus.
At the bus, the short circuit power is 1000 MVA and the voltage is 115 kV.
After synchronizing, the generator's power is increased to 100 MW without changing the excitation.
Calculate the generator's terminal voltage U_g and reactive power Q.

When synchronizing, the voltages of the generator are first tuned to same frequency, amplitude and phase order as in power system. Then connecting switch is then closed. At this moment the generator internal emf, terminal voltage and system bus V are same.

Question 3

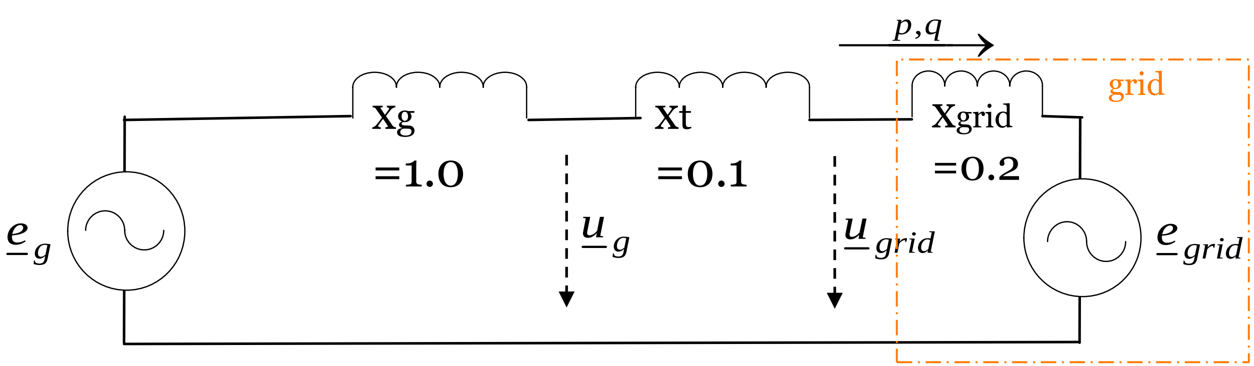
generator's terminal voltage U_g and reactive power Q.



Selecting base values: $S_b = 200\ \text{MVA} \quad , \quad U_b = 115\ \text{kV}$

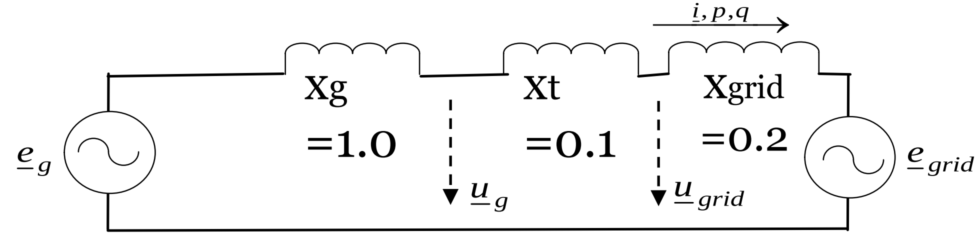
We get:

$$
\begin{cases}
p = \dfrac{P}{S_b} = \dfrac{100}{200} = 0.5 \\
x_g = 1.00 \\
x_t = 0.10 \\
x_{grid} = \dfrac{X_{grid}}{Z_b} = \dfrac{\left(\dfrac{U^2}{S_k}\right)}{\left(\dfrac{U_b^2}{S_b}\right)} = \dfrac{\left(\dfrac{115^2 \cdot 10^6}{1000 \cdot 10^6}\right)}{\left(\dfrac{115^2 \cdot 10^6}{200 \cdot 10^6}\right)} = 0.20
\end{cases}
$$



Question 3

generator's terminal voltage U_g and reactive power Q.



When connecting to the grid: $e_g = u_g = u_{grid} = e_{grid} = 1$

If magnetizing is not changed after the synchronization, the generator emf stays the same:

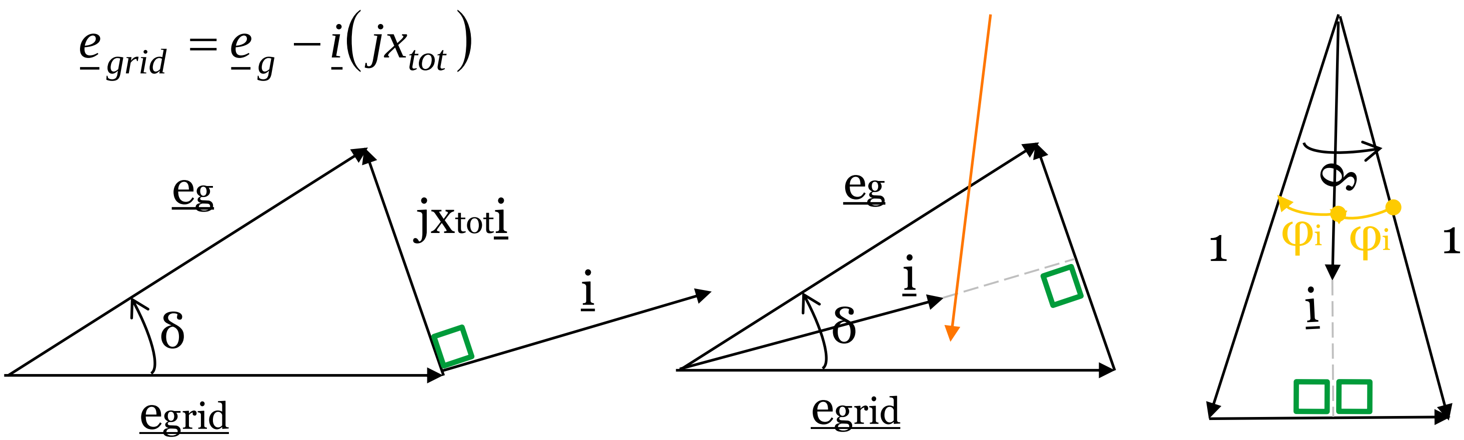
$$e_g = 1$$

$$p = \frac{e_g \cdot e_{grid}}{x_g + x_t + x_{grid}} \cdot \sin\delta \qquad 0.5 = \frac{1 \cdot 1}{1.00 + 0.10 + 0.20} \cdot \sin\delta \quad \Rightarrow \quad \sin\delta = 0.5 \cdot 1.30$$

$$\Rightarrow \quad \delta = 40.54°$$

Current leading load voltage → capacitive load

$$\underline{e}_{grid} = \underline{e}_g - \underline{i}\left(jx_{tot}\right)$$

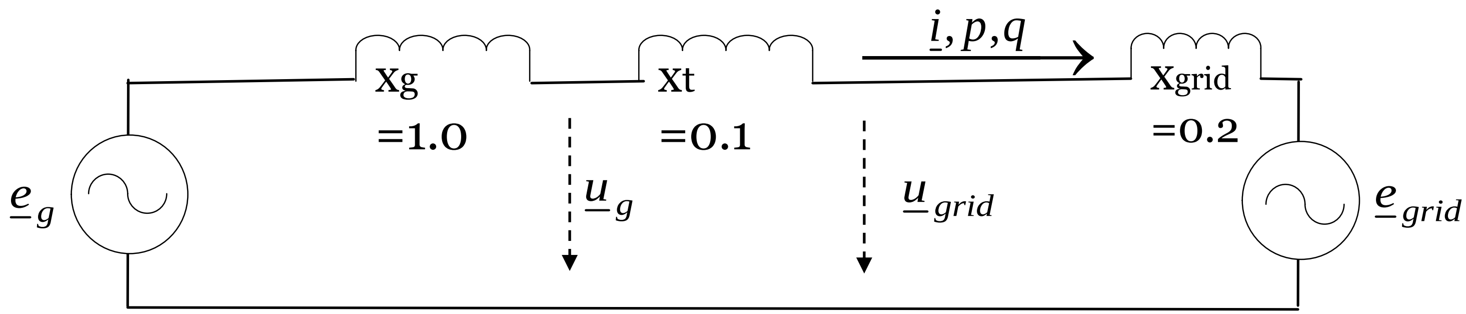


$$\varphi_i = \frac{\delta}{2} = \frac{40.54°}{2} = 20.27°$$

$$\cos\varphi_i = 0.938_{cap}$$

Question 3

generator's terminal voltage U_g and reactive power Q.



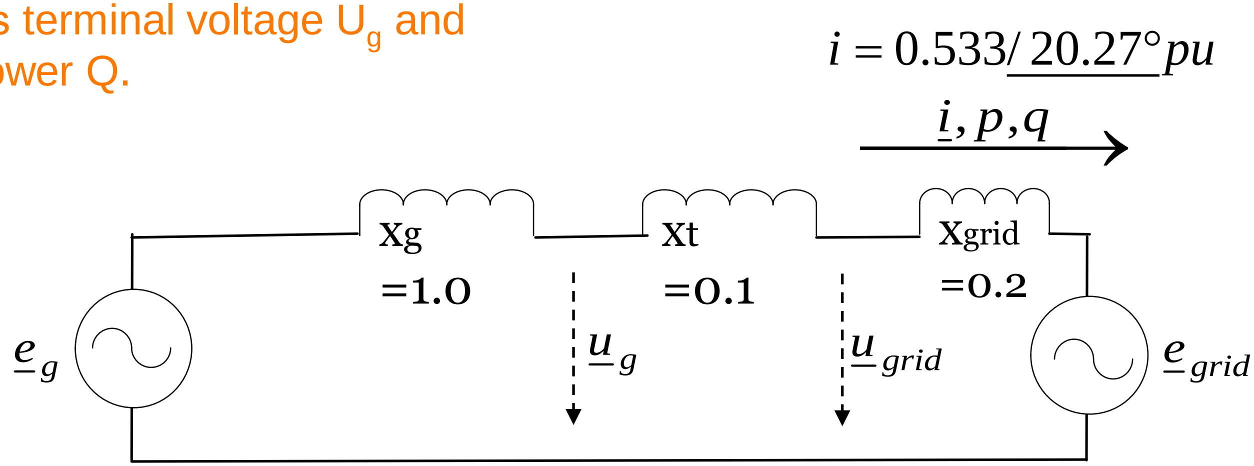
$$\underline{e}_{grid} = e_{grid}\underline{/0°} = 1\underline{/0°}$$

$$\underline{s} = \underline{u}\underline{i}^* = \frac{p}{\cos\varphi_i}\angle \underbrace{-\varphi_i}_{neg.(cap.)}$$

$$\underline{i} = \left(\frac{p}{e_{grid}\cos\varphi_i}\underline{/-\varphi_i}\right)^* = \frac{0.5}{1\cdot\cos 20.27°}\underline{/20.27°} = 0.533\underline{/20.27°}pu$$

Question 3

generator's terminal voltage U_g and reactive power Q.

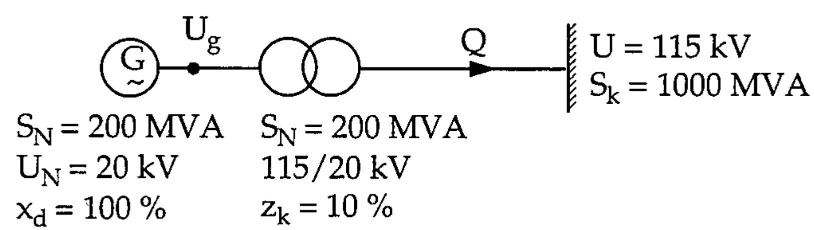


Generator terminal voltage is:

$$\underline{u}_g = \underline{e}_{grid} + j\left(x_t + x_{grid}\right)\cdot \underline{i}$$

$$= 1\angle 0° + (0.10 + 0.20)\angle 90° \cdot 0.533\angle 20.27°$$

$$= 0.956\angle 9.02° pu$$

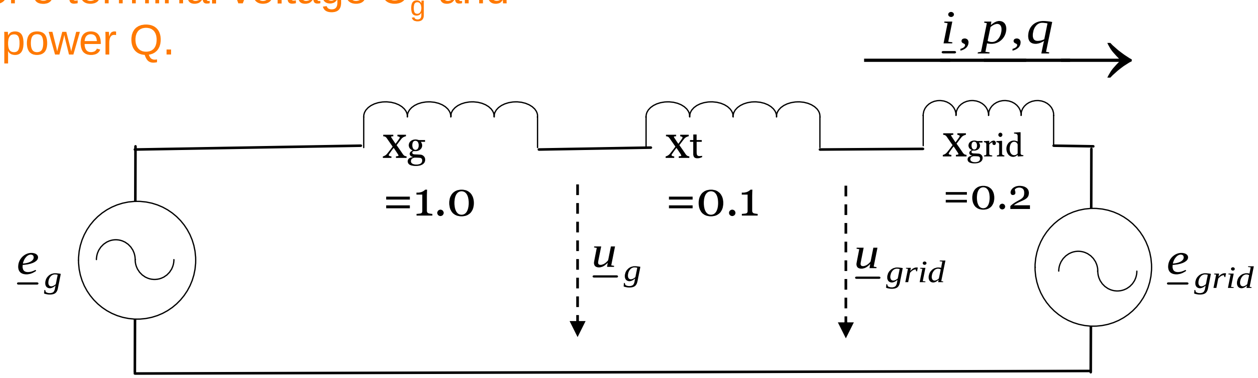


$$20\text{-kV level:} \quad U_b = 115\frac{20}{115} = 20\,\text{kV}$$

$$U_g = u_g \cdot U_b = 0.956 \cdot 20\ \text{kV} = 19.1\ \text{kV}$$

Question 3

generator's terminal voltage U_g and reactive power Q.

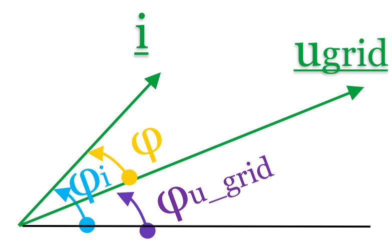


Voltage at the station is: $\underline{u}_{grid} = \underline{e}_{grid} + jx_{grid} \cdot \underline{i}$

$$= 1\angle 0° + 0.20\angle 90° \cdot 0.533\angle 20.27°$$

$$= 0.968\angle 5.93°\, pu$$

Reactive power flowing to the grid: $q = u_{grid} \cdot i \cdot \sin\left(\underbrace{\varphi_{u_{grid}} - \varphi_i}_{\varphi}\right)$

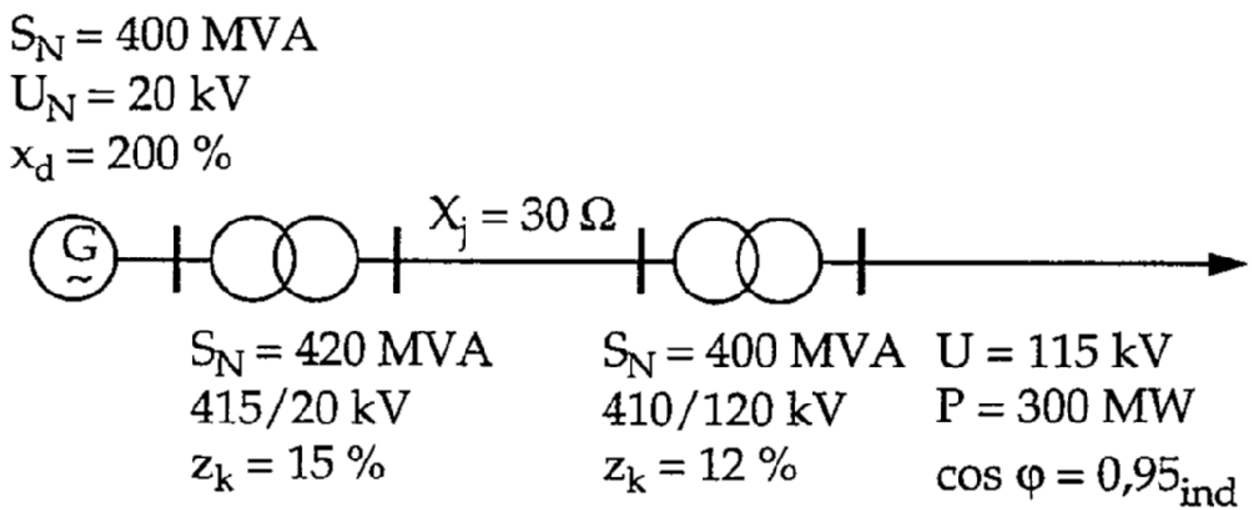


$$= 0.968 \cdot 0.533 \cdot \sin(5.93° - 20.27°)$$

$$= -0.128\, pu$$

$$\underline{\underline{Q}} = q \cdot S_b = -0.128 \cdot 200 = \underline{\underline{-25.6\,\text{Mvar}}}$$

Question 4



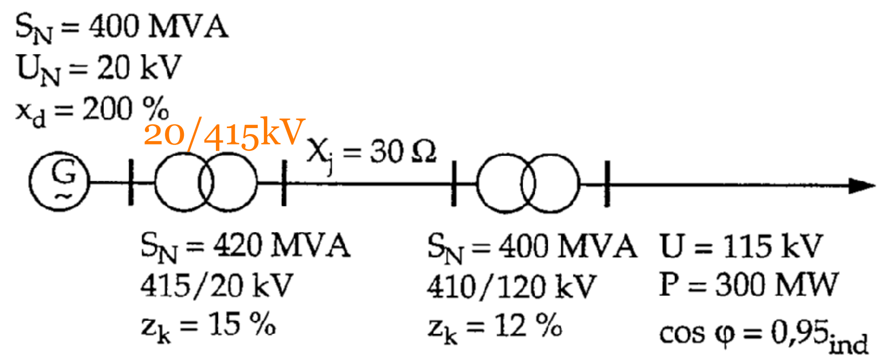
For the transmission system shown in the picture, calculate **generator's load current and terminal voltage**

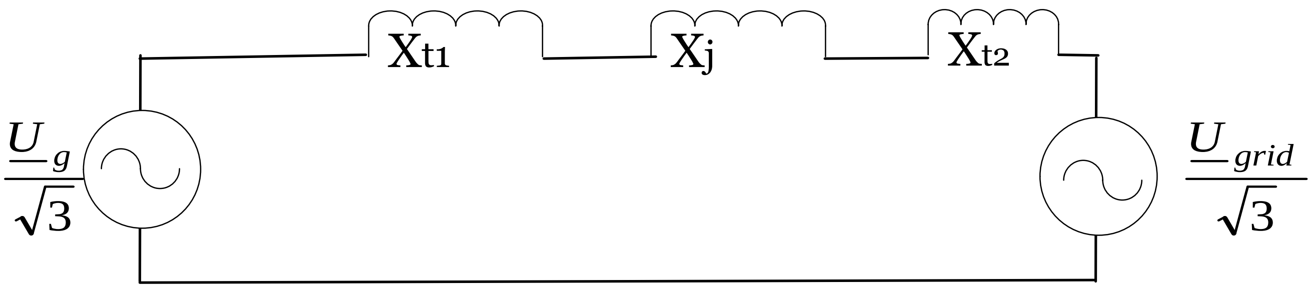
a) by reducing the network to generator's voltage level

b) by using per-unit values

Question 4

a) at generator voltage level





System reactances seen from the generator:

Transformer 1: $X_{t1} = z_k \frac{U_N^2}{S_N} = 0.15 \cdot \frac{20^2 \cdot 10^6}{420 \cdot 10^6}\ \Omega = \quad 0.143\Omega$

Line: $X_j = 30 \cdot \left(\frac{20}{415}\right)^2 \Omega = \quad 0.070\ \Omega$

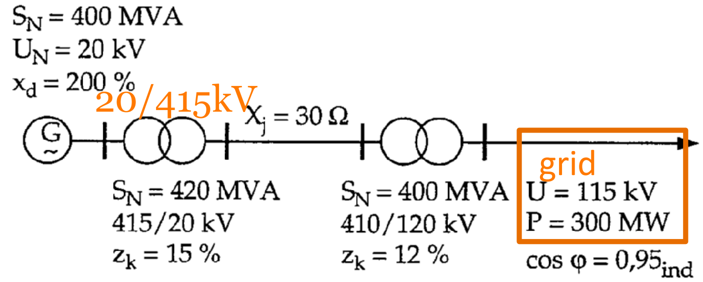
Transformer 2: $X_{t2} = 0.12 \cdot \frac{410^2 \cdot 10^6}{400 \cdot 10^6} \cdot \left(\frac{20}{415}\right)^2 \Omega = \quad 0.117\ \Omega$

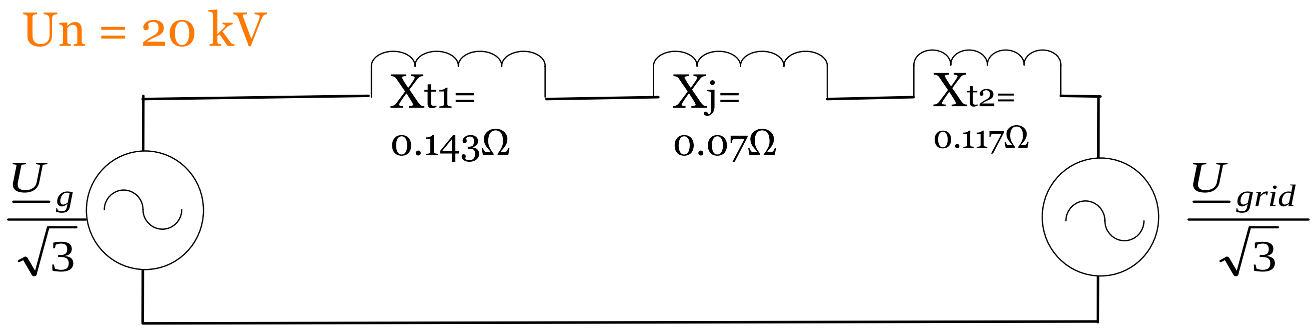
in secondary $Z_2 = \frac{U_2}{I_2}$

in primary $Z_2' = \frac{U_1}{I_1} = \frac{U_2 \cdot \frac{N_1}{N_2}}{I_2 \cdot \frac{N_2}{N_1}} = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_2$

Question 4

a) current and voltage at generator





Let's choose: $\underline{U}_{grid} = U_{grid}\angle 0°$

Jump over transf.2 and tranfs.1

Current at Generator:

Current at grid:

$$\underline{I} = \frac{P_{grid}}{\sqrt{3}\cdot U_{grid}\cdot\cos\varphi}\underline{/-\arccos 0.95}\cdot\frac{120}{410}\cdot\frac{415}{20}$$

$$\left(\underline{S} = 3\cdot\frac{\underline{U}}{\sqrt{3}}\underline{I}^* = \frac{P}{\cos\varphi}\right)$$

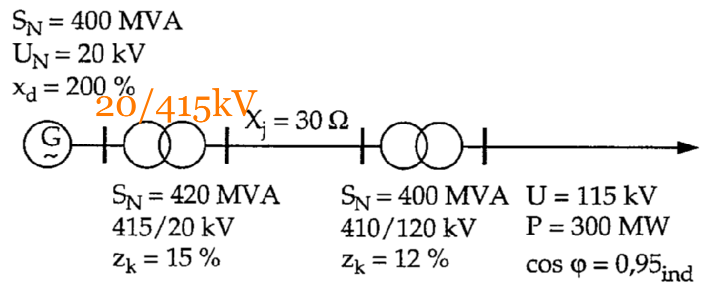
$$= \frac{300\cdot 10^6}{\sqrt{3}\cdot 115\cdot 10^3\cdot 0.95}\cdot\frac{120}{410}\cdot\frac{415}{20}\underline{/-\arccos 0.95}\ \text{A}$$

$$\left(\underline{I} = \frac{\underline{S}^*}{\sqrt{3}\underline{U}^*}\right)$$

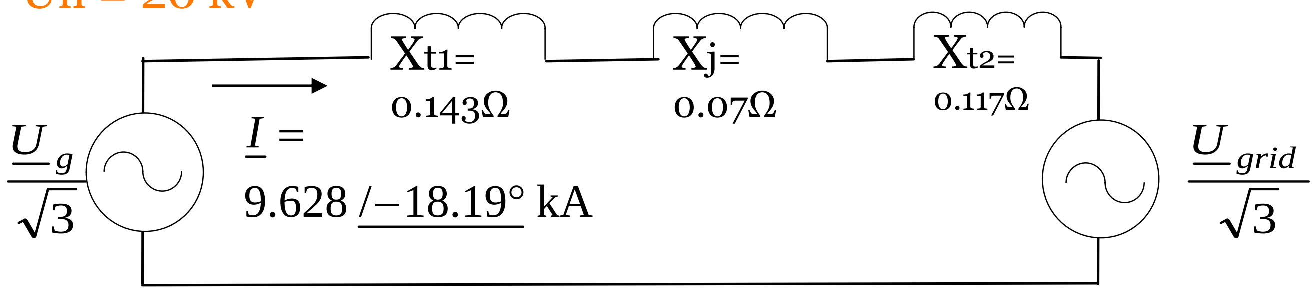
$$= 9628\ \underline{/-18.19°}\ \text{A} \approx \underline{9.6\underline{/-18.2°}\ \text{kA}}$$

Question 4

a) current and voltage at generator



Un = 20 kV



Grid voltage seen from the generator: $U_{grid} = 115 \cdot \frac{410}{120} \cdot \frac{20}{415}\ \text{kV} = 18.936\ \text{kV}$

Generator terminal voltage:

$$\frac{\underline{U}_g}{\sqrt{3}} = \frac{\underline{U}_{grid}}{\sqrt{3}} + \underline{I} \cdot j\left(X_{t1} + X_j + X_{t2}\right)$$

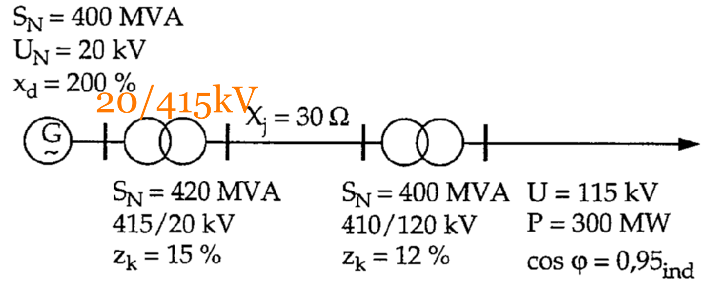
$$\underline{U}_g = 18.936\underline{/0°}\ \text{kV} + \sqrt{3} \cdot 9.628\underline{/-18.19°} \cdot (0.143 + 0.070 + 0.117)\underline{/90°}\ \text{kV}$$

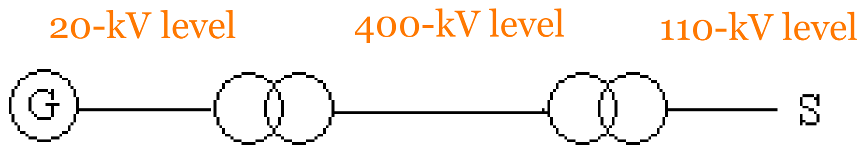
$$= (20.655 + j5.228)\ \text{kV} = 21.306\underline{/14.20°}\ \text{kV}$$

$$\approx \underline{\underline{21.3\underline{/14.2°}\ \text{kV}}}$$

Question 4

b) per unit approach





Let's choose base power: $S_b = 400\,\text{MVA}$

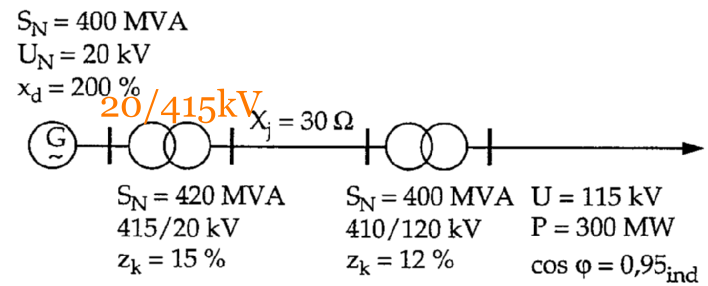
and base voltage level at line between the transformers: $U_b = 400\,\text{kV}$

The transformer ratios determine the other base voltages.

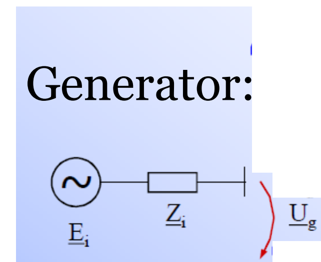
- 20-kV level: $U_b = 400\,\text{kV} \cdot \dfrac{20}{415} = 19.277\,\text{kV}$
- 110-kV level: $U_b = 400\,\text{kV} \cdot \dfrac{120}{410} = 117.073\,\text{kV}$

Question 4

b) per unit approach



20-kV level: has only generator Xd. Not needed because we are calculating generator terminal voltage which already includes voltage drop caused by Xd.



$$S_b = 400 \text{ MVA}, \quad U_b = 400 \cdot \frac{20}{415} = 19.277 \text{ kV}$$

400-kV level: $S_b = 400 \text{ MVA}, \quad U_b = 400 \text{ kV} \Rightarrow \quad Z_b = \frac{400^2}{400} = 400\,\Omega$

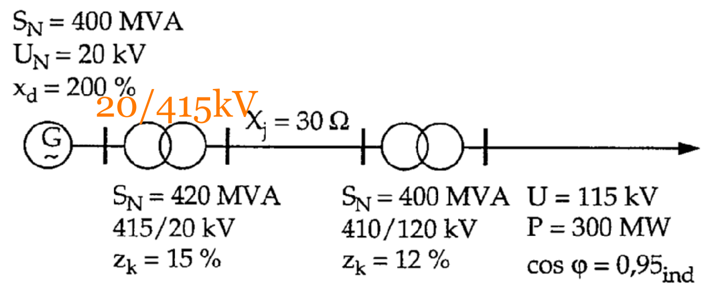
$$x_{t1} = \frac{z_{k1} \cdot \frac{U_N^2}{S_N}}{Z_b} = \frac{0.15 \cdot \frac{415^2}{420}}{400} = 0.154\,pu$$

$$x_j = \frac{X_j}{Z_b} = \frac{30}{400} = 0.075\,pu$$

$$x_{t2} = \frac{z_{k2} \frac{U_N^2}{S_N}}{Z_b} \cdot = \frac{0.12 \cdot \frac{410^2}{400}}{400} = 0.126\,pu$$

Question 4

b) per unit approach



110-kV level: $S_b = 400\text{ MVA}, \quad U_b = 400\frac{120}{410} = 117.073\,\text{kV}$ $\left(Z_b = \frac{117.073^2}{400} = 34.265\,\Omega \right)$

$$p_{grid} = \frac{P_{grid}}{S_b} = \frac{300}{400} = 0.75$$

$$u_{grid} = \frac{U_{grid}}{U_b} = \frac{115}{117.073} = 0.982 \quad ;choose \quad \underline{u}_{grid} = u_{grid}\,\underline{/0°}$$

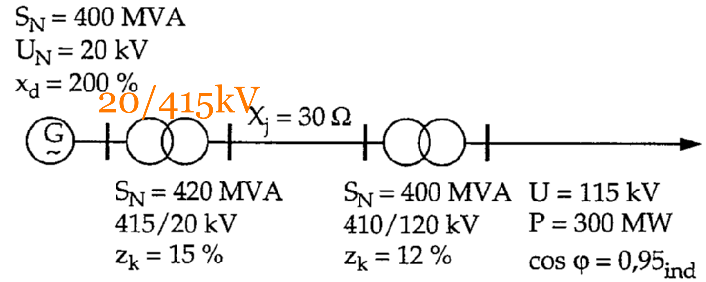
$$\underline{i} = \frac{p_{grid}}{u_{grid}\cdot\cos\varphi}\,\underline{/-\arccos 0.95} = \frac{0.75}{0.982\cdot 0.95}\,\underline{/-\arccos 0.95} = 0.804\,\underline{/-18.19°}$$

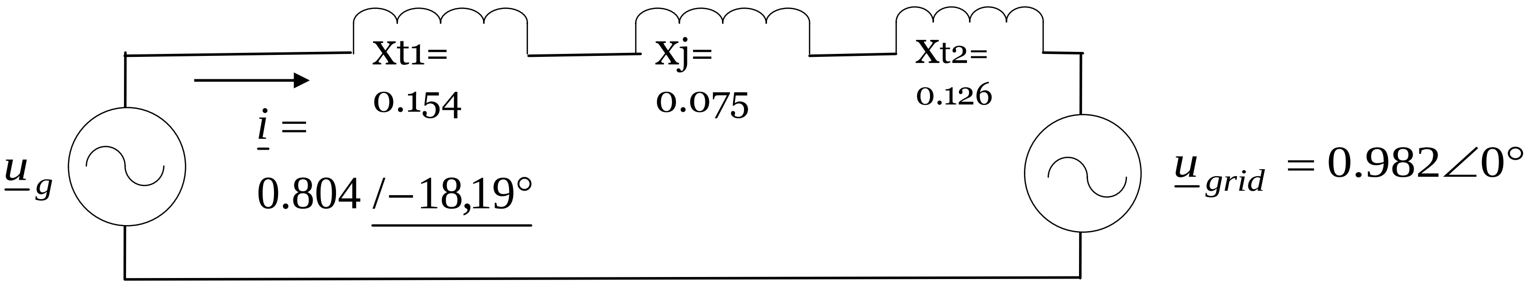
$$\underline{I} = \underline{i}\cdot\underline{I}_b = \underline{i}\cdot\frac{S_b}{\sqrt{3}U_b} = 0.804\,\underline{/-18.19°}\cdot\frac{400\cdot 10^6}{\sqrt{3}\cdot 19.277\cdot 10^3}\text{ A} \approx 9.6\angle -18.2°$$

$$I = \underline{\underline{9.6\text{ kA}}}$$

Question 4

b) per unit approach





$$\underline{u}_g = \underline{u}_{grid} + \underline{i} \cdot j\left(x_{t1} + x_j + x_{t2}\right)$$

$$= 0.982\,\underline{/0°} + 0.804\,\underline{/-18.19°} \cdot j\left(0.154 + 0.075 + 0.126\right)$$

$$= 1.105\,\underline{/14.21°}$$

$$\underline{U}_g = \underline{u}_g \cdot U_{b\,20\text{kV}} = 1.105\,\underline{/14.21°} \cdot 19.277\ \text{kV} \approx 21.3\,\underline{/14.2°}\ \text{kV}$$

$$U_g \approx 21.3\text{kV}$$