Aalto-yliopisto
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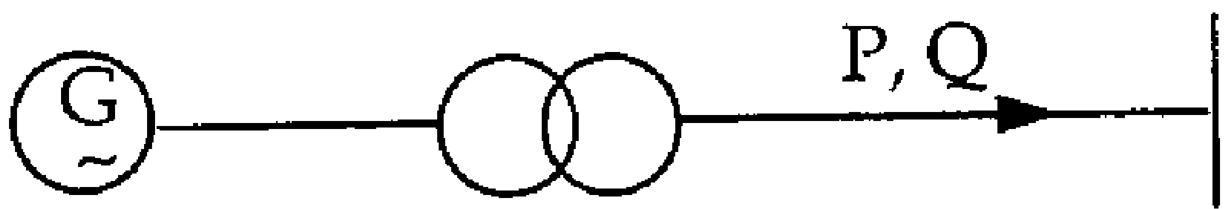
Exercise 8

Power systems

Question 1

The generator is feeding the network. P=250 MW and Q=0 Mvar. Calculate the generator terminal voltage when it is disconnected from the network.

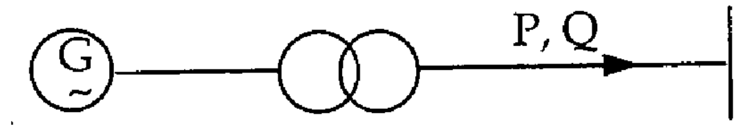


$S_N = 300$ MVA
$U_N = 20$ kV
$x_d = 200$ %

$S_N = 300$ MVA
400/20 kV
$z_k = 15$ %

$U = 410$ kV

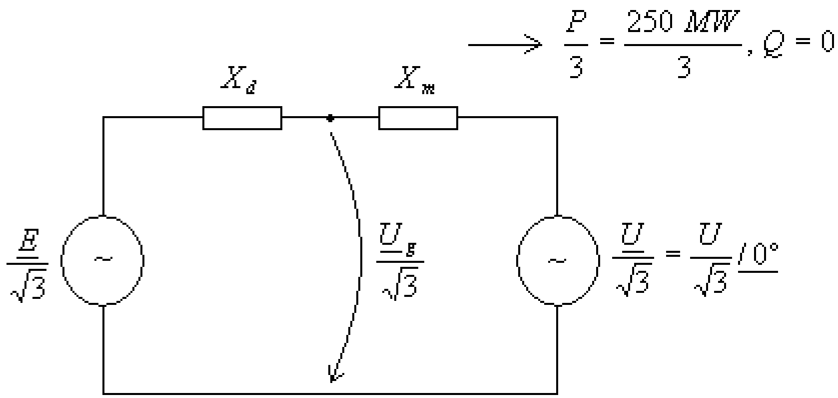
Question 1



$S_N = 300$ MVA, $U_N = 20$ kV, $x_d = 200$ %

$S_N = 300$ MVA, 400/20 kV, $z_k = 15$ %

$U = 410$ kV



When disconnected from the network, the terminal voltage Ug of the generator rises to the value of E generated

Generated voltage equals to receiving end voltage (U) + voltage drop across the reactance.

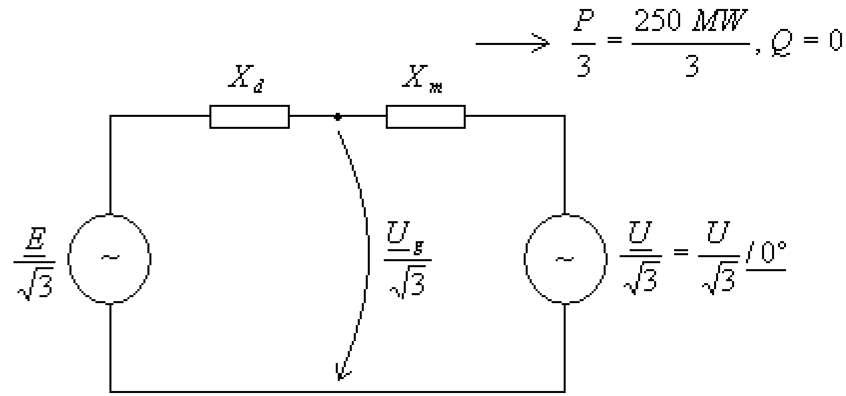
$$\frac{\underline{E}}{\sqrt{3}} = \frac{\underline{U}}{\sqrt{3}} + j\left(X_d + X_m\right)\cdot \underline{I} \qquad \underline{I} = \frac{\underline{S}^*}{\sqrt{3}\cdot \underline{U}^*} = \frac{P}{\sqrt{3}\cdot U}$$

Calculating reactance values on the side of the 20-kV Transformer

$$X_d = x_d \cdot \frac{U_N^2}{S_N} = 2.0 \cdot \frac{(20\ \text{kV})^2}{300\ \text{MVA}} = 2.67\ \Omega \qquad U = \frac{20\ \text{kV}}{400\ \text{kV}} \cdot 410\ \text{kV} = 20.5\ \text{kV}$$

$$X_m = z_k \cdot \frac{U_N^2}{S_N} = 0.15 \cdot \frac{(20\ \text{kV})^2}{300\ \text{MVA}} = 0.20\ \Omega$$

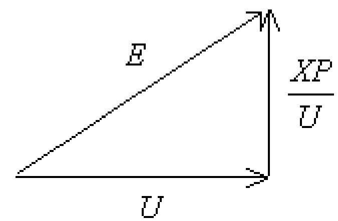
Question 1



$$\frac{\underline{E}}{\sqrt{3}} = \frac{\underline{U}}{\sqrt{3}} + j(X_d + X_m)\cdot \underline{I}, \quad \underline{I} = \frac{\underline{S}^*}{\sqrt{3}\cdot \underline{U}^*} = \frac{P}{\sqrt{3}\cdot U}$$

Converting further:

$$\underline{E} = U + j(X_d + X_m)\cdot \frac{P}{U} \quad \Rightarrow \quad E^2 = U^2 + \left((X_d + X_m)\cdot \frac{P}{U}\right)^2$$



Generator voltage and terminal voltage at the time of disconnection:

$$\Rightarrow \quad E = \sqrt{U^2 + \left((X_d + X_m)\cdot \frac{P}{U}\right)^2} = \sqrt{(20.5\ \text{kV})^2 + \left[(2.67 + 0.2)\,\Omega \cdot \frac{250\ \text{MW}}{20.5\ \text{kV}}\right]^2} = 40.5\ \text{kV}$$

Question 2

A highly capacitive circuit of capacitance per phase 100 μF is disconnected by a circuit breaker, the source inductance being 1 mH. The breaker gap breaks down when the voltage across it reaches twice the system peak line-to-neutral voltage of 38 kV. **Calculate the current flowing with the breakdown and its frequency and compare it with the normal (50-Hz) charging current of the circuit.**

Question 2

Energy is distributed half into the electric field and half in the magnetic field: Inductive energy equals to capacitive energy during a travelling wave.

$$\frac{1}{2}Li_0^2 = \frac{1}{2}Cu^2 \Leftrightarrow u = i_0\sqrt{\frac{L}{C}},$$

The breaker gap breaks down, when the voltage across it reaches twice the system peak line-to-neutral voltage of 38 kV

$$u = 2u_{\text{ph,peak}} = 2\times 38\text{kV} = 76\text{kV}$$

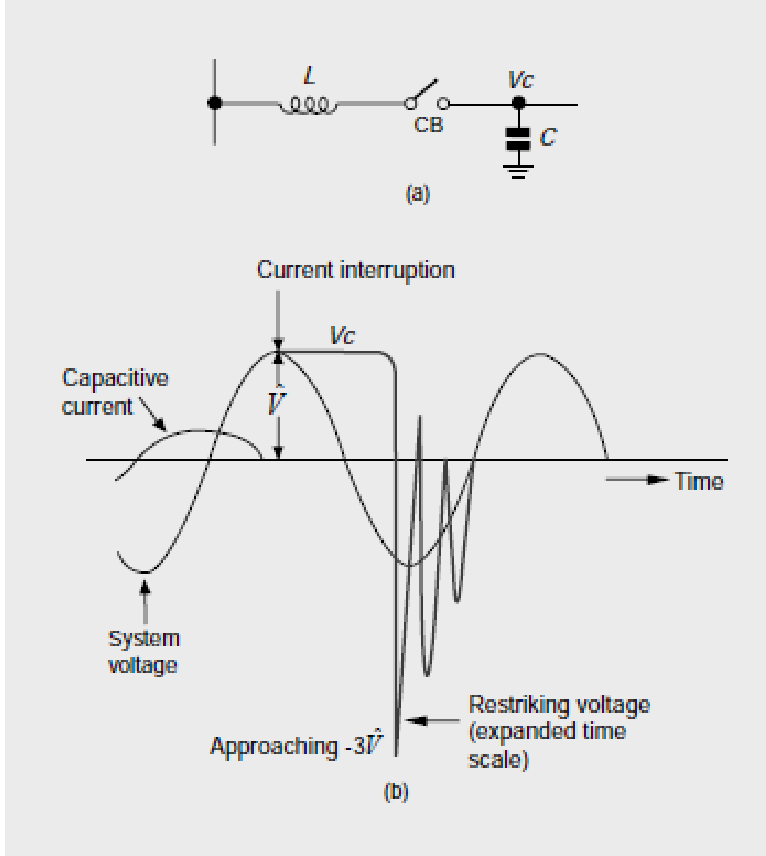


Figure 10.5 Voltage waveform when opening a capacitive circuit

Electric Power Systems, 5th ed., Weedy et al.

At this moment, the electricity starts to "travel", and thus we can convert the voltage into current:

$$i_0 = \frac{u}{\sqrt{\frac{L}{C}}} = \frac{76\times 10^3}{\sqrt{\frac{1\times 10^{-3}}{100\times 10^{-6}}}} = \underline{\underline{24.05\text{kA}}}$$

Question 2

Frequency:

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 100 \times 10^{-6}}} = 3162.3 \frac{\text{rad}}{\text{s}}$$

$\Rightarrow$

$$\boxed{f = \frac{\omega}{2\pi} = \frac{3162.3}{2 \times \pi} \text{Hz} = \underline{\underline{503.3\text{Hz}}}}$$

The normal charging current of the circuit is:

$$\underline{I} = \frac{\frac{\underline{U}_{ph,peak}}{\sqrt{2}}}{\underline{Z}} = \frac{38 \times 10^3}{\sqrt{2}\left(j\omega L - j\frac{1}{\omega C}\right)} = \frac{38 \times 10^3}{\sqrt{2} \cdot 31.52\angle -90°} \text{A} = \underline{\underline{852\text{A}\angle -90°}}$$

$$\text{Thus, } \frac{i_o}{I} \approx 28.2$$

Question 3

The effective inductance and capacitance of a faulted system as viewed by the contacts of a circuit breaker are 2 mH and 500 pF, respectively. The circuit breaker chops the fault current when it has an instantaneous value of 100 A. **Calculate the restriking voltage set up across the circuit breaker**. Neglect resistance.

Question 3

The breaker tends to open the circuit before the current natural-zero, and the electromagnetic energy present is rapidly converted to electrostatic energy

$$\frac{1}{2}Li_0^2 = \frac{1}{2}Cu^2 \quad \rightarrow \quad u = i_0\sqrt{\frac{L}{C}},$$

And the restriking voltage
(in addition to the system voltage) is

$$u_r = ue^{-\alpha t}\sin\omega_0 t, \quad \text{and now } \alpha = \frac{R}{2L} = \frac{0}{2L} = 0$$

$$\Leftrightarrow u_r = u\sin\omega_0 t = i_0\sqrt{\frac{L}{C}}\sin\omega_0 t$$

Where $\omega_0 = \dfrac{1}{\sqrt{LC}}$

and i_0 is the value of the current at the instant of chopping *(t=0)*.

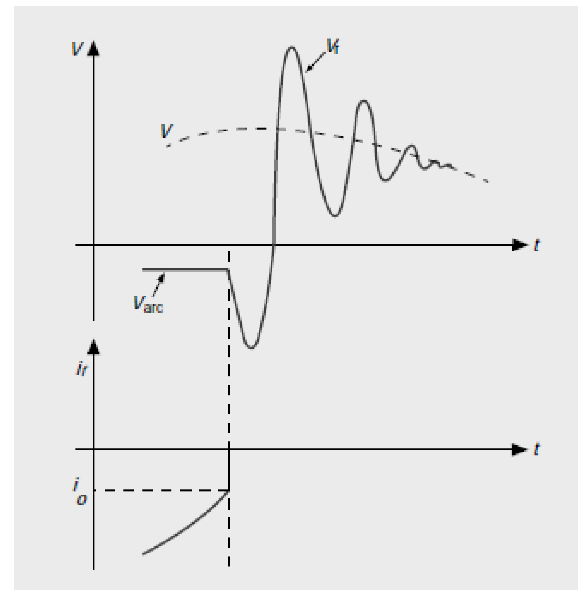


Figure 10.6 Voltage transient due to current chopping: i_f = fault current; v = system voltage, i_0 = current magnitude at chop, v_r = restriking voltage

Electric Power Systems, 5th ed., Weedy et al.

Question 3

$$u_r = i_0\sqrt{\frac{L}{C}}\sin\omega_0 t = 100\mathrm{A}\times\sqrt{\frac{2\times10^{-3}}{500\times10^{-12}}}\sin\frac{t}{\sqrt{2\times10^{-3}\times500\times10^{-12}}}$$
$$= 200\,\mathrm{kV}\,\sin\left(10^6 t\right)$$

This is the overvoltage transient that oscillates in the disconnected system behind the opened circuit breaker. The stress over the breaker is the difference of this voltage and power system voltage at the breaker terminals.

Question 4

A long overhead line has a surge impedance of 500Ω and an effective resistance at the frequency of the surge of 7Ω/km. If a surge of magnitude 500kV enters the line at a certain point, calculate the magnitude of this surge after it has traversed 100km and calculate the resistive power loss of the wave over this distance. The wave velocity is 3×10^5 km/s.

Question 4

Considering the power losses (dp) over a length (dx), where resistance and shunt are R(Ω) and G(Ω^{-1})

$$dp = i^2 Rdx + (u-du)^2 Gdx \approx i^2 Rdx + (i^2 {Z_0}^2 - 2i{Z_0}^2 di)Gdx = i^2 Rdx, \quad G \approx 0$$

incoming power :

$$p = ui = i^2 Z_0$$

power at the end :

$$p_l = (u-du)(i-di) = (i-di)^2 Z_0 \approx i^2 Z_0 - 2iZ_0 di$$

Therefore, power loss dp:

$$dp = p_l - p = -2iZ_0 di = i^2 Rdx$$

Electric Power Systems, 5th ed., Weedy et al.

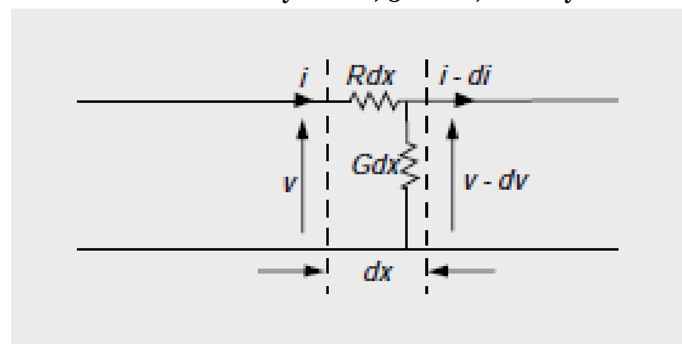


Figure 10.27 Lossy line section (L and C are not shown)

$$\Leftrightarrow \frac{di}{i} = -\frac{1}{2}\left(\frac{R}{Z_0}\right)dx \xrightarrow{\text{integral}} \ln i = -\frac{1}{2}\left(\frac{R}{Z_0}\right)x + C \quad \Leftrightarrow e^{\ln i} = e^{\frac{1}{2}\left(\frac{R}{Z_0}\right)x+C}$$

$$\Leftrightarrow i = e^C e^{-\frac{1}{2}\left(\frac{R}{Z_0}\right)x} \Leftrightarrow \quad i = i_0 e^{-\frac{1}{2}\left(\frac{R}{Z_0}\right)x}$$

Question 4

also voltage drops according to the equation :

$$u_x = u_0 e^{-\frac{1}{2}\left(\frac{R}{Z_0}\right)x}$$

Calculating magnitude of a surge at x=100 km

$$\boxed{u_{100} = u_0 e^{-\frac{1}{2}\left(\frac{R}{Z_0}\right)x} = 500 \times e^{-\frac{1}{2}\left(\frac{7}{500}\right)100} = \underline{\underline{248.3\text{kV} \approx 250\text{kV}}}}$$

Calculating resistive power loss:

$$i_{0\text{km}} = i_0 = \frac{u_0}{Z_0} = \frac{500 \times 10^3}{500} A = 1000A$$

$$i_{100\text{km}} = i_{100} = \frac{u_{100}}{Z_0} = \frac{250 \times 10^3}{500} \text{A} \approx 500\text{A}$$

$$\boxed{P_{\text{loss}} = P_{0\text{km}} - P_{100\text{km}} = u_0 i_0 - u_{100} i_{100} \approx 500\text{MW} - 125\text{MW} = \underline{\underline{375\text{MW}}}}$$