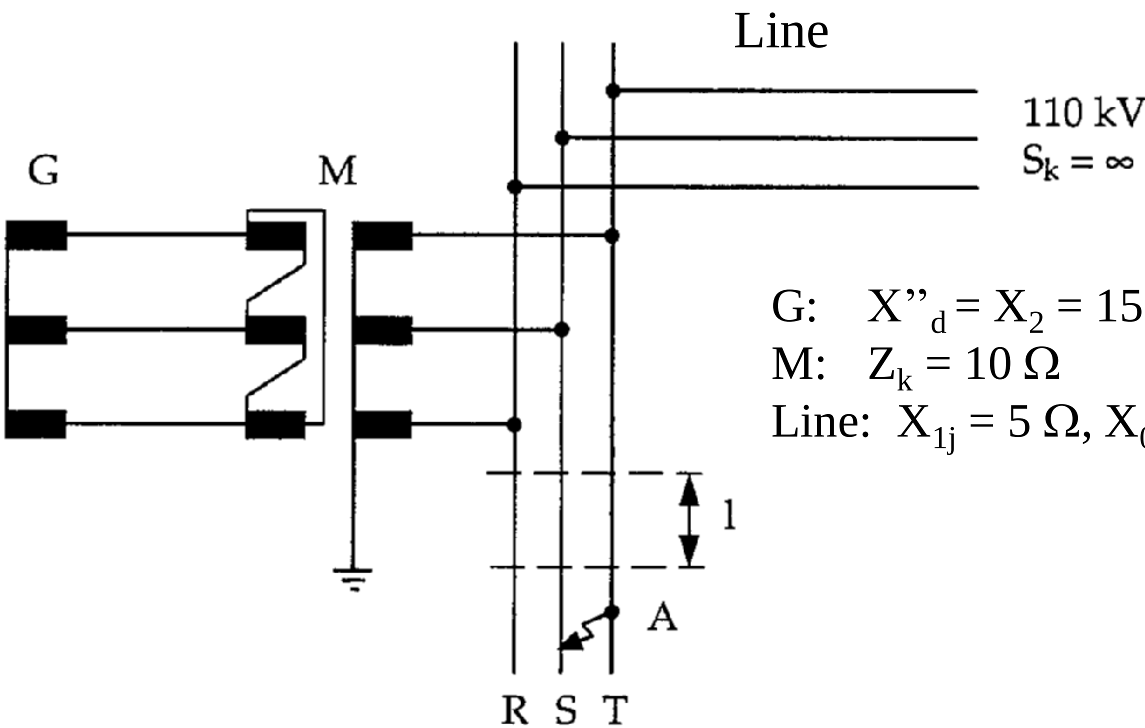
Aalto-yliopisto
Teknillinen korkeakoulu

Exercise 9

Power systems

Question 1



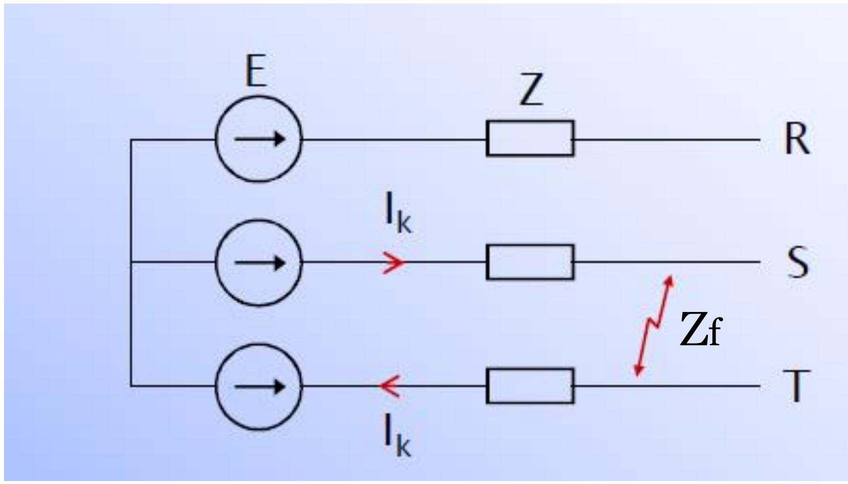
G: $X''_d = X_2 = 15\ \Omega$, $X_0 = 10\ \Omega$
M: $Z_k = 10\ \Omega$
Line: $X_{1j} = 5\ \Omega$, $X_{0j} = 16\ \Omega$

Voltage C Factor table		
Voltage Level	Cmax	Cmin
Low Voltage (< 1 kV)	1.05	0.95
High Voltage (> 1 kV)	1.1	1

http://help.easypower.com/ezp/9.6/content/06_IEC_Short_Circuit/Setting_the_Short_Circuit_Method.htm

A two-phase zero-impedance fault occurs at point A. The distance between phases in the bus bar system is 2.5 m. **Calculate the maximum peak force affecting each phase (per length) in area 1**. Apply the IEC recommended voltage correction factor (C factor) to calculate the maximum short circuit current.

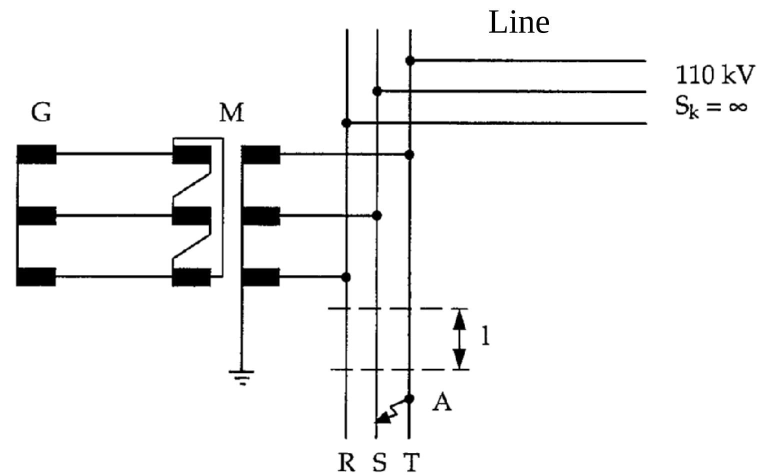
Question 1



$I_R = 0$

$I_T = - I_S$

$U_S = U_T$
$(Z_f = 0)$



$\underline{a} = 1\angle 120°$

Zero
Positive
Negative

$$\begin{vmatrix} I_0 \\ I_1 \\ I_2 \end{vmatrix} = \frac{1}{3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{vmatrix} \begin{vmatrix} I_R \\ I_S \\ I_T \end{vmatrix} \quad \begin{matrix} 1. \\ 2. \\ 3. \end{matrix}$$

1. $\Rightarrow \quad I_0 = 0$

2. & 3. & $I_T = -I_S \quad \Rightarrow \quad I_2 = -I_1$

Voltage source is symmetric:

$\Rightarrow \quad E_1 = E_R \ ; \quad E_2 = 0 \ ; \quad E_0 = 0$

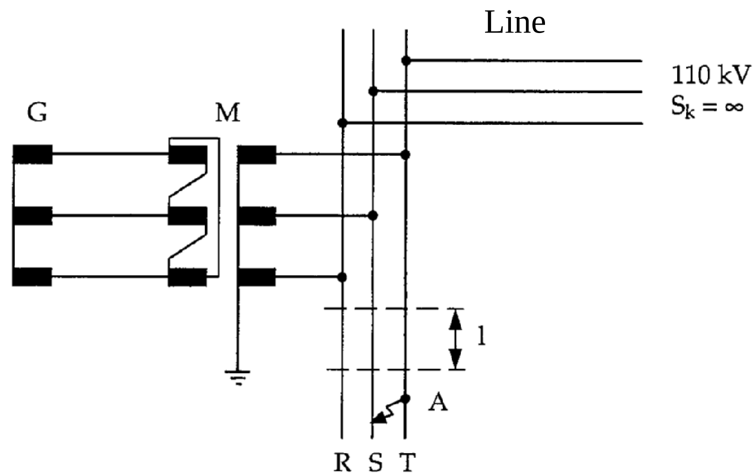
$$\begin{vmatrix} \underline{U}_R \\ \underline{U}_S \\ \underline{U}_T \end{vmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} \begin{bmatrix} \underline{U}_0 \\ \underline{U}_1 \\ \underline{U}_2 \end{bmatrix}$$

$$\underline{U}_S = \underline{U}_0 + \underline{a}^2 \underline{U}_1 + \underline{a} \underline{U}_2$$

$$\underline{U}_T = \underline{U}_0 + \underline{a} \underline{U}_1 + \underline{a}^2 \underline{U}_2$$

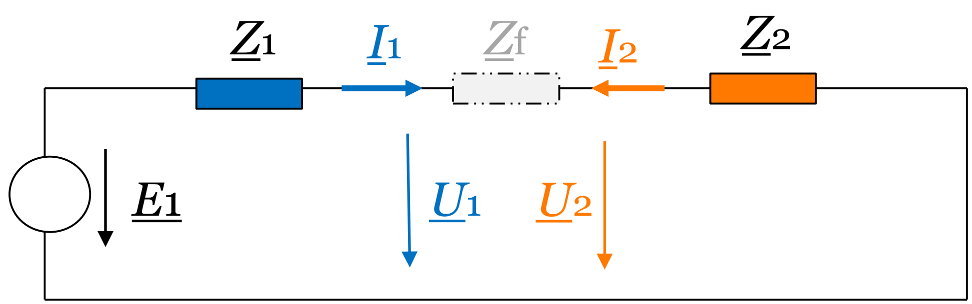
$$\underline{U}_S = \underline{U}_T \qquad \Rightarrow \underline{U}_1 = \underline{U}_2$$

Question 1

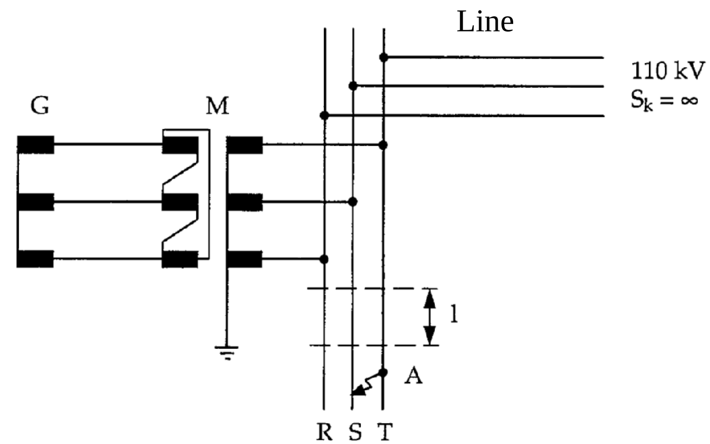


$$\begin{cases} \underline{U}_1 = \underline{U}_2 \\ \underline{I}_2 = -\underline{I}_1 \end{cases}$$

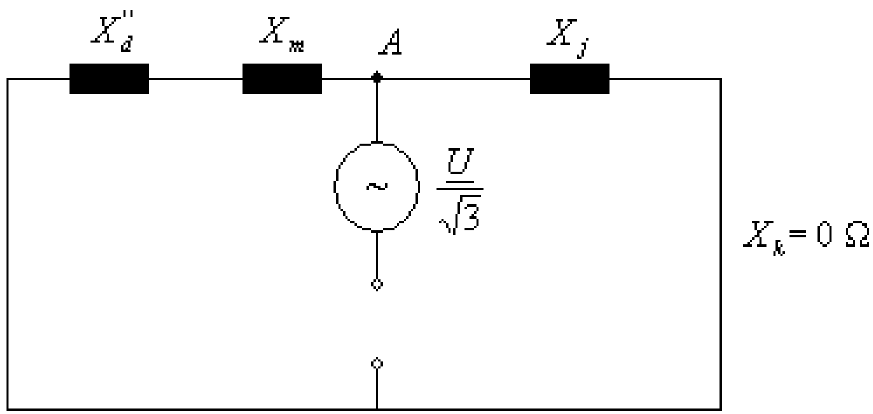
→ We can see that the voltages of the
positive and negative sequence are the same and
the currents are of the same magnitude but in different directions
→ We get the following equivalent circuit:



Question 1
Maximum peak force



Positive sequence network:



$$X_k = \frac{U^2}{S_k}$$

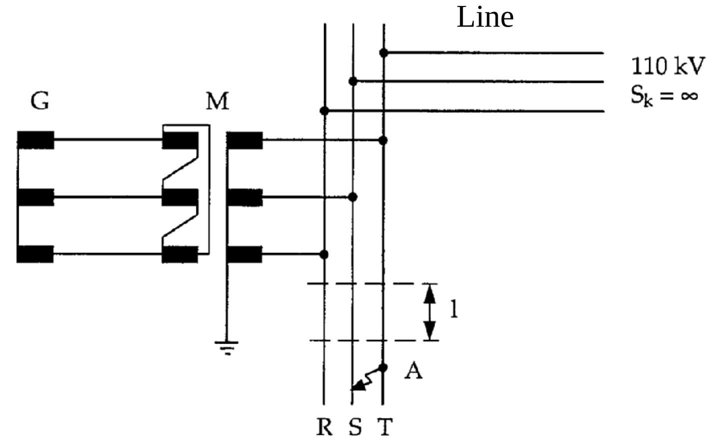
G: $X''_d = X_2 = 15\ \Omega$, $X_0 = 10\ \Omega$
M: $Z_k = 10\ \Omega$
Line: $X_{1j} = 5\ \Omega$, $X_{0j} = 16\ \Omega$

$$X_d'' = 15\ \Omega$$
$$X_m = Z_k = 10\ \Omega$$
$$X_j = 5\ \Omega$$

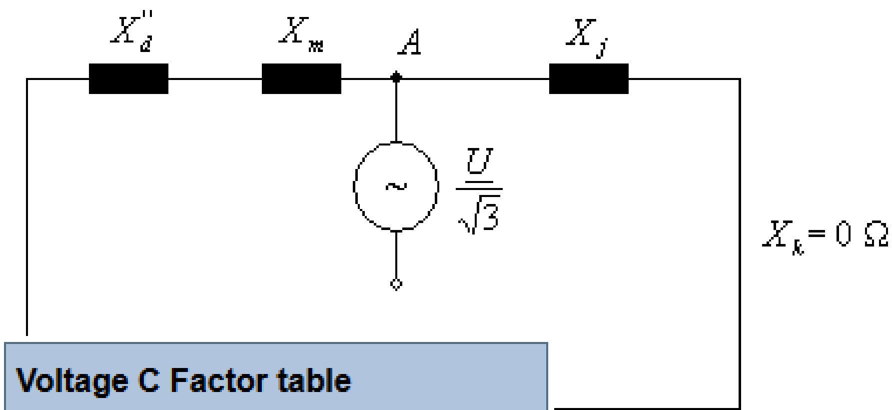
$$X_1 = \frac{\left(X_d'' + X_m\right)\cdot X_j}{X_d'' + X_m + X_j} = \frac{(15+10)\cdot 5}{5+10+5}\ \Omega = 4.17\ \Omega$$

Question 1

Maximum peak force



Positive sequence network:



$$X_k = \frac{U^2}{S_k}$$

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http://help.easypower.com/ezp/9.6/content/06_IEC_Short_Circuit/Setting_the_Short_Circuit_Method.htm

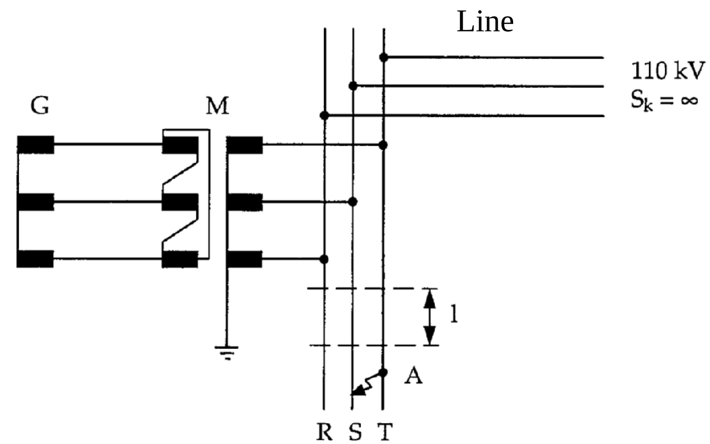
$$X_1 = \frac{\left(X_d'' + X_m\right)\cdot X_j}{X_d'' + X_m + X_j} = \frac{(15+10)\cdot 5}{5+10+5}\ \Omega = 4.17\ \Omega$$

$$U = c \cdot U_N = \boxed{1.10}\ 110\ \text{kV} = 121\ \text{kV}$$

$$\underline{U} = U\,\underline{/0^\circ}$$

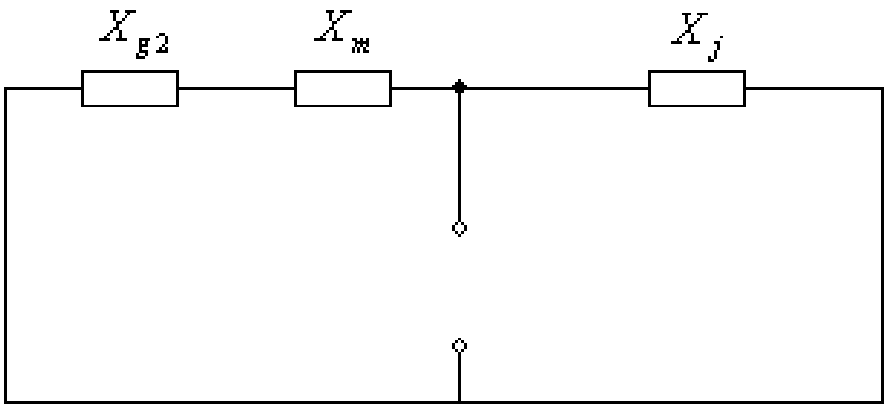
Question 1

Maximum peak force



G: $X''_d = X_2 = 15\ \Omega$, $X_0 = 10\ \Omega$
M: $Z_k = 10\ \Omega$
Line: $X_{1j} = 5\ \Omega$, $X_{0j} = 16\ \Omega$

Negative sequence network:



$$X_{g2} = 15\ \Omega$$

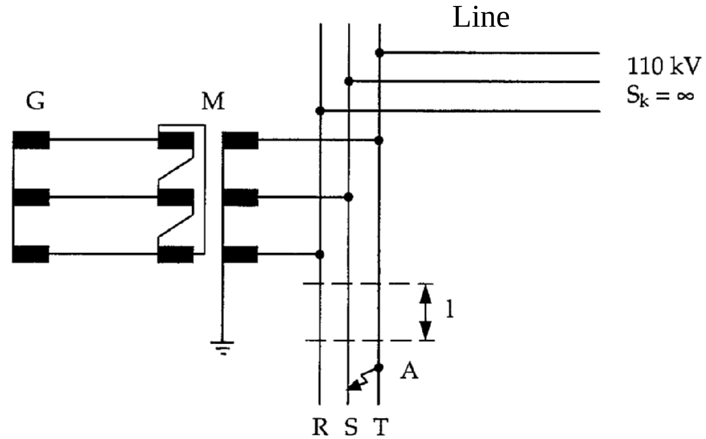
$$X_m = Z_k = 10\ \Omega$$

$$X_j = 5\ \Omega$$

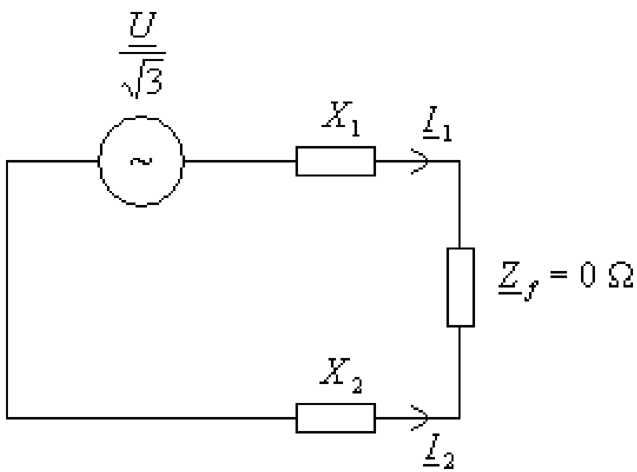
$$X_2 = \frac{(X_{g2} + X_m) \cdot X_j}{X_{g2} + X_m + X_j} = \frac{(15+10)\cdot 5}{5+10+5}\ \Omega = 4.17\ \Omega$$

Question 1

Maximum peak force



Two phase short circuit ($I_1 = -I_2$):



$$\underline{I}_1 = -\underline{I}_2 = \frac{\frac{U}{\sqrt{3}}\angle 0°}{j(X_1 + X_2) + \underline{Z}_f} = \frac{\frac{121\,\text{kV}}{\sqrt{3}}}{j(4.17 + 4.17)\,\Omega + 0\,\Omega} \approx 8.38\ \text{kA}\,\underline{/-90°}$$

$$\underline{a} = 1\angle 120°$$

$$\begin{vmatrix} \underline{I}_R \\ \underline{I}_S \\ \underline{I}_T \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{vmatrix} \begin{vmatrix} \mathrm{I}_0 \\ \mathrm{I}_1 \\ \mathrm{I}_2 \end{vmatrix}$$

$$\underline{I}_S = \underline{I}_0 + \underline{a}^2 \underline{I}_1 + \underline{a}\underline{I}_2 = 0 + (\underline{a}^2 - \underline{a}) \cdot \underline{I}_1$$

$$\underline{I}_T = \underline{I}_0 + \underline{a}\underline{I}_1 + \underline{a}^2 \underline{I}_2 = 0 + (\underline{a}^2 - \underline{a}) \cdot \underline{I}_2$$

$$\underline{a}^2 - \underline{a} = 1\underline{/240°} - 1\underline{/120°} = \sqrt{3}\,\underline{/-90°}$$

$$I_S = \sqrt{3} \cdot I_1 = \sqrt{3} \cdot 8.38\ \text{kA} = 14.52\ \text{kA}$$

Question 1

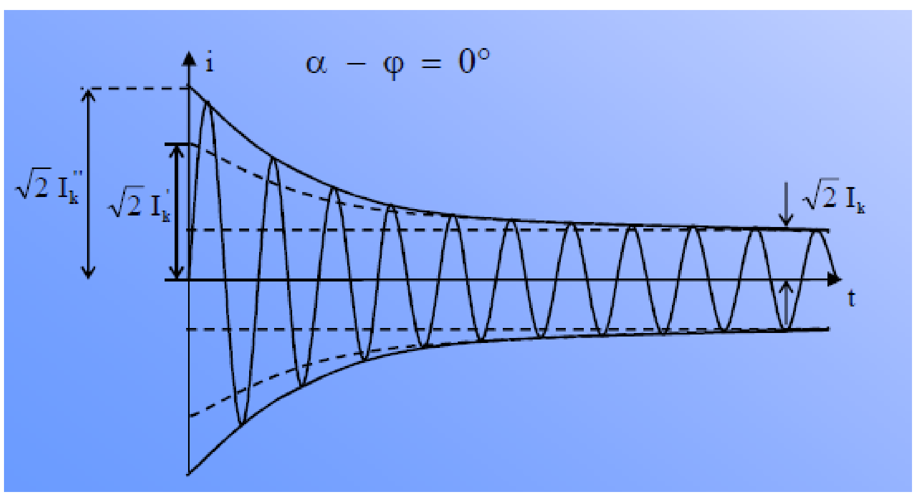
Maximum peak force

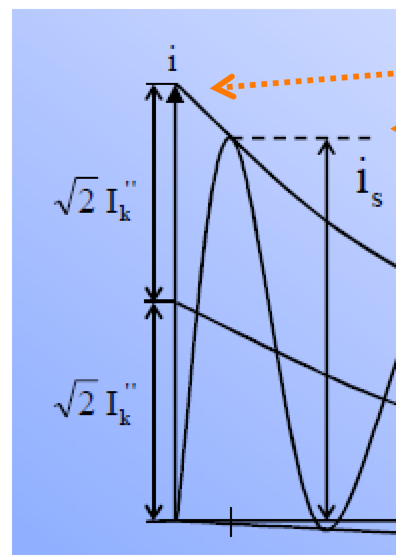
$$I_S = \sqrt{3} \cdot I_1 = \sqrt{3} \cdot 8.38 \text{ kA} = 14.52 \text{ kA}$$

Peak value of the alternating current: $\hat{i} = I_S \sqrt{2}$

In addition, there is a DC-component. If no attenuation, the amplitude of DC-component can be equal to the peak value of the alternating current component.

$$i_{\max} = \underline{2}\hat{i} = 2 I_S \sqrt{2} = 2 \cdot 14.52 \text{ kA} \cdot \sqrt{2} = 41.07 \text{ kA}$$



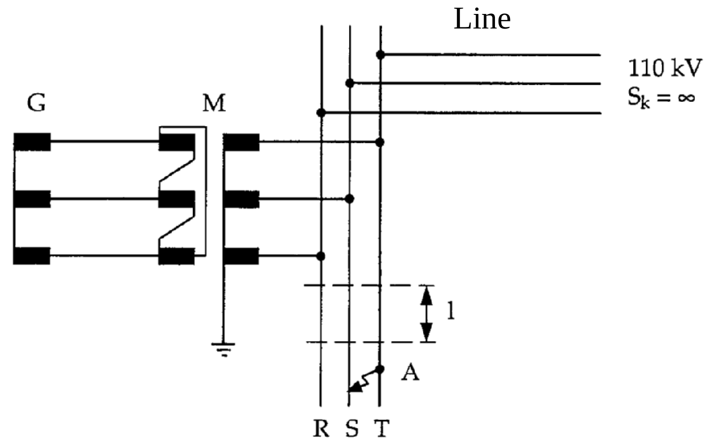


2

1.8 (used in real high-voltage systems which have losses, see lecture 9)

Question 1

Maximum peak force



$$i_{max} = 2\hat{i} = 2 I_S \sqrt{2} = 2 \cdot 14.52\ \text{kA} \cdot \sqrt{2} = 41.07\ \text{kA}$$

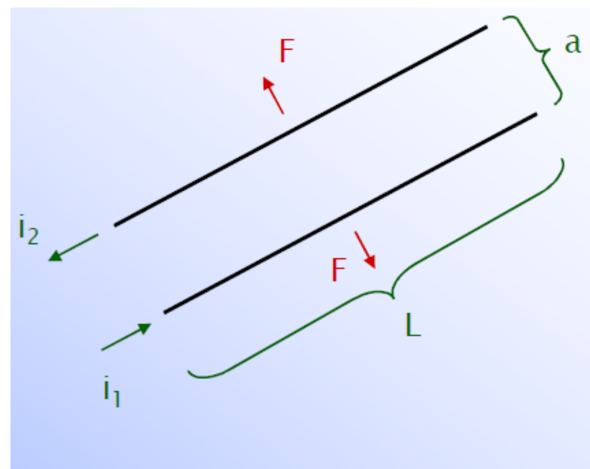
Max force is between lines S and T:

$$F = \frac{\mu_0}{2\pi} \frac{i_1\, i_2\, L}{a}$$

$$F_{max} = \frac{4\pi \cdot 10^{-7}}{2\pi} \cdot i_S \cdot i_T \cdot \frac{l}{a}$$

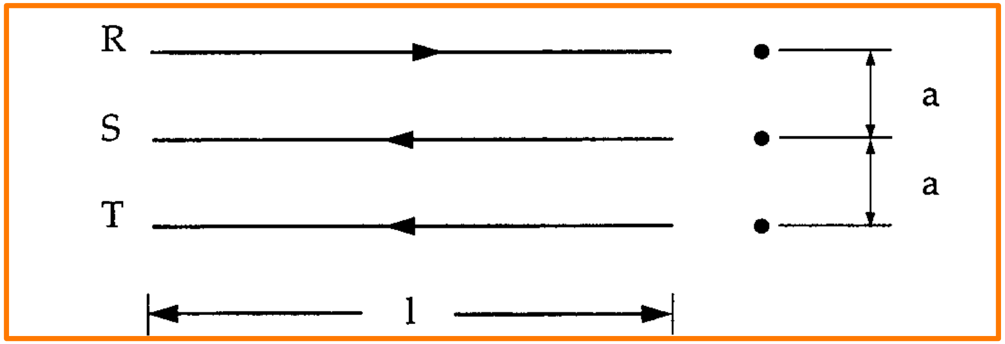
$$F_{max} = \frac{0.2}{10^6} \cdot i_{max}^2 \cdot \frac{l}{a}$$

$$\boxed{\underline{\underline{\frac{F_{max}}{l}}} = 0.2 \cdot \frac{i(\text{kA})_{max}^2}{a} = 0.2 \cdot \frac{(41.07\ \text{kA})^2}{2.5\ \text{m}} \frac{\text{N}}{\text{kA}^2} \approx \underline{\underline{134.9\ \frac{\text{N}}{\text{m}}}}}$$

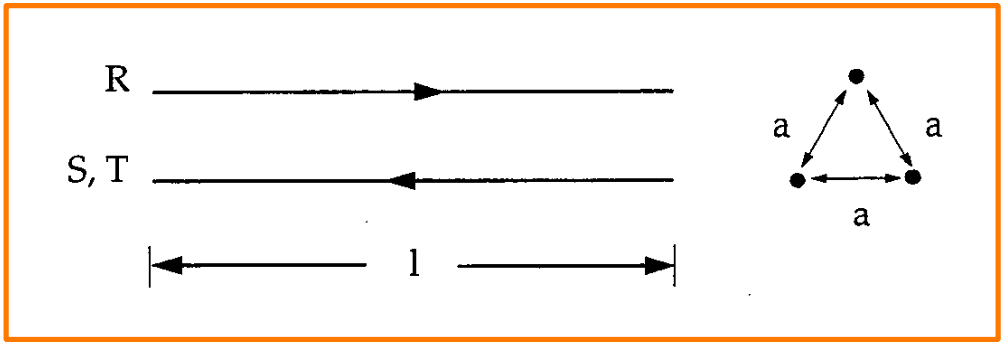


Question 2

a)



b)



$a = 0.3$ m

A short circuit occurs in a 24-kV bus bar system. The phase current instantaneous values are $i_R = 30$ kA, $i_S = 15$ kA and $i_T = 15$ kA.

Calculate the forces (per length) that affect each bus bar for the

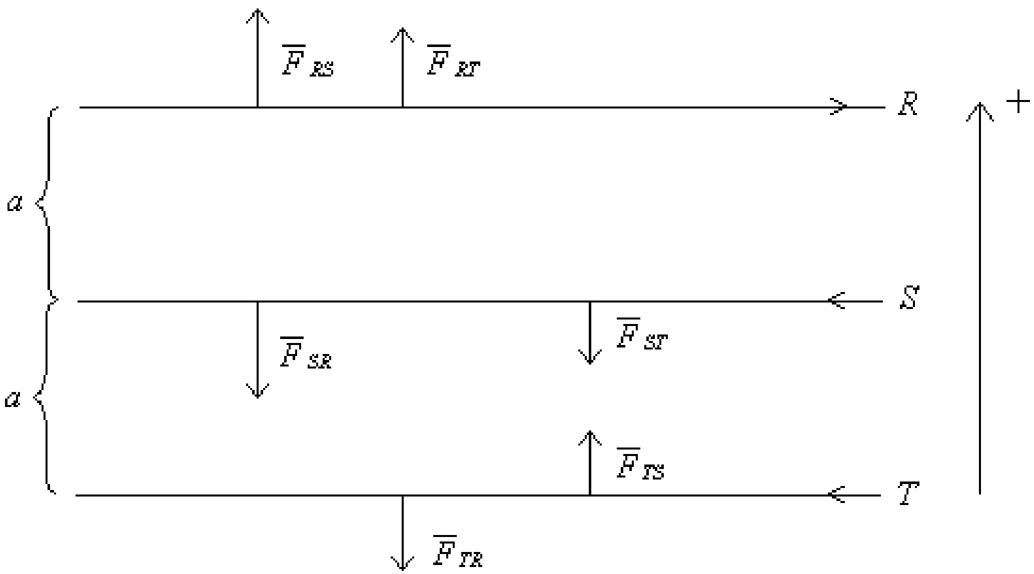
a) Upper system

b) Lower system

Question 2

a) forces per length in the upper system

$i_R = 30$ kA, $i_S=15$ kA and $i_T=15$ kA.



$$\boxed{a = 0.3\ \text{m}}$$

$$F = \frac{\mu_0}{2\pi}\frac{i_1\, i_2\, L}{a}$$

$$\boxed{F_{max} = \frac{0.2}{10^6}\cdot i_1 i_2 \cdot \frac{l}{a}}$$

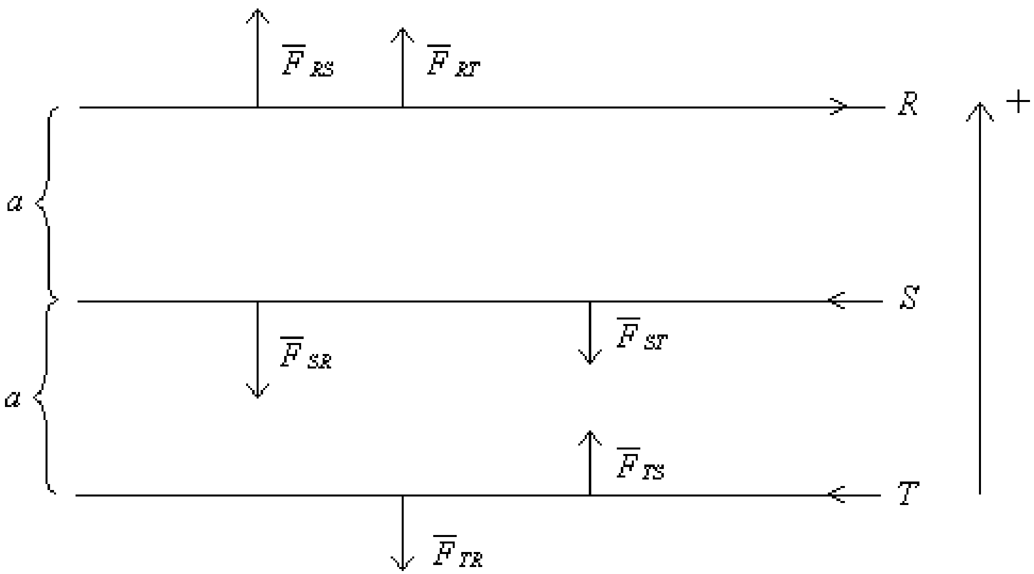
$$\frac{F_{RS}}{l} = \frac{F_{SR}}{l} = 0.2\cdot\frac{i_R i_S}{a} = 0.2\cdot\frac{30\cdot 15}{0.3\ \text{m}}\ \text{N} = \quad 300\ \frac{\text{N}}{\text{m}}$$

$$\frac{F_{RT}}{l} = \frac{F_{TR}}{l} = 0.2\cdot\frac{i_R i_T}{2a} = 0.2\cdot\frac{30\cdot 15}{2\cdot 0.3\ \text{m}}\ \text{N} = \quad 150\ \frac{\text{N}}{\text{m}}$$

$$\frac{F_{ST}}{l} = \frac{F_{TS}}{l} = 0.2\cdot\frac{i_S i_T}{a} = 0.2\cdot\frac{15\cdot 15}{0.3\ \text{m}}\ \text{N} = \quad 150\ \frac{\text{N}}{\text{m}}$$

Question 2

a) forces per length in the upper system



a=0.3 m

$$\underline{\underline{\frac{\overline{F}_R}{l}}} = \frac{\left(\overline{F}_{RS} + \overline{F}_{RT}\right)}{l} = (300+150)\,\frac{\mathrm{N}}{\mathrm{m}} = \underline{\underline{450\,\frac{\mathrm{N}}{\mathrm{m}}}}$$

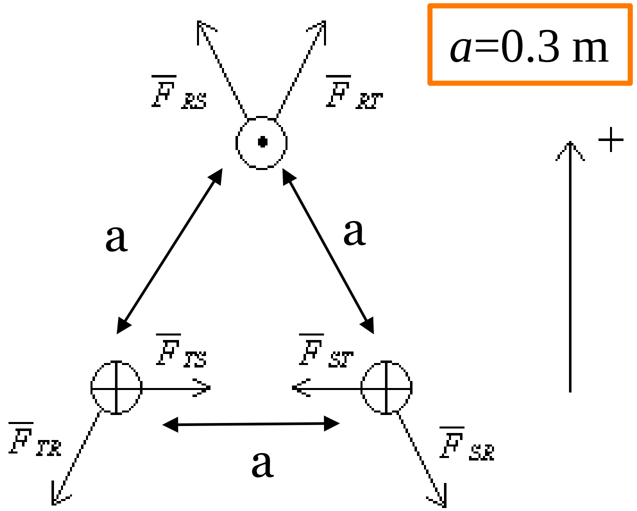
$$\underline{\underline{\frac{\overline{F}_S}{l}}} = \frac{\left(\overline{F}_{SR} + \overline{F}_{ST}\right)}{l} = (-300-150)\,\frac{\mathrm{N}}{\mathrm{m}} = \underline{\underline{-450\,\frac{\mathrm{N}}{\mathrm{m}}}}$$

$$\underline{\underline{\frac{\overline{F}_T}{l}}} = \frac{\left(\overline{F}_{TR} + \overline{F}_{TS}\right)}{l} = (-150+150)\,\frac{\mathrm{N}}{\mathrm{m}} = \underline{\underline{0\,\frac{\mathrm{N}}{\mathrm{m}}}}$$

Question 2

b) forces per length in the lower system

i_R= 30 kA, i_S=15 kA and i_T=15 kA.

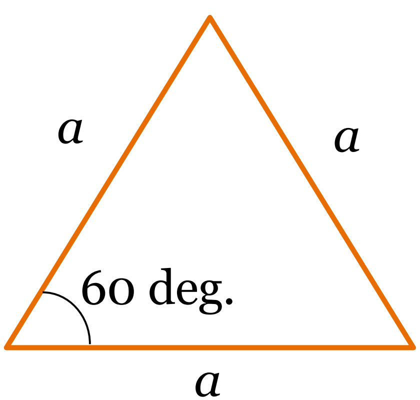


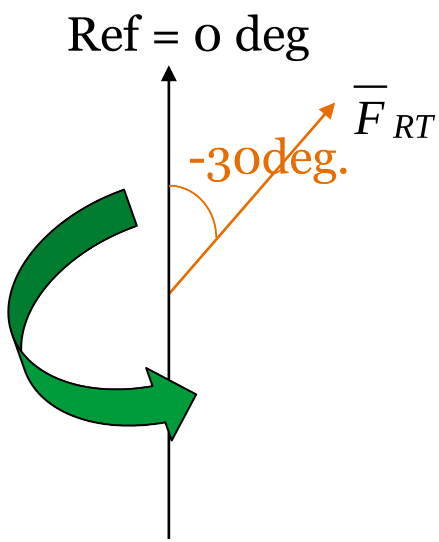
a=0.3 m

$$\frac{F_{RS}}{l} = \frac{F_{SR}}{l} = 0.2 \cdot \frac{i_R i_S}{a} = 0.2 \cdot \frac{30 \cdot 15}{0.3\ \mathrm{m}}\ \mathrm{N} = \quad 300\ \frac{\mathrm{N}}{\mathrm{m}}$$

$$\frac{F_{RT}}{l} = \frac{F_{RT}}{l} = 0.2 \cdot \frac{i_R i_T}{a} = 0.2 \cdot \frac{30 \cdot 15}{0.3\ \mathrm{m}}\ \mathrm{N} = \quad 300\ \frac{\mathrm{N}}{\mathrm{m}}$$

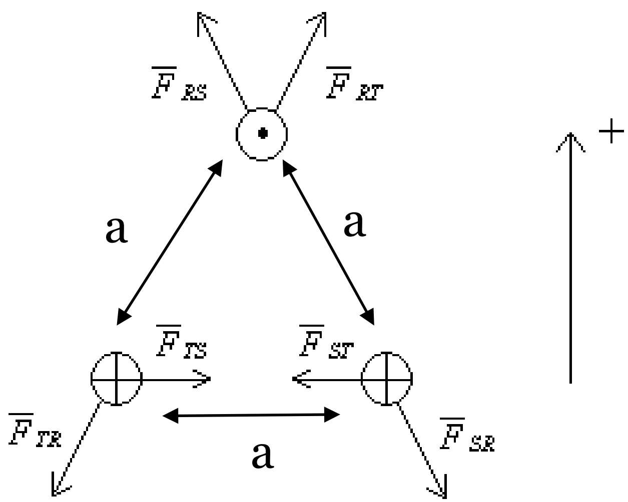
$$\frac{F_{ST}}{l} = \frac{F_{TS}}{l} = 0.2 \cdot \frac{i_S i_T}{a} = 0.2 \cdot \frac{15 \cdot 15}{0.3\ \mathrm{m}}\ \mathrm{N} = \quad 150\ \frac{\mathrm{N}}{\mathrm{m}}$$

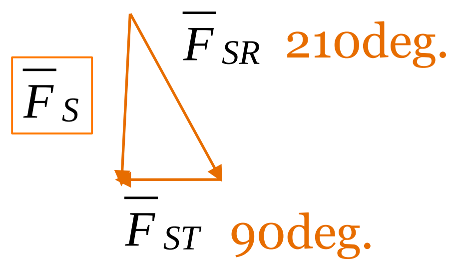


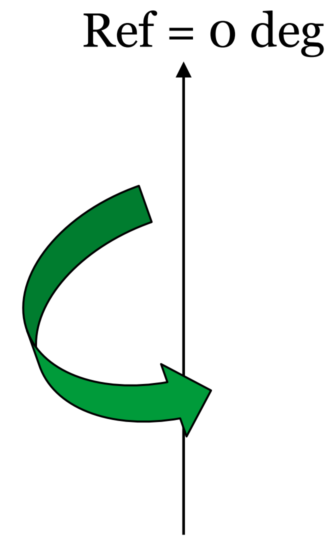


Question 2

b) forces per length in the lower system







$$\boxed{\underline{\underline{\frac{\overline{F}_R}{l}}} = \frac{\left(\overline{F}_{RS}+\overline{F}_{RT}\right)}{l} = \left(300\angle\underline{30^\circ} + 300\angle\underline{-30^\circ}\right)\frac{\text{N}}{\text{m}} = \underline{\underline{519.6\ \frac{\text{N}}{\text{m}}}}}$$

Directly up

$$\boxed{\underline{\underline{\frac{\overline{F}_S}{l}}} = \frac{\left(\overline{F}_{SR}+\overline{F}_{ST}\right)}{l} = \left(300\angle\underline{210^\circ} + 150\angle\underline{90^\circ}\right)\frac{\text{N}}{\text{m}} = \underline{\underline{-259.8\ \frac{\text{N}}{\text{m}}}}}$$

Directly down

$$\boxed{\underline{\underline{\frac{\overline{F}_T}{l}}} = \frac{\left(\overline{F}_{TR}+\overline{F}_{TS}\right)}{l} = \left(300\angle\underline{150^\circ} + 150\angle\underline{-90^\circ}\right)\frac{\text{N}}{\text{m}} = \underline{\underline{-259.8\ \frac{\text{N}}{\text{m}}}}}$$

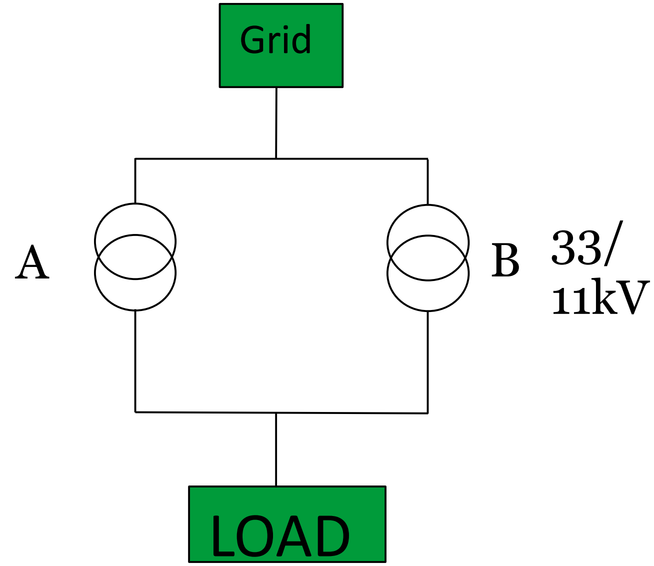
Directly down

Question 3

Two identical transformers each have a nominal or no-load ratio of 33/11 kV and a reactance of 2 Ω referred to the 11-kV side; resistance may be neglected. The transformers operate in parallel and supply a load of 9 MVA, 0.8 p.f. lagging. **Calculate the current taken by each transformer when they operate five tap steps apart (each step is 1.25 per cent of the nominal voltage).**

Question 3

Current by each transformer



S = 9 MVA, cos φ = 0.8ind

Let's select load side as a base reference:

$$Z_b = \frac{U_b{}^2}{S_b} = \frac{\left(11\times10^3\right)^2}{9\times10^6} = 13.44\Omega$$

$$I_B = \frac{S_B}{\sqrt{3}\times U_B} = \frac{9\times10^6}{\sqrt{3}\times11\times10^3} = 472.4\text{A}$$

Transformer reactance in per unit:

$$x_t = \frac{X_t}{Z_b} = \frac{j2\Omega}{13.44\Omega} = j0.149\,pu$$

Load current in per unit:

$$i_L = \left(\frac{s\angle\arccos(0.8)}{u\angle0^\circ}\right)^* = \frac{1}{1}\angle-36.87^\circ = (0.8-j0.6)\,pu$$

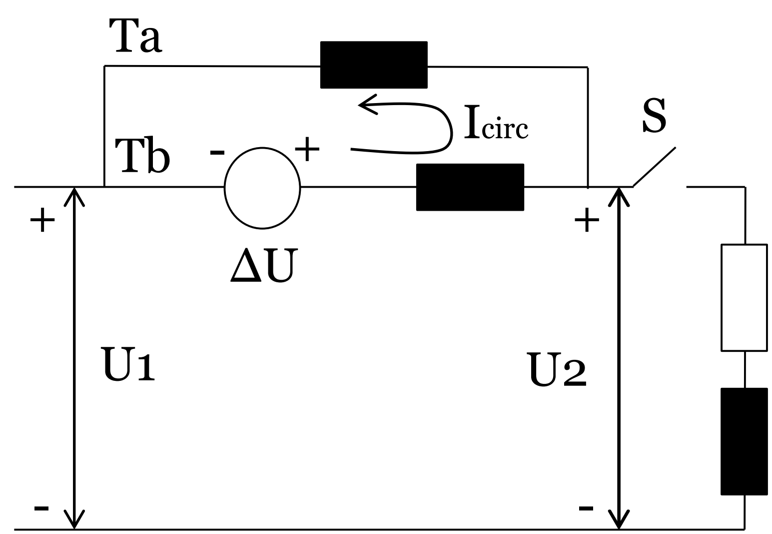
Question 3

Current by each transformer

An approximate solution is to use this equivalent circuit, where ΔU is the regulating transformer to accommodate the voltage tap change in the second transformer. This voltage creates a circulating current I_{circ} .

With switch ***S*** closed, only a very small fraction of that current goes through the load impedance, because it is much larger than the transformers impedance, then superposition principle is applied to ΔU and the source voltage. With ΔU short-circuited, the current in each path is half the load current. Then we just need to superimpose the circulating current.

Equivalent circuit



Question 3

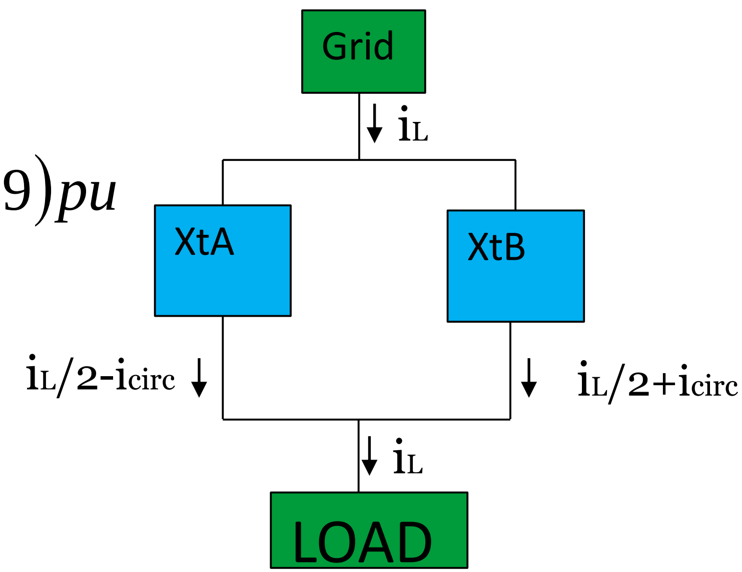
Current by each transformer, when 5 voltage taps of 1.25%

Voltage difference due to tap setting creates a current circulating through the parallel transformers:

$$\underline{i}_{circ.} = \frac{\Delta \underline{U}}{\underline{Z}} = \frac{5 \cdot 0.0125}{2 \cdot x_t} = \frac{0.0625}{j0.298} = -j0.210\,pu$$

$$\underline{i}_{Ta} = \frac{1}{2}(0.8 - j0.6) - (-j0.210) = (0.4 - j0.09)\,pu$$

$$= 0.41\angle -12.68°\,pu$$



$$\boxed{I_{Ta} = 0.41 \times 472.4A = \underline{\underline{194A}}}$$

$$\boxed{I_{Tb} = \left[\frac{1}{2}(0.8 - j0.6) - j0.210\right] \times 472.4\text{A} = \underline{\underline{306\text{A}}}}$$

Question 4

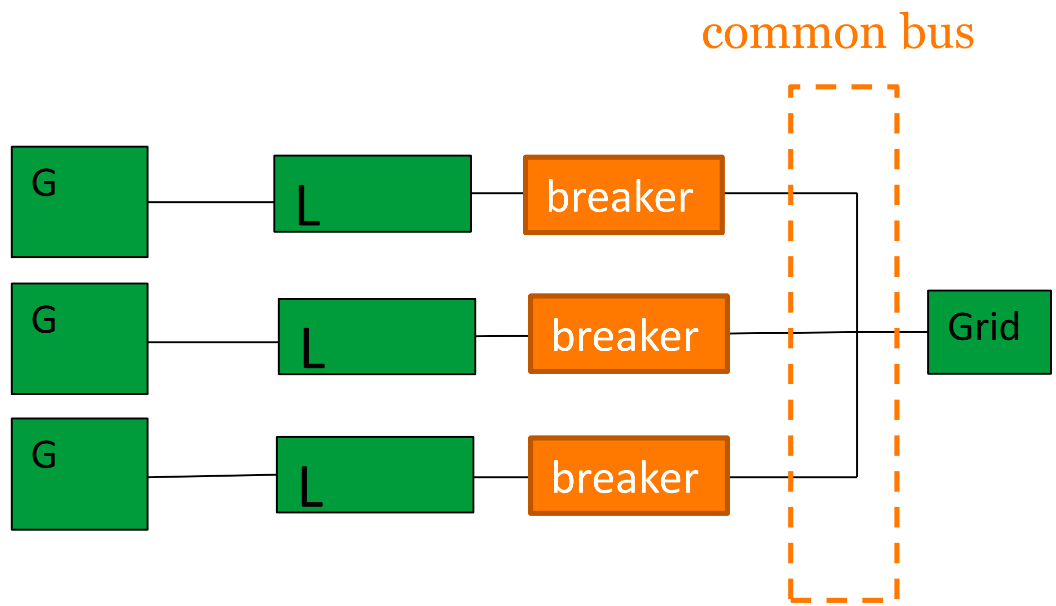
Three 11-kV, 100-MVA generators are connected to common busbars. Each is connected via a 100-MVA inductor and an identical circuit breaker. The inductors have reactances of 0.15pu, 0.20pu and 0.30pu.

If the generators each have a transient reactance of 0.25pu, **what is the minimum circuit-breaker rating to protect the generators against a fault on the common busbars**?

Question 4

Inductor reactances 0.15pu, 0.20pu and 0.30pu.

100-MVA generators each have a transient reactance of 0.25pu



G_1: $x_T = (0.25 + 0.15)\,pu = 0.4\,pu$

$$S_{SC} = \frac{S_B}{x_T} = \frac{100\text{MVA}}{0.4} = 250\text{MVA}$$

G_2: $x_T = (0.25 + 0.2)\,pu = 0.45\,pu$

$$S_{SC} = \frac{100\text{MVA}}{0.45} = 222\text{MVA}$$

G_3: $x_T = (0.25 + 0.3)\,pu = 0.55\,pu$

$$S_{SC} = \frac{100\text{MVA}}{0.55} = 182\text{MVA}$$

Therefore, the minimum circuit-breaker rating to protect the generators from common busbar fault, since they were said to be all identical, is **250 MVA**.