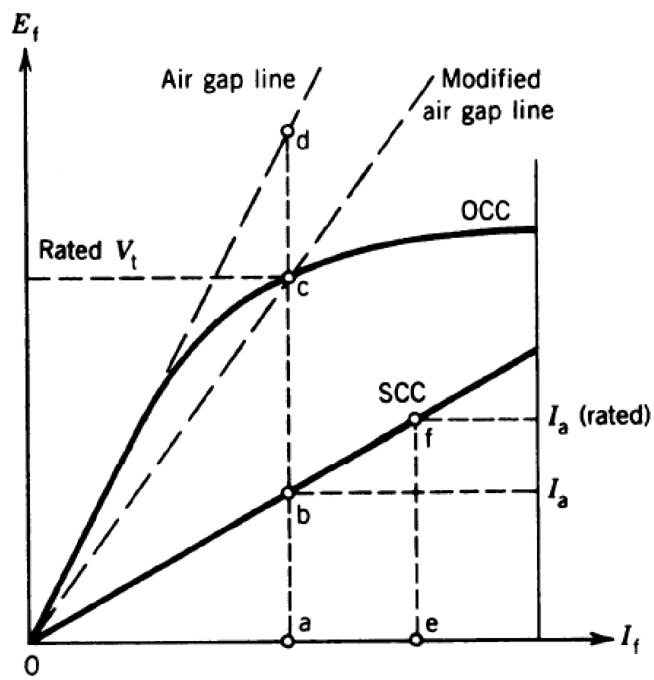
1)

a)

Open-circuit test:

I_f (A)	150	300	450	600	750	900	1200
U_l (kV)	3.75	7.5	11.2	13.6	15	15.8	16.5

Short-circuit test: I_f=750A, I_a=7000A.



- To draw the open circuit curve we will use the measured values for I_f and U_l .
- To draw the short circuit curve we will use the slope of the curve since we know the coordinates of point b that is (750,7000). The slope is: $m = 7000/750 = 9.33$. From the equation below, we can find the analytical equation of the line since we know the slope and coordinates of a point on the line.

$$y - y_1 = m(x - x_1)$$

$$y - 7000 = 9.33(x - 750)$$

$$y = 9.33x$$

Where x is the measured points of I_f.

▪ To draw Air-gap line we will use linear interpolation from the test results. The slope of the curve can be found from the first measured test values as:

$$m = 3.75 \cdot 10^3 / 150 = 25$$

$$y - y_1 = m(x - x_1)$$

$$y - 3750 = 25(x - 150)$$

$$y = 25x$$

Values of the y-axis for the line can be found as:

$$I_f(1:7) \cdot 25 = [\ 3.75 \quad 7.5 \quad 11.25 \quad 15 \quad 18.75 \quad 22.5 \quad 30] \cdot 10^3$$

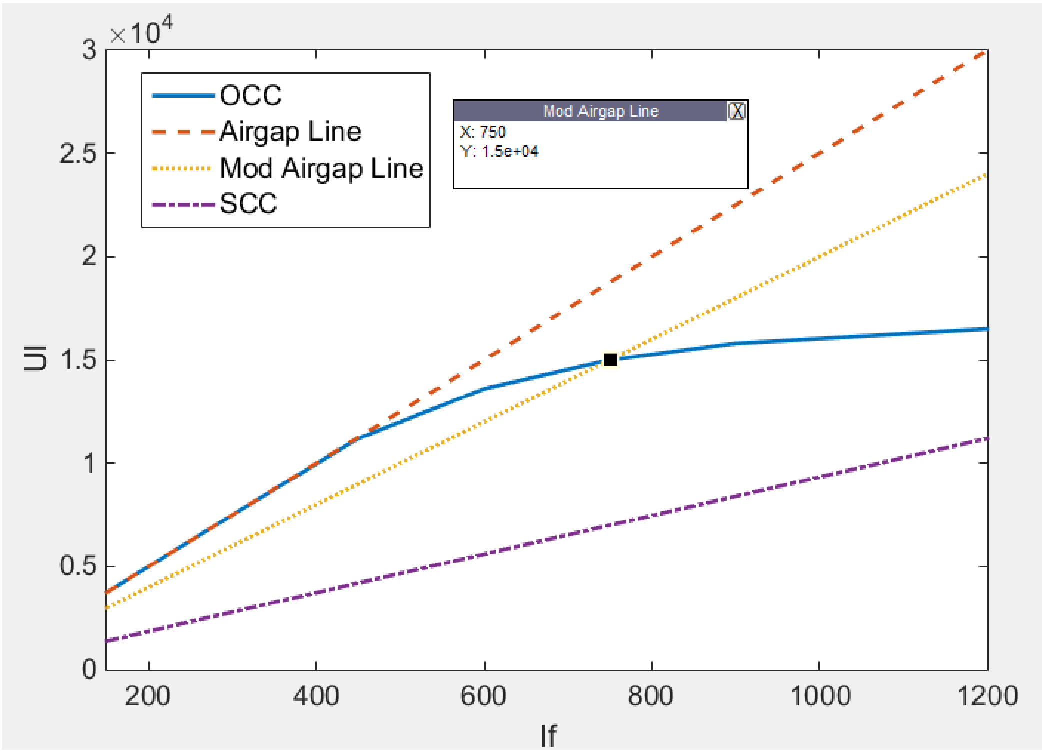
Then we can draw the line. Airgap line represents the unsaturated operation of the machine.

▪ To draw the modified air-gap curve first we need to find the coordinates of point c since we know the point a that is (750,0). If we place this in the open circuit test curve we find the coordinates of point c as (750,15000). Therefore, slope is $m = 15000/750 = 20$. From the equation below we can find the analytical equation of the line.

$$y - y_1 = m(x - x_1)$$

$$y - 15000 = 20(x - 750)$$
$$y = 20x$$

Using this expression and I_f values we can draw the line for the modified airgap.



b)

$$\dot{Z}_s = \cancel{R_a} + jX_s = jX_s$$

$$X_{s(unsat)} = \frac{E_{DA}}{I_{BA}} = \frac{18750}{\sqrt{3}\cdot 7000} = 1.546\Omega$$

$$X_{s(sat)} = \frac{E_{CA}}{I_{BA}} = \frac{15000}{\sqrt{3}\cdot 7000} = 1.237\Omega$$

$$U_b = \frac{15000}{\sqrt{3}} = 8660V$$

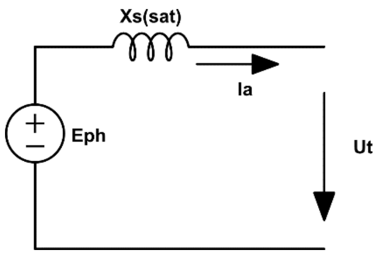
$$I_b = \frac{195\cdot 10^6}{\sqrt{3}\cdot 15\cdot 10^3} = 7506A$$

$$Z_b = \frac{U_b}{I_b} = \frac{8660}{7506} = 1.154\Omega$$

$$X_{s(unsat),\text{pu}} = \frac{X_{s(unsat)}}{Z_b} = 1.34\,pu$$

$$X_{s(sat),\text{pu}} = \frac{X_{s(sat)}}{Z_b} = 1.07\,pu$$

c)



$$S = 100MVA$$
$$\cos\varphi = 0.8 \text{ leading}$$
$$\varphi = 36.87°$$

$$I_a = \frac{S}{\sqrt{3}U_t} = \frac{100\cdot 10^6}{\sqrt{3}\cdot 15\cdot 10^3} = 3849A$$

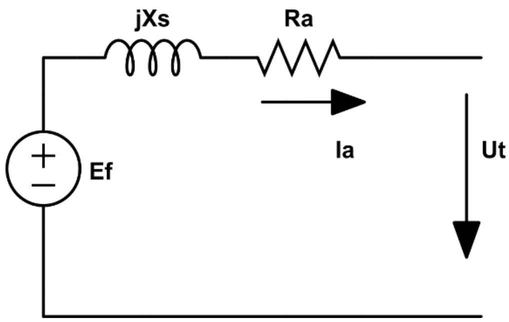
$$\dot{I}_a = 3849\angle 36.87°A$$

$$\dot{U}_T = \frac{15000}{\sqrt{3}}\angle 0° = 8660\angle 0°V$$

$$\dot{E}_{ph} = \dot{U}_T + jX_s\dot{I}_a = 8660 + j1.237 \cdot 3808.97 = 6942\angle 33.23°V$$

$$\dot{E}_{ph} = \frac{15000}{750} I_f \Rightarrow I_f = \frac{750}{15000} \cdot 6942 \cdot \sqrt{3} = 601.2A$$

2)



$U_l = 14kV$
$S = 10MVA$
$\cos\varphi = 0.85$
$\varphi = 31.79°$
$X_s = 20\Omega$
$R_a = 2\Omega$

a)

$$U_{ph} = \frac{U_l}{\sqrt{3}} = \frac{14 \cdot 10^3}{\sqrt{3}} = 8083V$$

$$I_a = \frac{S_n}{\sqrt{3} \cdot U_l} = \frac{10 \cdot 10^6}{\sqrt{3} \cdot 14 \cdot 10^3} = 412.4A$$

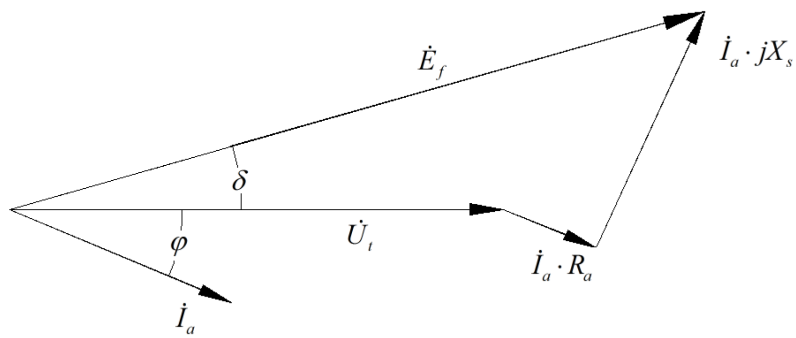
$$\dot{I}_a = 412.4\angle -31.76°$$

$$\dot{E}_f = \dot{U}_t + \dot{I}_a \cdot R_a + \dot{I}_a \cdot jX_s$$

$$\dot{E}_f = 8083\angle 0° + 412.4\angle -31.79° \cdot 2 + 412.4\angle -31.79° \cdot j20$$

$$\dot{E}_f = 13155.52 + j6573.66 = 14706\angle 26.6°V$$

$$\delta = 26.6°$$



b)

$$T = \frac{3}{w_s} \cdot \frac{|E_f||U_t|}{|X_s|} \sin\delta$$

$$n_s = \frac{60 f}{p} = \frac{60 \cdot 60}{1} = 3600 rpm$$

$$f_s = \frac{n_s}{60} = 60 Hz$$

$$w_s = 2\pi f = 377 rad/s$$

$$T = \frac{3}{377} \cdot \frac{14706 \cdot 8083}{20} \cdot \sin 26.6° = 21.18 kNm$$

c)

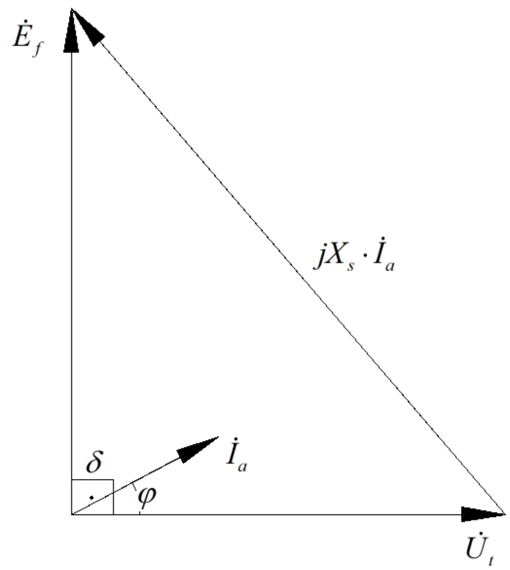
$$P_{\max} = 3 \frac{|E_f| \cdot |U_t|}{|X_s|} = 3 \frac{14706 \cdot 8083}{20} = 17.83 MW$$

d)

$$\dot{I}_a = \frac{\dot{E}_f - \dot{U}_t}{jX_s} = \frac{14706\angle 90° - 8083\angle 0°}{j20}$$

$$\dot{I}_a = 735 + j404 = 838.8\angle 28.8° A$$

$$\varphi = 28.8° \qquad \cos\varphi = 0.876 \text{ leading}$$



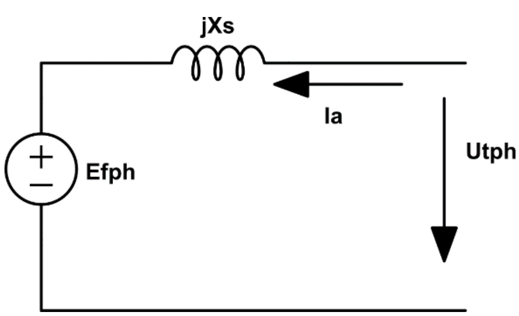
3) We are given :

$S = 10MVA$
$U_l = 2300V$
$f = 60Hz$
$X_s = 0.9\,pu$
$U_t = 2300V$
$E_f = 3450\angle -20°V$

a)

$$\delta = -20° < 0 - \text{Motor}$$

b)



$$E_{fph} = \frac{3450}{\sqrt{3}} = 1991.86V$$

$$U_{tph} = \frac{2300}{\sqrt{3}} = 1327.9V$$

$$I_l = I_{ph} = \frac{S}{\sqrt{3}U_l} = \frac{10 \cdot 10^6}{\sqrt{3} \cdot 2300} = 2510.2A$$

$$Z_b = \frac{U_{tph}}{I_{ph}} = \frac{1327.9}{2510.2} = 0.53\Omega$$

$$X_s = X_{spu} \cdot Z_b = 0.9 \cdot 0.53 = 0.477\Omega$$

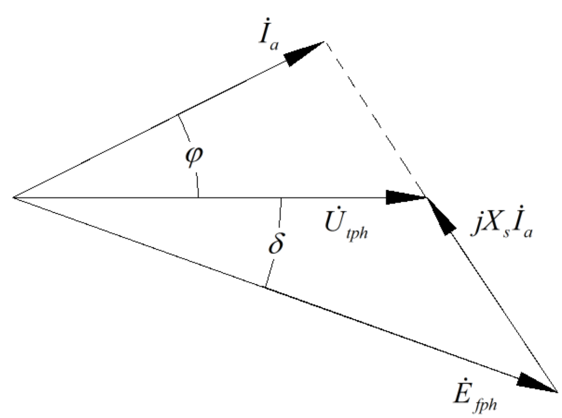
$$P = \frac{3 \cdot |U_{tph}| \cdot |E_{fph}|}{|X_s|} \cdot \sin\delta$$

$$P = \frac{3 \cdot 1327.9 \cdot 1991.86}{0.477} \cdot \sin 20° = 5.69MW$$

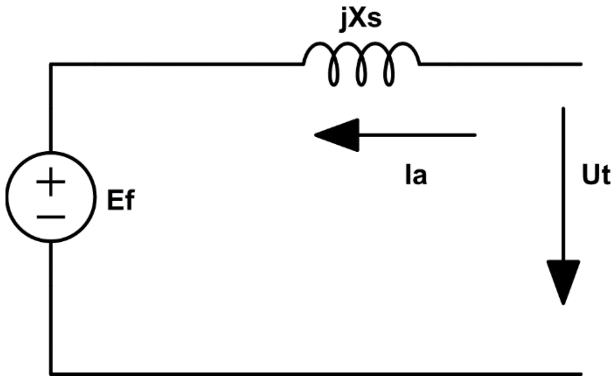
$$\dot{U}_{tph} = \dot{E}_{fph} + jX_s \dot{I}_a \Rightarrow \dot{I}_a = \frac{\dot{U}_{tph} - \dot{E}_{fph}}{jX_s}$$

$$\dot{I}_a = \frac{1327,9\angle 0 - 1991.86\angle -20°}{j0.477} = 1827\angle 38.6° A$$

$$\varphi = 38.6° \Rightarrow \cos\varphi = 0.7815 \text{ leading}$$



4)



We are given :

$S_n = 1MVA$
$U_t = 2300V$
$f = 60Hz$
$X_s = 0.8pu$
$P = 746kW$
$\cos\phi = 0.85(leading),\ \phi = 31.79°$
$2p = 10$

a)

$$U_t = \frac{2300}{\sqrt{3}} = 1328V$$

$$I_l = I_{ph} = \frac{S_n}{\sqrt{3}U_l} = \frac{10^6}{\sqrt{3}\cdot 2300} = 251A$$

$$Z_{ph} = \frac{U_{ph}}{I_{ph}} = \frac{1328}{251} = 5.29\Omega$$

$$X_s = 5.29\cdot 0.8 = 4.23\Omega$$

$$P = \sqrt{3}U_l \cdot I_a \cdot \cos\phi \Rightarrow I_a = \frac{P}{\sqrt{3}U_l \cdot \cos\phi} = \frac{746000}{\sqrt{3}\cdot 2300\cdot 0.85} = 220.3A$$

$$\dot{U}_t = \dot{E}_f + jX_s\dot{I}_a \Rightarrow \dot{E}_f = \dot{U}_t - jX_s\dot{I}_a$$

$$\dot{E}_f = 1328\angle 0 - j4.23\cdot 220.3\angle 31.79° = 1983.8\angle -23.53°V$$

b)

$$P_{\max} = \frac{3|U_t||E_f|}{|X_s|} = \frac{3\cdot 1328\cdot 1983.8}{4.23} = 1.868MW$$

$$n_s = \frac{60f}{p} = \frac{60\cdot 60}{5} = 720rpm$$

$$w_s = 2\pi \frac{n_s}{60} = 75.4 rad/s$$

$$T_{\max} = \frac{P_{\max}}{w_s} = 24.77 kNm$$

c)

$$P_{\max} = \frac{3|U_t||E_f^{'}|}{X_s} = \frac{3 \cdot 1328 \cdot E_f^{'}}{4.23} = 746 \cdot 10^3 W$$

$$E_f^{'} = 792V,\ E_f = 1983.8V$$

$$X = \frac{E_f^{'}}{E_f} \cdot 100 = \frac{792}{1983.8} \cdot 100 = 40\%$$

5) We are given:

$S = 100 MVA$

$U_l = 12 kV$

$f = 60 Hz$

$Xd = 1.0\, pu$

$Xq = 0.7\, pu$

$P = 72 MW$

$\cos\phi = 0.9 lagging$

$$U_{ph} = U_t = \frac{12000}{\sqrt{3}} = 6928V$$

$$I_{ph} = I_l = \frac{S}{\sqrt{3}U_l} = \frac{100 \cdot 10^6}{\sqrt{3} \cdot 12000} = 4811A$$

$$Z = \frac{U_{ph}}{I_{ph}} = \frac{6928}{4811} = 1.44\Omega$$

$$Xd = 1 \cdot 1.44 = 1.44\Omega\,,\;\; Xq = 0.7 \cdot 1.44 = 1\Omega$$

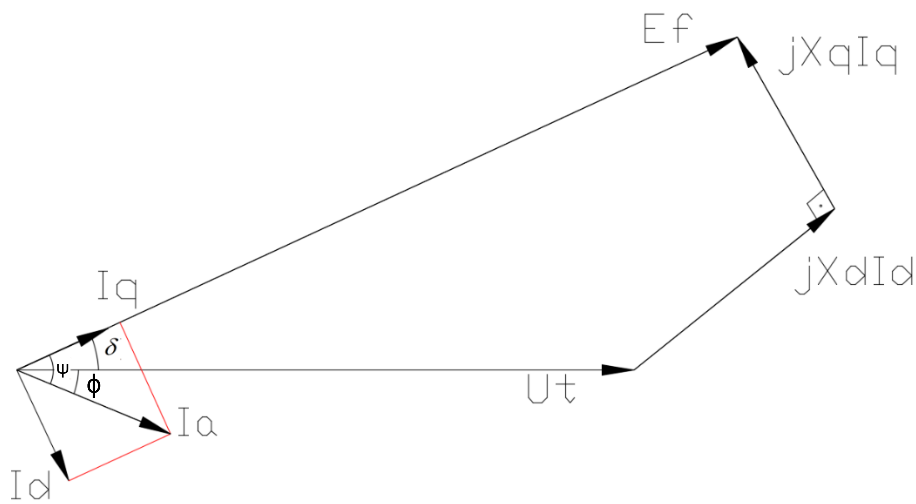
$$P = \sqrt{3} U_l I_a \cos\phi \Rightarrow I_a = \frac{P}{\sqrt{3} U_l \cos\phi} = \frac{72 \cdot 10^6}{\sqrt{3} \cdot 12000 \cdot 0.9} = 3849A$$

$$\phi = \cos^{-1}(0.9) = 25.84^\circ$$

$$\dot{I}_a = 3849\angle -25.84^\circ$$

a)

$$\dot{E}_f = \dot{U}_t + jX_d \dot{I}_d + jX_q \dot{I}_q$$



$$\sin\delta = \frac{X_q I_q}{U_t} \Rightarrow U_t \sin\delta = X_q I_q$$

$$\sin(90 - \psi) = \cos(\psi) = \frac{I_q}{I_a} \Rightarrow I_q = I_a \cos(\psi) = I_a \cos(\emptyset + \delta)$$

$$U_t \sin\delta = X_q I_q = X_q I_a \cos(\emptyset + \delta) = X_q I_a (\cos(\emptyset)\cos(\delta) - \sin\emptyset \sin\delta)$$

Divide both sides with $\cos\delta$

$$U_t \tan\delta = X_q I_a (\cos(\emptyset) - \sin\emptyset \tan\delta)$$

$$\tan\delta \left(U_t + X_q I_a \sin\emptyset\right) = X_q I_a \cos(\emptyset)$$

$$\tan\delta = \frac{X_q I_a \cos(\emptyset)}{\left(U_t + X_q I_a \sin\emptyset\right)} = \frac{1 \cdot 3849 \cdot 0.9}{6928 + 1 \cdot 3849 \cdot 0.436} = 0.4$$

$$\delta = 21.93°, \boldsymbol{\psi} = \emptyset + \delta = 47.77°$$

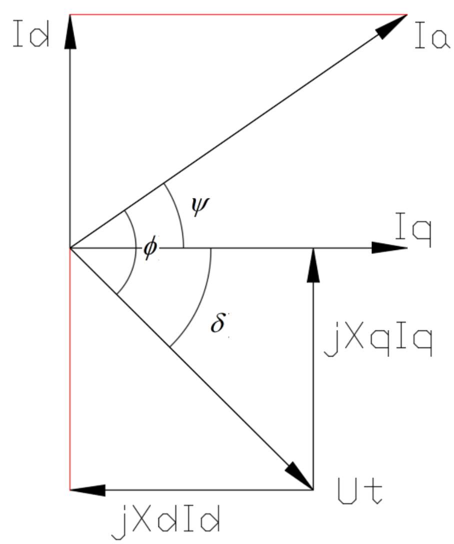
$$E_f = U_t \cos\delta + X_d I_d = 6928 \cdot \cos 21.93° + 1.44 \cdot 2850 = 10526V$$

$$I_d = I_a \sin(\psi) = 2850 \text{ A}$$

$$I_q = I_a \cos(\psi) = 2587 \text{ A}$$

b)

$$P = 3\overset{0}{\frac{U_t E_f}{X_d}}\sin\delta + 3\frac{U_t^2(X_d - X_q)}{2X_d X_q}\underset{1}{\sin 2\delta}$$



$$I_f = 0 \Rightarrow E_f = 0$$

$$\sin 2\delta = 1 \Rightarrow 2\delta = 90°, \quad \delta = 45°$$

$$P_{\max} = 3\frac{U_t^2(X_d - X_q)}{2X_d X_q} = 3\frac{6928^2(1.44-1)}{2 \cdot 1.44 \cdot 1} = 22MW$$

$$\sin 45 = \frac{X_q I_q}{U_t} \Rightarrow I_q = \frac{U_t \sin 45}{X_q} = 4898.8A$$

$$\cos 45 = \frac{X_d I_d}{U_t} \Rightarrow I_d = \frac{U_t \cos 45}{X_d} = 3402A$$

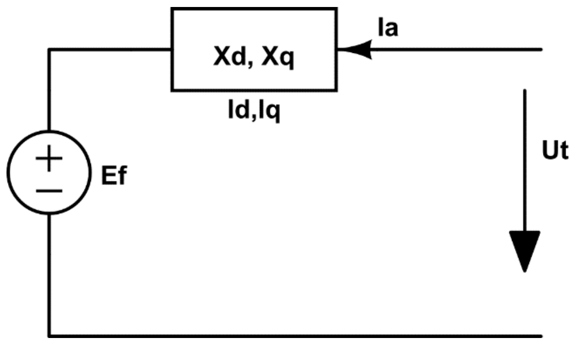
$$I_a = \sqrt{I_d^2 + I_q^2} = 5964A$$

$$\tan\psi = \frac{I_d}{I_q} \Rightarrow \psi = 34.78°$$

$$\phi = \psi + \delta = 34.78 + 45 = 79.78°$$

$$\cos\phi = 0.177 \ leading$$

6)



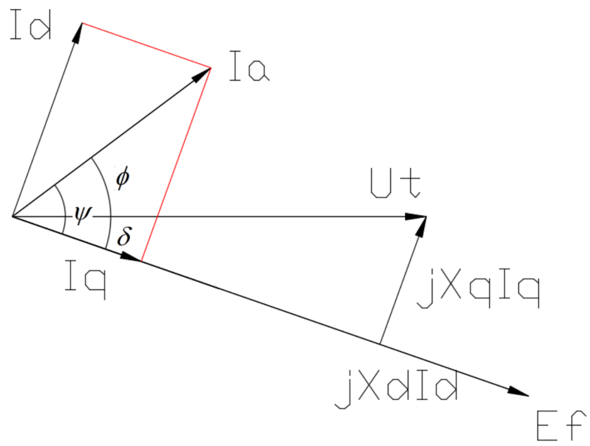
$$X_d = 1.2\,pu$$
$$X_q = 0.6\,pu$$

a)

$$P = 0.8\,pu$$

$$\cos\phi = 0.8 leading$$

$$\dot{U}_t = \dot{E}_f + jX_d\dot{I}_d + jX_q\dot{I}_q \quad \text{(M)}$$



$$\psi = \phi + \delta$$

$$I_d = I_a \sin\psi = I_a \sin(\phi + \delta)$$

$$\tan\delta = \frac{I_a X_q \cos\phi}{U_t + X_q I_a \sin\phi} = \frac{1 \cdot 0.6 \cdot 0.8}{1 + 0.6 \cdot 1 \cdot 0.6} = 0.35 \Rightarrow \delta = 19.3°$$

$\rightarrow$ the derivation is the same as in the previous example

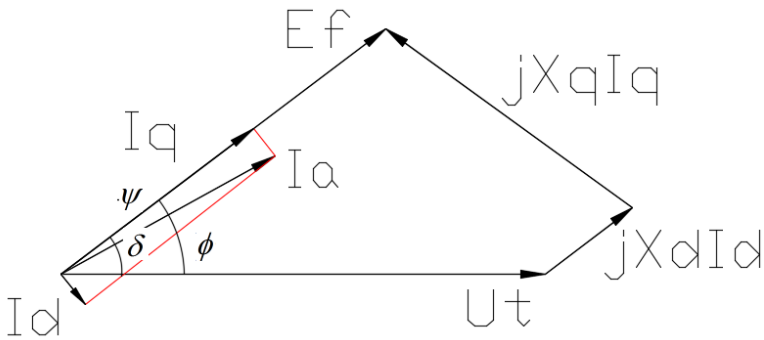
$$E_f = U_t \cos\delta + X_d I_d = U_t \cos\delta + X_d I_a \sin(\phi + \delta)$$

$$E_f = 1 \cdot \cos 19.3° + 1.2 \cdot 1 \cdot \sin(36.86 + 19.3) = 1.94\, pu$$

$$P_f = \frac{|E_f| \cdot |U_t|}{|X_d|} \sin\delta = \frac{1.94 \cdot 1}{1.2} \sin 19.3° = 0.53\, pu$$

$$P_s = \frac{|U_t|^2 \cdot (X_d - X_q)}{2 X_d X_q} \sin 2\delta = \frac{1^2 \cdot 0.6}{2 \cdot 1.2 \cdot 0.6} \cdot \sin(2 \cdot 19.3) = 0.26\, pu$$

b)



$$\delta > 0 \text{ Generator}$$

$$\psi = \delta - \phi$$

$$U_t \sin\delta = X_q I_q$$

$$I_q = I_a \cos\psi = I_a \cos(\delta - \phi)$$

$$U_t \sin\delta = X_q I_a \cos(\delta - \phi) = X_q I_a (\cos\delta \cos\phi + \sin\delta \sin\phi) \cdot \frac{1}{\cos\delta}$$

$$U_t \tan\delta = X_q I_a \cos\phi + X_q I_a \tan\delta \sin\phi$$

$$\tan\delta \cdot (U_t - X_q I_a \sin\phi) = X_q I_a \cos\phi$$

$$\tan\delta = \frac{X_q I_a \cos\phi}{U_t - X_q I_a \sin\phi} = \frac{1 \cdot 0.6 \cdot 0.8}{1 - 0.6 \cdot 1 \cdot 0.6} = 0.75 \Rightarrow \delta = 36.87°$$

$$\psi = \delta - \phi = 36.87 - 36.87 = 0°$$

$$\Rightarrow I_d = I_a \sin\psi = 0$$

$$E_f = U_t \cos\delta + I_d X_d = 1 \cdot 0.8 + 0 \cdot 1.2 = 0.8 pu$$