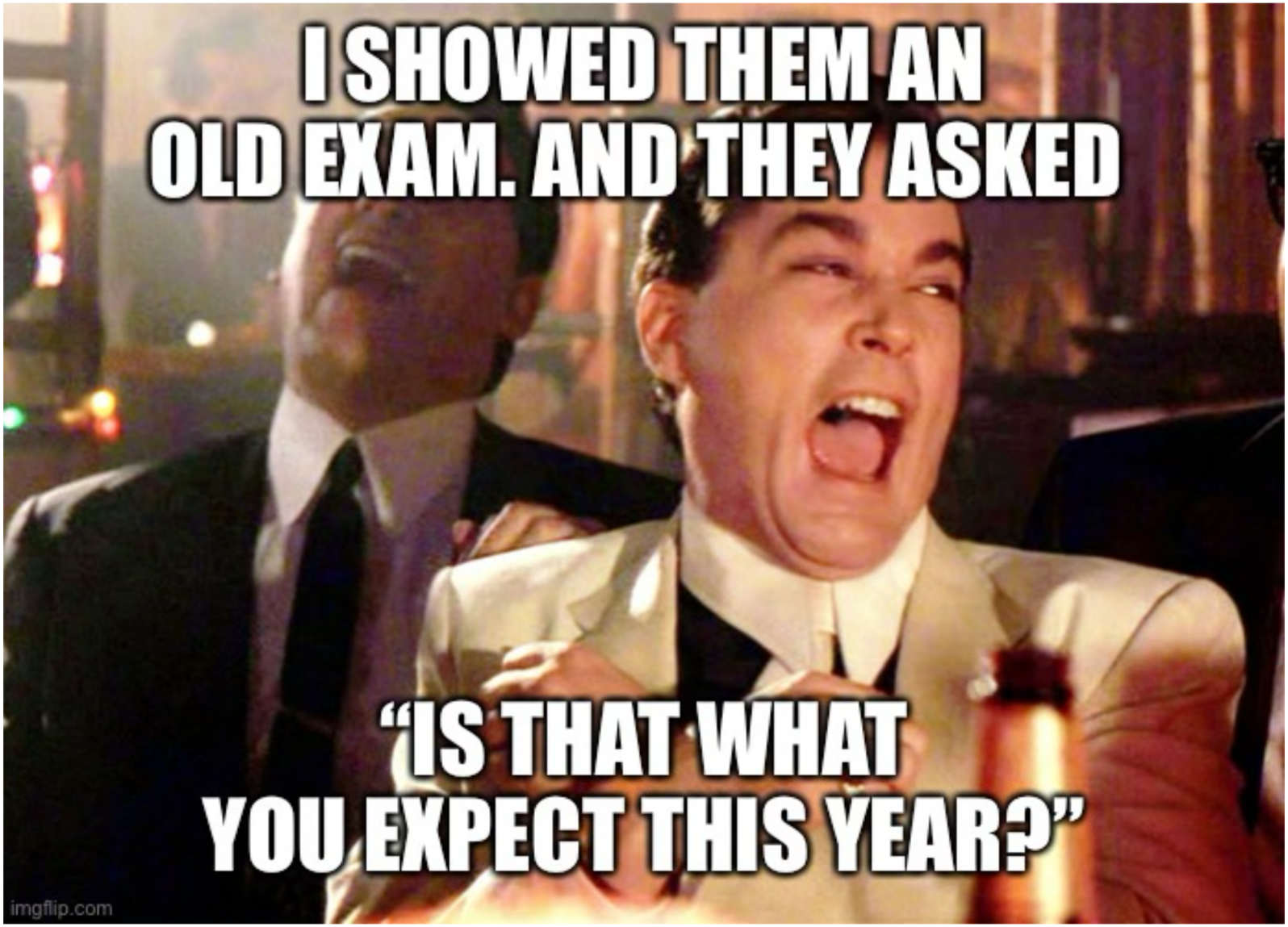

I SHOWED THEM AN
OLD EXAM. AND THEY ASKED
"IS THAT WHAT
YOU EXPECT THIS YEAR?"
imgflip.com

COMPLEX NUMBERS

$$\mathbb{C} = \{ (x,y) \mid x,y \in \mathbb{R} \}$$

$$z_1 = (x_1, y_1), \quad z_2 = (x_2, y_2)$$

$$z_1 + z_2 = (x_1 + x_2, \; y_1 + y_2)$$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2, \; x_1 y_2 + x_2 y_1)$$

EXAMPLE $(0,1) \cdot (0,1)$

$$= (0 \cdot 0 - 1 \cdot 1, \; 0 \cdot 1 + 0 \cdot 1)$$

$$= (-1, 0)$$

Notation: $i = (0,1)$

$$z = (x,y) = (x,0) + (0,y)$$

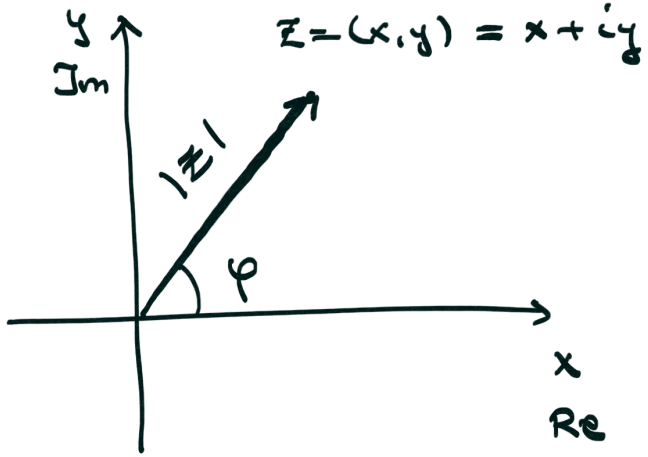
$$= (x,0) + (0,1) \cdot (y,0)$$

$$= x + iy \quad , \; x,y \in \mathbb{R}$$

We recognise:

$$i^2 = -1$$

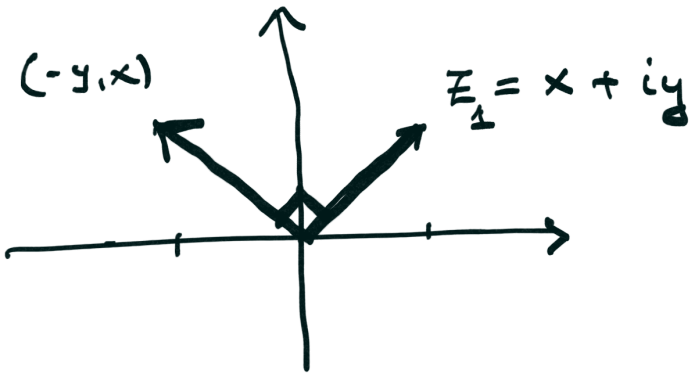
Polar coordinates:



$$z = |z| (\cos\varphi + i \sin\varphi)$$

$$= |z| e^{i\varphi} = r e^{i\varphi} \quad \left(\begin{matrix} \text{Euler} \\ \text{notation} \end{matrix} \right)$$

EXAMPLE



$z_2 = i$

$z_2 z_1 = i(x+iy)$

$= ix + i^2 y = -y + ix$

EXAMPLE

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$; 2^{nd} order ODE :

Try e^{rx}. Q: What if r_1, r_2 are complex?

$$e^{i\varphi} = \sum_{n=0}^{\infty} \frac{(i\varphi)^n}{n!}$$

$$= 1 + i\varphi - \frac{1}{2}\varphi^2 + i^3 \frac{\varphi^3}{3!} + \dots - \dots$$

$$= \underline{1 + i\varphi - \frac{1}{2}\varphi^2} - i\frac{\varphi^3}{3!} + \dots - \dots$$

$$\otimes \quad \mathrm{Re}\, e^{i\varphi} = 1 - \frac{1}{2}\varphi^2 + \frac{1}{4!}\varphi^4 - \dots$$

$$\otimes\otimes \quad \mathrm{Im}\, e^{i\varphi} = \varphi - \frac{\varphi^3}{3!} + \frac{1}{5!}\varphi^5 - \dots$$

$$\otimes = \cos\varphi$$

$$\otimes\otimes = \sin\varphi$$

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

$$\Gamma \quad e^{i\pi} = \cos\pi + i\sin\pi = -1$$

$$\Leftrightarrow \quad e^{i\pi} + 1 = 0$$

EXAM

Remote! Distribution via MyCo...

Open book, it is timed.

↳ no proctoring ← no monitoring

If you have requested special arrangements, everything will be taken care of.

Remember:

Your name, Student number, Course code

Structure:

Course exam: 5 problems

Exam exam: 6 "

Three Foundational Topics:

(1) Taylor

(2) Integration by parts

(3) 2nd Order ODE (with constant coefficients)