Principles of Economics I 2021
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Problem Set 2 (Due 1.10.2021 at 10:00)

1. Let's practice a bit more with indifference curves and budget sets. Draw the indifference curves for a consumer with the following types of preferences over oranges and apples.

    (a) A consumer that considers apples and oranges to be interchangeable, i.e. an apple is equally good as two oranges to her.

    (b) A consumer that is allergic to apples but likes oranges (assume that the consumer can discard any apples that she has if she wants).

    (c) A consumer that likes to eat fruit but only in a fruit salad where she puts exactly the same amount of apples and oranges.

    (d) Draw the budget set for a consumer that spends her budget I on apples and oranges. Put apples x on the x-axis and oranges y on the y-axis in a plane. Let p_x denote the price of apples and p_y the price of oranges so that the cost of consuming x, y is $p_x x + p_y y$. Draw the budget set for $I = 200, p_x = 4, p_y = 2$ draw also the budget set for $I = 100, p_x = 2, p_y = 1$. In the latter case, we say that oranges are the numeraire good (since $p_x = 2$ indicates that an apple costs the same as two oranges and the total budget is equal to 100 oranges). What do you observe when you compare the two budget sets and how do you explain this?

    (e) (Extra credit) As in the previous part, let x be the consumption of apples and y the consumption of oranges. Consider the consumer whose MRS between apples and oranges depends on the ratio of her consumption so that MRS $= \frac{y}{x}$. This means simply that for example at $x = 30, y = 60$ she considers each apple to be equally desirable as two oranges. From the two equations $MRS = MRT$ and the budget constraint $p_x x + p_y y = I$, solve the optimal consumption x^*, y^* of apples and oranges. (Hint: treat p_x, p_y and I as parameters in the problem, i.e. treat them as you would treat fixed numerical values).

2. At current wages of EUR 10 per hour Ann chooses to work for 8h per day. To reward her for good performance, her boss gives her a raise to

EUR 15 per hour. Ann is lucky enough to be in a job where she can pick her own working hours.

(a) Can you say with certainty what will happen to Ann's working hours as a result of this raise?

(b) Ann computes that at her old working hours, the boss ends up paying EUR 40 more per day. She is tempted to go to the boss and ask for a different wage contract. A flat payment of EUR 40 per day and the old wage of EUR 10 on top. Draw the budget constraint for the alternative wage contract and for the EUR 15 per hour wage (without flat payments).

(c) What is your advice to Ann: should she ask for this alternative contract?

(d) Suppose that the boss wants to induce Ann to work more. Rather than raising the wage, the boss gives a bonus EUR 10 per hour for each extra hour of overtime work (i.e. if Ann works for $t > 8$ hours, then her pay is $EUR80 + (t - 8)20$, for $t \leq 8$, the pay is unchanged). Draw Ann's budget set in this case.

(e) Continuing on the previous problem, draw Ann's indifference curves in such a way that is consistent with the choice of $t = 8$ in the original budget set and with $t = 10$ in the new budget set. What is Ann's average pay per hour in the new wage scheme at her optimally chosen working hours? How would Ann choose her working hours if she got paid this average wage for each of the hours that she works and no overtime bonus?

3. Consider next a simple game theoretic situation. There is a crossroads of two one-way roads. One of the roads runs from south to north while the other runs from west to east. Two drivers come to the crossroads simultaneously from different directions. Ann comes from the south while Bob arrives from the west. Ann and Bob must decide simultaneously whether to continue driving or wait.

(a) Who are the players, what are the strategies, what are the outcomes and what are reasonable payoffs? In other words, draw a game matrix representing this situation.

(b) Does either of the drivers have a dominant strategy? Are there Nash equilibria?

(c) If there are many equilibria, how should the players know which one to play? Consider various traffic rules and arrangements to help in choosing a Nash equilibrium. Why do we see different practical solutions to the problem?

4. Here is a simple example of a classic bargaining situation. Ann and Bob have altogether 10 (identical) apples and they need to decide how to divide them. Both like apples so getting more apples is preferred to fewer apples.

   (a) One possible way of doing this is to let Ann and Bob submit simultaneous demands: let x_A and x_B be the demand of Ann and Bob respectively and restrict the demands to be integers between 0 and 10. If $x_A + x_B \leq 10$, then Ann and Bob get the number of apples that they demanded. If $x_A + x_B > 10$, then the game ends in conflict and both players get 0 apples. Assume that the payoff of each player is the number of apples received at the end of the game. Does either of the players have a dominant strategy? What are the Nash-equilibria of the game? (Hint: you do not have to draw the 10 x 10 matrix to answer this, but just think carefully what you can say about best responses).

   (b) Do you think the material payoff (the number of apples received) is a reasonable preference for such bargaining situations? What other motivations might the payoffs take into account?

   (c) One often likes procedures that result in equal treatment of identical players. Comment on the previous method from this perspective and consider an alternative procedure where Ann divides the apples in two sets of apples and Bob chooses one of the sets. How should Bob choose? How should Ann divide? (This is called the divide and choose method).

5. Two friends meet for dinner at a restaurant that serves 4 different menus. The prices of the menus are EUR 30, EUR 40, EUR 50, EUR 60. Both friends value the menus (in the same order) at EUR 35, EUR 50, EUR 58, EUR 60. This means that the menu priced at EUR 50 is worth EUR 58 to the diner and therefore a relatively good deal.

   (a) When dining, you choose a single menu from the list. How would each of the two friend choose if they were eating alone in the restaurant?

   (b) Suppose now that the friends choose simultaneously their menus and they have agreed to split the bill, i.e. each pays half of the sum of the menus that were chosen. In this case, the material payoff to each friend is the value of the meal minus the cost, i.e. half of the sum of the prices of the two menus. Draw the game matrix for this case.

(c) Do the players have dominant strategies? If yes, is the dominant strategy equilibrium socially desirable? If no, what kinds of Nash equilibria does the game have?

(d) What would the players choose if the restaurant has a policy that all diners at the same table must order the same menu?

(e) Can you think of reasons why the material payoffs may not be the only concern to the diners?

6. (Extra credit) Consider the following modification to the bargaining game of Problem 4. Ann and Bob still submit simultaneous demands x_A, x_B and if $x_a + x_B \leq 10$, then each gets the demand they made. If $x_A + x_B > 10$, the conflict is resolved in arbitration and the rules of the arbitration are as follows: If one of the players was less greedy, she gets her demand (and the other player gets the remaining apples). If the demands were the same, then each player gets 5 apples.

(a) Can either player receive more apples than she demanded?

(b) Is there a way for either of the players to secure at least 5 apples?

(c) What is Ann's best response if $x_B > 6$?

(d) Can there be Nash equilibria where at least one of the players makes a demand strictly higher than 6?

(e) What is the division of apples in any Nash equilibrium of the game?

(f) (No question here) Since you did not have to draw the matrix, you can extend the argument above to any number of apples (say 100 or 1000) with simple modifications to the numbers 5 and 6 in the argument above to get the equivalent conclusion.