1. (**Addition to last week's warm up**) (4p)

   Calculate the electron gyro-frequency and gyro-radius (assume that the velocity has perpendicular direction and average magnitude) for the following common plasmas.

   (a) Fusion experiment: $T_e = 10\,\mathrm{keV}, B = 5\,\mathrm{T}$

   (b) High-latitude ionosphere at 150 km altitude: $T_e = 0.1\,\mathrm{eV}, B = 50\,000\,\mathrm{nT}$

   (c) Solar wind at 1 AU: $T_e = 10\,\mathrm{eV}, B = 5\,\mathrm{nT}$

   (d) Core of the Sun: $T_e = 1\,\mathrm{keV}$, no magnetic field

   (e) Neutron star: $T_e = 100\,\mathrm{keV}, B = 10^8\,\mathrm{T}$

   Now that we have considered the magnetization of plasmas, comment on the following. Is the electron plasma frequency still the highest frequency in a plasma? How does the gyro-radius compare to the characteristic length scale given in the lecture slides for each plasma?

2. (**A quick mathematical comparison**) (2p)

   Show that the magnetic moment μ, introduced in the second lecture, is exactly the same as the magnetic moment you calculate for a current loop corresponding to an electron on a circular orbit.

3. (**Recalling how to integrate coupled differential equations**) (6p)

   Integrate the charged particle motion (find the position as a function of time) in a homogeneous magnetic field, $\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$,

   (a) for $\boldsymbol{E} = \boldsymbol{0}$ (no electric field).

   (b) for $\boldsymbol{E} = E_0\hat{\boldsymbol{x}}$.

4. (**Real physics**) (4p)

   Derive the condition (lecture slides) for particle reflection in a magnetic mirror with $B = B_0$ at the center and the maximum field strength $B = B_{\mathrm{max}}$ at the ends of the device.

5. (**More real physics**) (6p)

   Consider charged particles in Earth's magnetic field, with field lines running (at the moment...) from the south pole to the north pole. Take $3 \times 10^{-5}\,\mathrm{T}$ for its magnitude at Earth's equator, and assume that it falls off with radial distance r from the Earth's center as $1/r^3$, as for a perfect dipole. The protons have an energy of 1 eV and the electrons 30 keV, each with a density of $n = 10^7\,\mathrm{m}^{-3}$ on the equatorial plane at a distance of $r = 5R_{\mathrm{E}}$, where R_{E} is the Earth's radius.

   (a) Compute the maximum ion and electron ∇B-drift velocities.

   (b) Does an electron drift east- or westward?

   (c) How long does it take an electron to encircle the Earth?

   (d) Compute the ring current density in units of $\mathrm{A\,m^{-2}}$.

   In (c) and (d), use the maximum drift velocity(/-ies) obtained in (a).

6. (**Food for thought**)

   Let's consider the significance of Earth's magnetic field on Earth's surface. Put an electric charge in between two capacitor plates that produce a strong electric field with straight field lines perpendicular (east-west) to the prevailing (weak) Earth's magnetic field. According to common sense (and experience), the particle is accelerated towards one of the plates. However, during the lectures we learned that if the particle experiences electric and magnetic fields perpendicular to each other, it moves in direction perpendicular to both fields, i.e., up or down in this case! What makes things even stranger is that the magnitude of the drift is given by E/B, i.e., it is particularly strong for LOW magnetic fields. Consider carefully the motion of the particle and show that there is no contradiction and the particle follows the electric field. Return your short write-up in MyCourses before the next lecture.