PHYS-E0461 Introduction to plasma physics for fusion and space applications

Exercise set 3

- 1. (A little warm-up). (4p)
 - (a) Find the normalization coefficient A_1 for a 1-dimensional Maxwellian distribution, $f(v) = A_1 e^{-mv^2/2T}$. (1p)
 - (b) Using the result from (a), find the normalization coefficients for a 2-dimensional Maxwellian $f(\boldsymbol{v}) = A_2 e^{-mv^2/2T}, \boldsymbol{v} = v_x \hat{\boldsymbol{x}} + v_y \hat{\boldsymbol{y}}$, and a 3-dimensional Maxwellian $f(\boldsymbol{v}) = A_3 e^{-mv^2/2T}, \boldsymbol{v} = v_x \hat{\boldsymbol{x}} + v_y \hat{\boldsymbol{y}} + v_z \hat{\boldsymbol{z}}$. (1p)
 - (c) Use spherical coordinates and recalculate the normalization coefficient for the 3-dimensional Maxwellian distribution. (2p)

2. (More on the Maxwellian distribution) (4p)

Using the speed distribution g(v), derived during the lectures, calculate

- (a) the most probable speed.
- (b) the average speed.

3. (More Maxwellian distributions) (4p)

- (a) Using the speed distribution g(v), derived during the lectures, obtain the *energy* distribution h(E). Make sure the normalization is right.
- (b) Calculate the average particle energy in a Maxwellian distribution. You may choose which presentation of the Maxwellian you use.

4. (Food for thought: planets and atmospheres)

Planets have a so-called *escape velocity*: if a particle (or body) is moving away at a speed exceeding the escape velocity, the gravitational pull is not strong enough to withhold it. The escape velocity is given by $v_{\rm esc} = \sqrt{2GM/r}$, where G is the gravitational constant, M is the mass of the planet to escape from, and r is the distance of the particle/body from the center of the planet. Assume that particles at the top of an atmosphere follow the Maxwellian distribution. Discuss the existence of atmospheres around different kinds (size, mass, temperature) of planets, keeping in mind that the particles collide exchanging energy and momentum. Return your short write-up in MyCourses before the next lecture.