## ELEC-E8101 Digital and Optimal Control Exercise 0

The problems marked with an asterisk ( $\star$ ) are not discussed during the exercise session. The solutions are given in MyCourses and these problems belong to the course material.

1. Find the inverse Laplace transform of the following functions
a) $F(s)=\frac{s+3}{s^{2}+3 s+2}$
b) $F(s)=\frac{1}{s^{3}+s}$
c) $F(s)=\frac{10 s^{2}+4}{s(s+1)(s+2)^{2}}$
d) $F(s)=\frac{16 s+16}{\left(s^{2}+16\right)\left(s^{2}+4 s+20\right)}$
2. Consider a system represented by the differential equation

$$
\frac{d^{2} y(t)}{d t^{2}}+4 \frac{d y(t)}{d t}+3 y(t)=2 u(t)
$$

where the initial conditions are $y(0)=1, \frac{d y}{d t}(0)=0$, and $u(t)$ is a unit step $(u(t)=1, t \geq 0)$. Find the steady-state response of this system.
*3. Assume that the mass of the inverted pendulum shown in Fig. 1 is $m$ and is evenly distributed along the length of the rod. (The center of gravity of the pendulum is located at the center of the rod.) Assuming that $u$ is small, derive mathematical models for the system in the forms of differential equations and transfer functions.


Figure 1: Inverted pendulum on a cart.

