ELEC-E8101 Digital and Optimal Control

Exercise 1

The problems marked with an asterisk ($\star$) are not discussed during the exercise session. The solutions are given in MyCourses and these problems belong to the course material.

1. Find the z-transform of the following sequences using the definition:

   a) $y[k] = 1,\ k = 0, 1, 2, 3, \ldots$.

   b) $y[k] = a^k,\ k = 0, 1, 2, 3, \ldots$ and a is a constant.

   c) $y[k] = e^{-kh/T},\ k = 0, 1, 2, 3, \ldots$, and h and T are constants. Is this the same result as in part b) of the exercise?

2. Given that

$$Y(z) = \frac{0.792z^2}{(z-1)(z^2 - 0.416z + 0.208)}$$

   a) Determine the value of $y[k]$ as $k \to \infty$.

   b) Verify your result using MATLAB.

3. Find the inverse z-transform of the following expression

$$Y(z) = \frac{(1 - e^{-ah})z}{(z-1)(z - e^{-ah})},\ a \text{ is a constant.}$$

4. Using z-transforms solve for $y[k]$ from the following difference equation:

$$y[k+2] - 1.5y[k+1] + 0.5y[k] = u[k+1],$$

   where $u[k]$ is the unit step starting at $k = 0$, i.e.,

$$u[k] = \begin{cases} 1 & \text{if } k \geq 0 \\ 0 & \text{if } k < 0. \end{cases}$$

   Initial conditions: $y[-1] = 1, y[0] = 0.5$.

*5. Prove that the following holds:

$$\mathcal{Z}\left\{\frac{1}{2}(kh)^2\right\} = \frac{h^2 z(z+1)}{2(z-1)^3}.$$

   Hint: Begin by transforming $\mathcal{Z}\{kh\}$.