ELEC-E8101 Digital and Optimal Control

Exercise 2

The problems marked with an asterisk ($\star$) are not discussed during the exercise session. The solutions are given in MyCourses and these problems belong to the course material.

1. The pulse transfer function of a system with ZOH is:

$$H(z) = \frac{0.2}{z - 0.8}.$$

   a) Determine the *pulse* response.

   b) Determine the *step* response.

   c) Show the step and pulse responses of the system on MATLAB.

2. Consider the following difference equation:

$$y[k+2] - 1.3y[k+1] + 0.4y[k] = u[k+1] - 0.4u[k].$$

   a) Determine the pulse transfer function.

   b) Is the system stable? Justify your answer.

3. The considered continuous-time process has first-order dynamics and is given by

$$\tau\dot{y}(t) + y(t) = u(t).$$

   a) Find the Laplace transfer function of the system.

   b) Discretize the process using the step-invariance method with a sampling time of h.

   c) Determine the unit step response of the discretized system.

4. Let us consider the following scalar state-space model of system $\mathcal{G}$:

$$\mathcal{G}: \begin{cases} \dot{x}(t) = ax(t) + bu(t) \\ y(t) = cx(t) \end{cases}$$

   It is assumed that parameters c and b are positive, and a is non-zero.

   a) Find the Laplace transfer function of the system. For what parameter values of a is the continuous-time process stable?

   b) Discretize the process (sampling time h) using the Tustin transformation. For what parameter values of a is the discrete-time process stable?

c) The continuous-time process is controlled by a proportional controller. For what values of gain K_P is the controlled system stable?

d) The discrete-time process is controlled by a proportional controller. For what values of gain K_P is the controlled system stable?

⋆5. Compute the inverse z-transform of the following transfer function:

$$X(z) = \ln\left(\frac{1}{1 - az^{-1}}\right), \quad |z| > |a|.$$

Hint: Note that $\ln(1 - x) = \sum_{k=1}^{\infty} \frac{1}{k} x^k, \ |x| < 1.$