31E11100 - Microeconomics: Pricing
Autumn 2020
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**Some extra problems**

1. A monopoly produces output at constant marginal cost $c > 0$. There is a linear demand $Q(p) = a - bp$. Suppose that $c < a/b$.

   (a) Formulate and solve the pricing problem of the monopolist.

   (b) Suppose that the producer is an upstream firm that does not sell directly to consumers, but sells through a retailer, who is a monopolist in the downstream market. First, the producer sets the wholesale price $w$. Then, the retailer sets the retail price $p$. The retailer makes profit $pQ(p) - wQ(p)$ and the producer makes profit $wQ(p) - cQ(p)$. Solve the equilibrium prices $w$ and $p$ by backward induction, starting from the optimal retail price price as a function of the wholesale price, and then proceed to the optimal wholesale price.

   (c) Discuss the nature of the solution (key-word: double-marginalization). What could be done to improve efficiency in a setup like this?

2. A paper mill uses wood as an input for paper. There are two ways to procure wood: the mill can purchase $q(w)$ units from the domestic market by paying a unit price $w$ (the mill is a monopsony in the domestic market). Alternatively, it can import wood with a constant unit price $w'$ (the mill is a price taker in the import market).

   (a) Suppose the wood suppliers in the domestic market cannot sell to the import market, whereas the mill has access to both markets. If the mill wants to buy q units of wood, how much from this amount should the mill import to minimize its cost of supply? What is the price ratio $w/w'$? Provide intuition. Derive the marginal cost $MC(q)$ as a function of buying $q$ units.

(b) Consider now that the domestic suppliers can export wood at price $w'$. What price is now offered by the mill? How does the marginal cost function change?

3. A pharmaceutical company sells a given drug in two geographically separated markets, denoted $A$ and $B$. The demands are given by $Q_A(p_A) = 1 - p_A$ and $Q_B(p) = \frac{1}{2} - p_B$. For simplicity, the transport and production costs are assumed to be zero.

   (a) Suppose that the firm sets a uniform price across the two markets. What is the profit-maximizing uniform price, and what are the quantities sold at that price in the two markets?

   (b) Suppose that the firm can set different prices in the two markets. What are the profit-maximizing prices and what are the quantities sold in the two markets?

   (c) Compute the producer's and consumers' surpluses under a uniform price and under geographical price discrimination. Compare the two situations and discuss.

   (d) Do the insights of c) hold generally? What if the demand in market $B$ is changed to $Q_B(p) = \frac{1}{3} - p_B$?

4. Two firms, Firm 1 and Firm 2, sell a homogenous good at zero marginal cost. There are three types of consumers. Mass $B > 0$ of the consumers are informed of the prices charged by both firms, $p_1$ and $p_2$. Mass $F_1 > 0$ know only the price $p_1$ charged by Firm 1, and mass $F_2 > 0$ know only price $p_2$ charged by Firm 2. Assume that $F_2 > F_1$. All consumers have unit demand with some reservation value $R > 0$ and if they observe two prices, they buy from the firm with the lowest price offer (in case of a tie, they flip a coin).

   (a) Suppose that the Firm 1 sets the price before Firm 2 (And Firm 2 observes the price offer of Firm 1 before choosing its own price offer). Characterize the sub-game perfect equilibrium of the game. Hint: use backward induction.

(b) Suppose that the Firm 2 sets the price before Firm 1, otherwise the same question.

(c) Argue that Firm 2 is indifferent between setting price first or second, but Firm 1 has a second-mover advantage.

5. Suppose buyers have logarithmic utility functions

$$u(\theta, q, t) = \theta \ln(q + 1) - t.$$

Assume that the cost of production is linear so that marginal cost is constant:

$$c(q) = cq, \text{ with } 0 < c < 1.$$

(a) Find the first-best level of $q$ as a function of $\theta$.

(b) Suppose there are two types of buyers: $\theta^H = 3$, $\theta^L = 1$. Let $\lambda$ denote the fraction of buyers of type $\theta^H$. Write down the IC and IR constraints for both types of buyers. How large is the information rent of type $\theta^H$ (as a function of $q^L$)?

(c) Which of the IC and IR constraints bind? Solve the optimal menu $\{(q^H, t^H), (q^L, t^L)\}$. For what values of $\lambda$ will the monopolist sell positive amounts to $\theta^L$?

6. A cable TV operator considers how to sell packages consisting of several TV channels to its customers. For concreteness, assume that the operator has three channels available and must decide a good way to sell. Suppose that each buyer has either a high valuation $v^H = 3$ or a low valuation $v^L = 2$ for each of the channels. These valuations are private information to the buyer (i.e. the seller does not know the individual valuations. For each channel and each buyer, the probability of high valuation is $\frac{1}{2}$ and therefore probability of a low valuation is also $\frac{1}{2}$. Assume that these valuations are statistically independent across buyers and across the channels.

(a) Assume that the monopolist sells the channels individually (i.e. sets a price for each channel and allows the buyers to choose the channels they want. How does the monopolist set the optimal prices? How large is the expected profit per channel?

(b) Suppose next that the monopolist sells all three channels together. What is the distribution of valuations that the buyers have for this package? What is the optimal price? What is the expected optimal pro t per channel in this case?

(c) (Harder) Suppose that there are 100 such channels. Use approximation of binomial distributions by Normal distribution to get an estimate for the number of buyers willing to pay 240 for the package of all channels.

7. Consider a monopolist selling in a market with 2 types of buyers with $\theta \in \{\theta^1, \theta^2\}$. Let $\lambda^i = \Pr(\theta = \theta^i)$ for $i \in \{1, 2\}$. To give a bit more realism to the solution, suppose that the monopolist can only offer a menu $\{(f^1, p^1), (f^2, p^2)\}$ of two part tariffs, where $f^i$ stands for the fixed fee and $p^i$ stands for the linear usage fee in the $i^{th}$ two-part tariff. Compare the solution in the case with $v(q) = \sqrt{q}$ and $c(q) = cq$ to the solution in the lecture notes where the monopolist may use arbitrary nonlinear offers $\{(q^1, t^1), (q^2, t^2)\}$. In particular, determine if the amount sold to the low type is higher or lower with two-part tariffs than in the solution in the lecture notes.

8. A monopoly retailer has exactly 2 units of a good available, and it wants to set prices to those two units in order to maximize its revenue. Three consumers will visit the store within the relevant time period. Each consumer has a unit demand, and one of the consumers has a reservation utility $r = 10$ while two of the consumers have a reservation utility $r = 6$. The three consumers arrive at the shop in random order and each of them either buys one unit upon entering (if there is any left) or just walks away. The problem of the retailer is to set the prices of the two products optimally in order to maximize expected revenue.

   (a) Suppose that the retailer must set the same price for both of the units. What is the optimal uniform price?

(b) Suppose that the retailer can set a different price for each of the two units. What is the optimal pricing scheme in such case?

(c) Compare the profits and total welfare across the cases a) and b).

9. A firm has either a high- or low-quality product. There is a mass 1 of consumers. Each consumer has a unit demand. There are two types of consumers: Fraction 2/5 of consumers have a higher valuation. Such a consumer has a reservation utility 10 for a high-quality product and a reservation utility 5 for a low-quality product. Fraction 3/5 of consumers have a lower valuation. Such a consumer has a reservation utility 6 for a high-quality product and a reservation utility 3 for a low-quality product. The unit cost of production for the firm is 0 if the quality is low, and 2 if the quality is high (Note: the quality of the firm's product is given, i.e. the firm cannot choose the quality).

   (a) Suppose that the consumers can tell the quality before they buy. Determine the optimal price that each type of firm will choose.

   (b) Suppose that the consumers cannot tell the quality before they buy. The goal here is to construct a separating equilibrium, where the high type can signal credibly its high quality level. If such an equilibrium exists, what is the price and associated profit of the low quality firm? Given this, what would be the price that the high quality firm needs to charge in order to separate from the low type, i.e. to discourage the low type from mimicking the high type? Finally, show that charging such a price is indeed profitable for the high-quality firm (To show this, you may assume that if the high type firm charges some other price than this, then the buyers believe that it is a low quality firm).