ELEC-E8101 Digital and Optimal Control

Exercise 3

The problems marked with an asterisk (⋆) are not discussed during the exercise session. The solutions are given in MyCourses and these problems belong to the course material.

1. For the following system:

$$y[k] + 0.5y[k-1] = u[k-1].$$

   a) Determine the pulse transfer function and pulse response.

   b) Is the system stable? Justify your answer.

2. Apply the forward difference approximation of the derivative to the second-order analog filter

$$C_a(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

   and examine the stability of the resulting digital filter for a stable underdamped analog filter ($0 < \zeta < 1$).

3. The continuous time PI-controller has the following transfer function:

$$G(s) = K\left(1 + \frac{1}{T_i s}\right).$$

   a) Determine a sampled version by Tustin's approximation and find the correspondence of the parameters K and T_i.

   b) Determine a sampled version by Euler's method and form the so-called incremental algorithm.

⋆4. a) Transform:

$$G(s) = \frac{6}{s^2 + 5s + 6}$$

   into pulse transfer function the step-invariance method. Assume ZOH and unit sampling time.

   b) Verify that the final value of the unit step response in the discrete case is the same as in continuous case.
   Hint: Use the Final Value Theorem.