Microeconomic Theory II
Helsinki GSE
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Problem Set 1, Due November 17, 2021

1. Consider a social choice problem where five agents vote over three choices. The alternatives are ranked by a series of ballots where each agent votes for one choice on each ballot. Assume sincere voting so that in the first round, agents vote for their favorite option and the option receiving the fewest votes is eliminated. If two alternatives are tied, a run-off with the same rules is arranged between them.

   In the second ballot, agents vote (sincerely) between the remaining two alternatives and the winner of this second ballot is put first in social preference, the loser is put the second and the option eliminated in the first ballot is put in the third place in social preferences.

   (a) What is the social ordering over x, y, z if the agents' preferences are as in the table below?

	1	2	3	4	5
first	x	x	z	z	y
second	y	y	y	y	x
third	z	z	x	x	z

Figure 1: Agents ranking of the alternatives.

   (b) Suppose Agent 1 reverses her ordering of x and y but z remains in the third place. What is the new social preference?

   (c) Which of the conditions in Arrow's Theorem is violated in this ballot process?

2. Consider an economy with n agents. Let X be the set of alternatives available in this economy. For each pair $(x, y) \in X \times X$, define the

variable d_i for each $i \in \{1, ..., n\}$ as follows:

$$d_i = \begin{cases} 1 \text{ if } x \succ y, \\ 0 \text{ if } x \sim y, \\ -1 \text{ if } y \succ x. \end{cases}$$

A social choice function is a function $f : \{-1, 0, 1\}^n \rightarrow \{-1, 0, 1\}$ (with the same interpretation as above). Let $d = (d_1, ..., d_n)$. The majority decision rule is defined as follows:

$$f(d_1, ..., d_n) = \begin{cases} 1 \text{ if } \Sigma_{i=1}^n d_i > 0, \\ 0 \text{ if } \Sigma_{i=1}^n d_i = 0, \\ -1 \text{ if } \Sigma_{i=1}^n d_i < 0. \end{cases}$$

Let $n^+(d) = \#\{i \text{ such that } d_i = 1\}$ and $n_-(d) = \#\{i \text{ such that } d_i = -1\}$. A social choice function is said to be *anonymous* if $f(d) = f(d')$ whenever $n^+(d) = n^+(d')$ and $n_-(d) = n_-(d)$. In other words, the rule treats all individuals in the same manner. A social choice function is *neutral* if $f(-d) = -f(d)$. A social choice function is *responsive* if we $f(d) \geq 0$ and $d' > d$ imply that $f(d') = 1$.

(a) Show that the majority rule is anonymous, neutral and responsive.

(b) Show that whenever f is anonymous and neutral, $n^+(d) = n_-(d)$ implies that $f(d) = 0$.

(c) Prove that whenever f is anonymous, neutral and responsive, it is given by the majority rule.

3. Suppose three memebers of a club decide the color for the flag of their club: yellow (y), blue (b), green (g) or red (r). The preferences are as given in the table below (again better alternatives at the top).

   (a) Suppose the members vote sincerely according to Borda rule. What is the outcome?

   (b) Would any member have an incentive to misrepresent her preferences if the other two vote sincerely?

1	2	3
r	b	g
y	r	y
b	g	b
g	y	r

Figure 2: Members' ranking of the colors.

4. Give an example of a market for which removing one of the individuals, together with the house she initially owns, makes one of the remaining individuals better off and another of the remaining individuals worse off.

5. In the assignment model, construct an example of a society and an allocation that is not Pareto efficient but no pair of agents want to swap their houses.

6. (Harder) Consider an economy with a countable infinity of generations. Each generation can obtain a utility $x_i \in \{0,1\}$. An outcome in this economy is then a sequence $x = (x_1, x_2, ...)$ with $x_i \in \{0,1\}$ for each i. Thus $X = \{0,1\}^{\mathbb{N}}$ Suppose that a social planner for the economy has preferences that satisfy two properties:

   i) Pareto-principle: For all $x, y \in X$, $x \geq y \Rightarrow x \succ y$.

   ii) Intergenerational Equity: For all $x, y \in X$, if $\exists i, j$ such that $x_i = y_j$ and $y_i = x_j$ and $x_k = y_k$ for $k \notin \{i, j\}$, then $x \sim y$.

   It can be shown that such rational preferences do exist. Show that the planner's preferences cannot have a utility representation. Hint: can you relate this to lexicographic individual preferences?