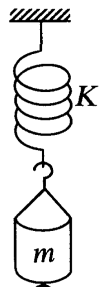
Problem 1

A mass of 500 g is attached to a massless spring. (6 p.)



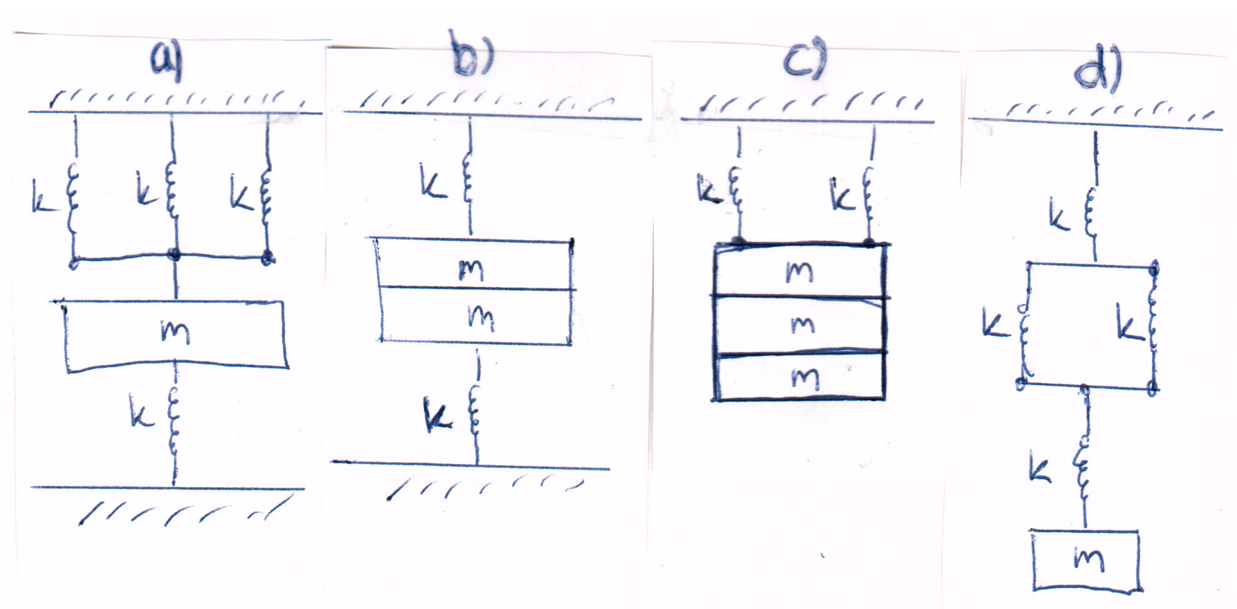
1. How much elongation does an extra mass of 200 g cause on the spring, if the undamped natural frequency f_0 of the original system without the extra mass is 5 Hz?
2. What is the mechanical resistance of the original system without the extra mass, if the frequency of its damped oscillation f_d is 4 Hz?

The extra mass is removed and this sets the system to oscillate.

1. How long does it take for the amplitude to halve?
2. What is the initial amplitude of the motion?
3. What is the initial phase of the motion?

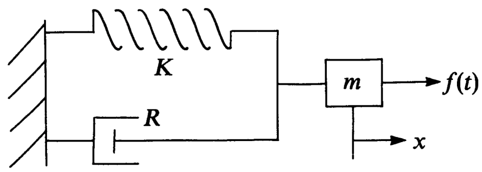
Problem 2

In the following figures the stiffness of all springs is K and the mass of the objects is m. The springs are assumed to be massless. Solve for the natural undamped angular frequency in each case. (10 p.)



Problem 3

A mass ($m = 10$ kg) is attached to a spring ($K = 1$ N/m). The system is excited with a sinusoidal excitation of constant force. Calculate the frequency of maximum magnitude for i) the displacement and ii) velocity of the mass, when the mechanical resistance is $R = 1$ kg/s and $R = 500$ kg/s. Check if the oscillation condition ($\alpha < \omega_0$) is met for each case. (4 p.)

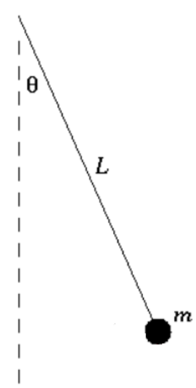


Hints:
i) start with the expression of the transfer function of the complex displacement of a system driven by a sinusoidal excitation of magnitude F: $\frac{|\tilde{x}|}{|\tilde{F}|} = \frac{1}{\sqrt{(K-\omega^2 m)^2+(\omega R)^2}}$
ii) start with the expression of the transfer function of the complex velocity of a system driven by a sinusoidal excitation of magnitude F: $\frac{|\tilde{v}|}{|\tilde{F}|} = \frac{1}{\sqrt{R^2+(\omega m-K/\omega)^2}}$

Bonus

Consider a simple pendulum with a length L and a mass m, as shown in the figure below. The pendulum is displaced and then released from an angle θ and thus oscillates in simple harmonic motion. We consider small oscillation amplitudes. (5 p.)

1. What are the forces applied to the mass? Give their expression.
2. Use Hooke's law for springs to find the expression for the force constant k.
3. Express the period of oscillation T of the simple pendulum.
4. In the light of your previous answer, what would be the natural frequency of the motion if the angle of displacement θ was doubled while still considering small oscillation amplitudes?



Hints:
- For small angles: $\sin\theta \approx \theta$.
- For questions 1. and 2., you could also write the differential equation for the motion of the pendulum using Newton's second law.